Chapter 4

Fiscal policy and sovereign risk premia under monetary union

The role of fiscal policy in stabilising economic conditions has been of particular relevance to countries that belong to a monetary union given their inability to (independently) use monetary policy to absorb country-specific shocks. A number of recent theoretical papers has shown that fiscal policy can play an important role in enhancing welfare by offsetting country-specific shocks, especially when price rigidities are strong (e.g. Beetsma and Jensen, 2005; Galí and Monacelli, 2008). Furthermore, according to traditional Keynesian theory, fiscal expansions can be very effective in stimulating aggregate demand if they are not offset by a monetary contraction, suggesting that fiscal multipliers can be quite large in small member states of a monetary union.

The recent sovereign debt crisis in Europe has, however, exposed the limits of fiscal policy. Figure 4.1, for instance, suggests that countries may face a certain threshold for sovereign debt, beyond which sovereign bond holders no longer perceive fiscal policy to be on a sustainable path and sharply raise sovereign risk premia. This has been particularly the case for euro area periphery countries during the sovereign debt crisis (indicated by the dark (blue) circles), whereas before the crisis sovereign risk premia seemed to respond little to changes in sovereign indebtedness. The extent to which higher sovereign

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Figure 4.1: The debt-to-output ratio vs. the sovereign risk premium

(a) Euro area periphery countries

(b) Euro area core countries

Notes: The figure shows quarterly data for long-term (10 year) government bond yields, minus the yield on a similar German bond, and gross government debt as a share of nominal GDP. Source: OECD National Accounts Statistics.
risk premia affect the ability of fiscal policy to stabilise the economy depends \textit{inter alia} on how they affect private borrowing conditions (and, subsequently, consumption and investment decisions). Figure 4.2 shows that, during the sovereign debt crisis, there has been a remarkable strong and positive relationship between sovereign credit default swap (CDS) rates and the CDS rates of the two largest domestic commercial banks within the corresponding country, suggesting that the perceived riskiness of banks has been moving in tandem with the perceived riskiness of the home sovereign.\footnote{See Harjes (2011), Demirgüç-Kunt and Huizinga (2013), Zoli (2013), and Acharya et al. (2014), among others, for a more detailed analysis on the relationship between sovereign and private risk premia.} Fiscal policy may therefore have significant crowding-out effects on private spending through its effect on sovereign and private risk premia. If that is the case, (1) can fiscal policy still prove successful in stabilising macroeconomic conditions and (2) what should be the optimal fiscal policy stance? The objective of this chapter is to answer these questions.

Our point of departure is a New Keynesian model for a two-country monetary union (much like Benigno, 2004). The two countries in this union are identical in terms of preferences and technology, but may differ in size and face different shocks. There is a single monetary authority which sets the policy interest rate in order to stabilise union-aggregate inflation and output. Furthermore, in each country there is a fiscal authority which sets a distortionary income tax as a function of domestic output and sovereign debt. Following Davig et al. (2011) and Bi (2012), we allow for the presence of a ‘fiscal limit’, which determines the maximum sustainable level of sovereign debt. Although the fiscal limit is unknown, agents are aware of its distribution and form expectations about future sovereign default probabilities. Rather than modelling the fiscal limit explicitly, we follow Daniel and Shiamptanis (2013), Locarno et al. (2013), Roeger and in ’t Veld (2013) and Schabert and van Wijnbergen (2014), among others, and assume a sovereign risk premium arises once agents believe the economy is near its fiscal limit. The sovereign risk premium is assumed to be a function of sovereign indebtedness and we calibrate this function using the data presented in Figure 4.1. Also, as in Erceg and Lindé (2010) and Corsetti et al. (2013b), we let the sovereign risk premium affect private borrowing conditions so as to capture the link between sovereign and
Figure 4.2: Sovereign CDS rates vs. domestic bank CDS rates, 2008-2012

(a) Euro area periphery countries

(b) Euro area core countries

Notes: The figure shows daily data between 2008 and 2012 for sovereign and bank CDS rates. The latter is determined by taking the (unweighted) average of the CDS rates of the two largest banks within the corresponding country (Italy: Intesa Sanpaolo and Unicredito; Portugal: Banco Comercial Portugês and Banco Espírito Santo; Spain: Banco Santander and Banco Bilbao Vizcaya Argentaria; Ireland: Allied Irish Banks and Bank of Ireland; Austria: Erste Bank and Raiffeisen International Bank; Belgium: Dexia and KBC Group; France: Société Générale and BNP Paribas; Netherlands: ABN Amro Bank and ING Group). Source: Datastream.
private risk premia as displayed in Figure 4.2.

We show that, in general, national fiscal authorities of small member states are able to stabilise macroeconomic conditions through use of countercyclical policies. Pro-cyclical fiscal policies, however, tend to intensify the business cycle and raise output variability. Furthermore, smaller member states tend to enjoy larger government spending multipliers than do larger member states, since smaller countries elicit a weaker offsetting monetary response by the central bank. We therefore confirm the results of traditional Keynesian theory on the effects of fiscal policy, at least under the assumption that sovereign risk premia are absent.

However, when sovereign risk premia become sensitive to changes in government indebtedness, fiscal policy can be a source of instability and raise output variability. In particular, when the fiscal authority reduces taxes in response to economic crises, the level of sovereign debt rises which triggers an increase in the sovereign risk premium when bond holders expect the fiscal limit to be breached. The higher risk premium raises public borrowing costs and further raises sovereign debt, which again raises the risk premium, etc. In order to avoid a sovereign debt crisis and prevent debt dynamics from spiralling out of control, future tax liabilities need to rise, which reduces the expected life-time disposable income of households. In addition, the rise in the sovereign risk premium leads to an increase in private borrowing costs, prompting households to borrow less and save more. Hence, when responding to economic crises, countercyclical fiscal policy can lead to negative wealth and crowding-out effects and a reduction in private consumption, thereby exacerbating the crisis. On the other hand, pro-cyclical policies aimed at reducing the stock of sovereign debt have the opposite effect and can stabilise economic conditions, even under very persistent negative shocks to output.

Furthermore, we show that government spending multipliers under sovereign risk premia are typically lower than when sovereign risk premia are absent, especially when the central bank is unable to offset the rise in the risk premium by reducing the policy interest rate. In fact, we find that when monetary policy keeps the interest rate fixed, the government spending multiplier can even be negative. This result contrasts earlier theoretical studies on the effects of fiscal policy when monetary policy is constrained by the zero lower bound, as these studies typically find multipliers to be very large (e.g.
Eggertsson, 2001; Christiano et al., 2009; Woodford, 2011).

The implications of sovereign risk premia for the effects of fiscal policy are reflected by the optimal fiscal policy stance. Using a utility-based welfare criterion (as in Woodford, 2003), we show that, in normal times, fiscal policy can be welfare-improving if the government responds to changes in domestic output and absorbs country-specific aggregate shocks. The optimal fiscal response to a negative productivity shock should be countercyclical when the country is relatively small, especially when the degree of price stickiness is large, yet pro-cyclical when the country is large (in which case monetary policy is more accommodative). However, in the presence of sovereign risk premia, optimal fiscal policy in a small member state dictates a pro-cyclical response to output contractions in order to prevent prolonged episodes of high distortionary taxes in the future that are otherwise needed to force sovereign debt back to its sustainable level. On the other hand, when the member state is relatively large, the rise in the sovereign risk premium is partially offset by the central bank and so optimal fiscal policy requires a countercyclical stance. Our results therefore indicate that the optimal fiscal stance depends crucially on the perceived riskiness of sovereign debt and the relative size of the member state within the monetary union.

The present chapter is on the verge of at least two branches of the recent macroeconomics literature. Firstly, the chapter is related to the literature on ‘expansionary fiscal contractions’, pioneered by Giavazzi and Pagano (1990), Sutherland (1997), Alesina and Ardagna (1998) and Perotti (1999), in which it is hypothesised that increases in the budget surplus can stimulate private spending if the fiscal authority manages to reduce long-term real interest rates and thereby also expected future tax liabilities. Similarly, we show that in times of elevated sovereign risk premia, tax hikes can stabilise aggregate output and increase welfare in small member states of a monetary union. However, whereas this literature is centred around the effects of exogenous fiscal

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2The expansionary fiscal contraction hypothesis has recently been tested empirically with mixed results. Jordà and Taylor (2013) and Leigh et al. (2014), for instance, find that fiscal contractions lead to a reduction in output growth, whereas Perotti (2012) shows that contractions are associated with economic expansions due to an accompanying exchange rate depreciation (see also Bonam and Lukkezen, 2014b). Furthermore, Coenen et al. (2008) argue that fiscal contractions lead to non-negligible adjustment costs in the short run, but can be expansionary in the long run if they reduce (and thereby mitigate the adverse effects of) distortionary taxes.
shocks, we also focus on the (optimal) endogenous fiscal response to changes in output and explicitly take into account the adverse effects of sovereign risk premia on private spending. Second, this chapter contributes to the literature on optimal fiscal policy rules in countries that belong to a monetary union (Kirsanova et al., 2007; Pappa and Vassilatos, 2007; Ferrero, 2009). Although not completely resolved, many authors agree that countercyclical fiscal policy can enhance welfare if economic shocks across member states are uncorrelated. Our results confirm this conjecture, yet only for member states that are relatively small. Furthermore, we show that the optimal fiscal stance also depends on the sensitivity of sovereign risk premia to changes in sovereign debt, an avenue that (to the extent of our knowledge) has not yet been explored carefully in previous studies.

The rest of this chapter is organised as follows. In Section 4.1, we lay out the main building blocks of the model and calibrate the structural parameters. The performance of fiscal stabilisation policy and the implications of sovereign risk premia are discussed in Section 4.2. The implications for the optimal design of fiscal policy are discussed in Section 4.3. Finally, Section 4.4 concludes.

## 4.1 A two-country monetary union

### 4.1.1 Overview of the model

We use a dynamic stochastic general equilibrium model for a two-country monetary union along the lines of Benigno (2004). The monetary union consists of two countries (or regions), ‘Home’ and ‘Foreign’, that interact with each other on international goods and asset markets. The union as a whole is assumed to be a closed economy. Each country is inhabited by a continuum of households. Households that live in Home are indexed by \( h \in [0, s] \), while those that live in Foreign are indexed by \( f \in (s, 1] \). The parameter \( s \) measures the relative size of the Home country.

Each household consumes domestically produced and imported goods, saves by investing in domestic nominal sovereign bonds, and supplies labour to monopolistically competitive firms in which it is a shareholder. Households may also issue or hold an internationally traded non-state contingent bond. Unlike Benigno (2004), we assume that households face a cost when taking foreign
asset positions, which reflects intermediation costs and is proportional to both private and public indebtedness.

Each firm produces a differentiated intermediate good and faces a price-setting constraint as in Calvo (1983). We assume only domestic labour is used in the production process of intermediate goods (hence, there is no mobility of labour across countries). Domestic producers charge the same price for their goods at home and abroad; together with the assumption of symmetric household preferences and absence of home bias in consumption, this implies that purchasing power parity holds at all times.

Monetary policy is conducted at the supranational level by a single monetary authority (or ‘central bank’) that sets the interest rate to target (a weighted average of) union-aggregate inflation and output. Fiscal policy, on the other hand, is conducted at the national level. In particular, each country has its own fiscal authority (or ‘government’) which is responsible for stabilising domestic output and the sustainability of sovereign debt. We assume that the fiscal authority may face fiscal solvency concerns by holders of sovereign bonds, reflected by a sovereign risk premium which is increasing in the public debt-to-output ratio.

The two countries are symmetric, yet are allowed to differ in country size and face idiosyncratic shocks. In the following, we focus on the Home environment; the behavioural and identity equations of the Foreign country are similar, unless specified otherwise. Variables expressed in Foreign currency are denoted with a * superscript. Wherever needed, demand for and prices of Home and Foreign goods are indexed by $H$ and $F$, respectively.

### 4.1.2 Households

In each period $t$, a representative household in the Home country chooses consumption, $c_t$, and labour supply, $n_t$, in order to maximise expected lifetime utility, expressed by

$$
E_t \sum_{k=0}^{\infty} \beta^k \left( \frac{c_{t+k}^{1-\sigma}}{1 - \sigma} - \frac{n_{t+k}^{1+\varphi}}{1 + \varphi} \right),
$$

where $E_t$ is the expectations operator conditional on the information available at $t$, $\beta \in (0, 1)$ denotes the household’s discount factor, $1/\sigma > 0$ is the inter-
temporal elasticity of substitution and $1/\varphi > 0$ the Frisch elasticity of labour supply. The household receives labour income, $W_t n_t$ where $W_t$ denotes the nominal wage rate. Firm profits made by domestic intermediate goods firms, $\psi_t = \int_0^s \psi_t (h) \, dh$, are paid out as dividends to the household sector. The household must pay a tax proportional to its labour and dividend income to the home fiscal authority; the tax rate is denoted by $\tau_t$.

Asset markets are assumed to be incomplete and the household may hold only two types of one-period (non-state contingent) nominal bonds: a domestically traded sovereign bond, $B_t$, and an internationally traded foreign bond, $F^*_t$. The sovereign bond offers a gross nominal return of $R_{g,t}$, which is equal to the product of the (risk-free) policy rate set by the central bank and a sovereign risk premium which arises when holders of sovereign bonds grow concerns about fiscal insolvency (see below). Regarding the foreign bond, we assume that the household is a net debtor and, following Turnovsky (1985), must pay a ‘private risk premium’, denoted by $\Xi^*_h,t$, over the risk-free foreign interest rate $R^*_t$ when borrowing abroad.\(^3\)

In line with recent literature, the private risk premium represents financial intermediation costs and is increasing in the country’s private external liabilities. Technically speaking, having $\Xi^*_h,t$ be determined by private external debt allows one to uniquely pin down $F^*$, i.e. the steady-state net foreign asset position of the household (see Schmitt-Grohé and Uribe, 2003). However, a reduced form financial friction of this kind also has a straightforward economic justification: as the household issues more and more debt and $F^*_t$ rises (relative to income), lenders become more concerned about the household’s ability to honour outstanding debt obligations and will therefore demand a higher risk premium to bear the additional credit risk.

In modelling the private risk premium, we also take into account the strong positive correlation between private credit risk and sovereign risk observed during the recent sovereign debt crisis in Europe (see Figure 4.2). Although causality may run in both directions (see Alter and Schüler, 2012), empirical evidence on the adverse effects of sovereign risk on private borrowing conditions is extensive (e.g. Borensztein and Panizza, 2009; Balteanu et al., 2011; Panetta et al., 2011; Bofondi et al., 2013; Demirgüç-Kunt and Huizinga, \(^3\)A similar international financial friction has been applied more recently by Benigno (2009) and De Paoli (2009).
4 Fiscal policy and sovereign risk premia under monetary union

2013; Zoli, 2013; Popov and Van Horen, 2013; Albertazzi et al., 2014). Such ‘sovereign risk pass-through’ is often explained by a significant exposure to sovereign default risk of financial intermediaries that hold large amounts of domestic sovereign bonds on their balance sheets, thereby making the financial sector more susceptible to a banking crisis following sovereign default.\(^4\)

The theoretical literature on sovereign risk pass-through is more limited, yet growing. For example, Bolton and Jeanne (2011), Gennaioli et al. (2014) and Sosa-Padilla (2014) use stylized models to capture the empirically observed relationship between sovereign default and bank lending rates and credit supply. Furthermore, Erceg and Lindé (2010) and Corsetti et al. (2013c) allow for the possibility that the risk of sovereign default, rather than default itself, can affect private borrowing conditions. This type of sovereign risk pass-through can be motivated by the fact that sovereign bonds are often used by banks as collateral when tapping the wholesale market for funds. An increase in sovereign risk then reduces the price, and hence collateral value, of sovereign bonds, which in turn impairs the banks’ balance sheets and raises private interest rate spreads, even in the absence of sovereign default. Since we abstain from sovereign default events, and only focus on scenarios in which sovereign bonds can be perceived as more risky, we follow the latter approach and postulate the following function for \(\Xi_{h,t}^*\):

\[
\Xi_{h,t}^* = \exp \left( \chi_{h,1} \frac{F_t^*}{P_t y_t} \right) \exp \left( \chi_{h,2} \frac{B_t}{P_t y_t} \right), \tag{4.2}
\]

with \(\chi_{h,1} > 0\) and where \(P_t\) denotes the Home consumer price index (CPI) and \(y_t\) real domestic aggregate output. Equation (4.2) implies that the degree of private and public indebtedness are used as proxy’s for private credit risk and sovereign risk, respectively, and both are used to determine the private risk premium. The extent to which investors expect sovereign risk to affect the household’s ability to repay is captured by the coefficient \(\chi_{h,2} \geq 0\), which, as in Erceg and Lindé (2010) and Corsetti et al. (2013c), therefore serves as a reduced form representation of sovereign risk pass-through.\(^5\)

\(^4\)According to another channel of sovereign risk pass-through, heavily strained public finances may adversely affect private borrowing conditions when a country’s sovereign is forced to raise future taxes significantly, or even appropriate private property, thereby limiting the ability of firms and households to honour their own debt obligations.

\(^5\)Since we refrain from the analysis of financial shocks, we ignore the possibility of conta-
4.1 A two-country monetary union

Let $P_{H,t}$ denote the aggregate domestic price index. The household’s flow budget constraint can then be written as

$$P_t c_t + B_t + \Xi_{h,t}^* R_{t-1}^* F_t^* = (1 - \tau_t) \left( W_t n_t + P_{H,t} \psi_t \right) + R_{g,t-1} B_{t-1} + F_t^*. \quad (4.3)$$

Maximising (4.1), subject to (4.3) and appropriate transversality conditions, while taking prices, the wage rate, the tax rate, the interest rates on sovereign and foreign bonds, the private risk premium and initial asset holdings, $B_{-1}$ and $F_{-1}^*$, as given, yields the following intratemporal first-order condition:

$$n_t^\phi = (1 - \tau_t) \frac{W_t}{P_t} c_t^{-\sigma}, \quad (4.4)$$

which reflects that, in any point in time, the household equates the marginal disutility of working to the marginal utility of consumption times the effective real wage, and two Euler equations:

$$c_t^{-\sigma} = \beta E_t \left[ r_{g,t} c_{t+1}^{-\sigma} \right], \quad (4.5)$$
$$c_t^{-\sigma} = \beta E_t \left[ \Xi_{h,t}^* r_t^* c_{t+1}^{-\sigma} \right], \quad (4.6)$$

with $r_{g,t} \equiv R_{g,t} (P_t / P_{t+1})$ and $r_t^* \equiv R_t^* \left( P_t^* / P_{t+1}^* \right)$, which determine the household’s optimal intertemporal allocation of consumption and savings by equating expected consumption growth to the real return of the assets available to the household.

A representative Foreign household has similar preferences and faces the following period budget constraint:

$$P_t^* c_t^* + B_t^* + F_t^* = (1 - \tau_t^*) \left( W_t^* n_t^* + P_{F,t} \psi_t^* \right) + R_{g,t-1} B_{t-1}^* + R_{t-1}^* F_{t-1}^* + T_t^*, \quad (4.7)$$

where $T_t^*$ represents the intermediation profits. In equilibrium, the optimality conditions for labour supply, consumption and savings of the Foreign household

\begin{enumerate}
\item From private credit risk to sovereign risk; hence, the household sector might be exposed
\item To deteriorating financial conditions in the public sector, but not the other way around.
\end{enumerate}
are given by

\[(n_t^*)^{\sigma^*} = (1 - \tau_t^*) \frac{W_t^*}{P_t^*} (c_t^*)^{-\sigma^*}, \quad (4.7)\]

\[(c_t^*)^{-\sigma^*} = \beta E_t \left[ r_{g,t}^* \left( E_{t+1} c_{t+1}^* \right)^{-\sigma^*} \right], \quad (4.8)\]

\[(c_t^*)^{-\sigma^*} = \beta E_t \left[ r_t^* \left( E_{t+1} c_{t+1}^* \right)^{-\sigma^*} \right]. \quad (4.9)\]

The household’s consumption bundle is a composite of domestically produced goods, \(c_{H,t}\), and imported goods, \(c_{F,t}\), as defined by the following Cobb-Douglas function:

\[c_t \equiv \frac{c_{H,t}^{1-s} c_{F,t}^s}{s^s (1-s)^{1-s}}. \quad (4.10)\]

Given the optimal choice of \(c_t\), and taking the aggregate Home and Foreign price indices as given, the household chooses the optimal allocation of Home and Foreign goods so as to minimise total consumption expenditures. The resulting demand schedules are given by

\[c_{H,t} = s \frac{P_t}{P_{H,t}} c_t, \quad c_{F,t} = (1-s) \frac{P_t}{P_{F,t}} c_t, \quad (4.11)\]

where the CPI is given by \(P_t = P_{H,t}^s P_{F,t}^{1-s}\). Similarly, Foreign demand for Home goods, \(c_{H,t}^*\), and Foreign goods, \(c_{F,t}^*\), is given by

\[c_{H,t}^* = s \frac{P_t^*}{P_{H,t}^*} c_{F,t}^*, \quad c_{F,t}^* = (1-s) \frac{P_t^*}{P_{F,t}^*} c_{F,t}^*. \quad (4.12)\]

Home and Foreign intermediate goods, \(c_{H,t}(h)\) and \(c_{F,t}(f)\), are aggregated according to the following CES composite indices:

\[c_{H,t} \equiv \left[ \left( \frac{1}{s} \right)^{\frac{1}{\epsilon}} \int_0^s c_{H,t}(h)^{\frac{1}{1-s}} dh \right]^{\frac{1}{1-\epsilon}}, \quad c_{F,t} \equiv \left[ \left( \frac{1}{1-s} \right)^{\frac{1}{\epsilon}} \int_s^1 c_{F,t}(f)^{\frac{1}{1-s}} df \right]^{\frac{1}{1-\epsilon}}, \quad (4.13)\]

where \(\epsilon > 1\) measures the elasticity of substitution between differentiated intermediate goods produced in the same country. Let \(P_{H,t}(h)\) and \(P_{F,t}(f)\) be the Home and Foreign price of the intermediate good produced by firm \(h\) and \(f\). The optimal allocation of expenditure within each variety of goods then
implies the following demand schedules:

\[ c_{H,t}(h) = \frac{1}{s} \left( \frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\epsilon} c_{H,t}, \quad c_{F,t}(f) = \frac{1}{1 - s} \left( \frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\epsilon} c_{F,t}, \]  

(4.14)

Similarly, for Foreign households we find

\[ c_{H,t}^*(h) = \frac{1}{s} \left( \frac{P_{H,t}^*(h)}{P_{H,t}^*} \right)^{-\epsilon} c_{H,t}^*, \quad c_{F,t}^*(f) = \frac{1}{1 - s} \left( \frac{P_{F,t}^*(f)}{P_{F,t}^*} \right)^{-\epsilon} c_{F,t}^*, \]  

(4.15)

where we have assumed that \( \epsilon = \epsilon^* \).

Since the Law of One Price holds, and because both countries share the same currency, prices are the same in each country, i.e. \( P_{H,t}(h) = P_{H,t}^*(h) \) and \( P_{F,t}(f) = P_{F,t}^*(f) \). The Home and Foreign aggregate price indices are given by

\[ P_{H,t} = P_{H,t}^* = \left[ \frac{1}{s} \int_0^s P_{H,t}(h)^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}}, \quad P_{F,t} = P_{F,t}^* = \left[ \frac{1}{1-s} \int_s^1 P_{F,t}(f)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}}. \]  

(4.16)

### 4.1.3 Firms

Each household owns an intermediate goods firm which produces a differentiated good \( y_t(h) \) using the following constant returns to scale production technology:

\[ y_t(h) = a_t n_t(h), \]  

(4.17)

where \( n_t(h) \) is the amount of labour demanded by firm \( h \). The variable \( a_t \) denotes a shock to labour productivity.

The firm aims to minimise its labour costs, \( W_t n_t(h) \), subject to the technology constraint (4.17). The resulting optimality condition for labour demand is given by

\[ \Lambda_t(h) = \frac{W_t}{a_t}, \]  

(4.18)

where \( \Lambda_t(h) \) denotes the Lagrange multiplier to (4.17) and is referred to as the firm’s nominal marginal costs, i.e. \( \Lambda_t(h) \equiv MC_t(h) \). Equation (4.18) holds for all firms and therefore \( MC_t(h) = MC_t \).

Although firms have monopolistic power over prices and are able to set
prices in excess of marginal costs, they face a price-setting constraint à-la Calvo (1983). In particular, only a share of $1 - \theta$ randomly selected firms is free to adjust prices in response to changing economic conditions, whilst remaining firms are forced to keep prices constant. Flexible price firms choose the optimal reset price, $P_{H,t}$, in order to maximise expected discounted profits, i.e.

$$\max E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left( P_{H,t} y_{t+k} (h) - W_{t+k} n_{t+k} (h) \right),$$

subject to the production function and the function determining the demand for the firm’s goods, which is given by $y_{t} (h) = \left( P_{H,t} / P_{H,t} \right)^{-\varepsilon} y_t$. $Q_{t,t+k}$ is a $k$-step ahead equilibrium pricing kernel satisfying $Q_{t,t+k} = \beta E_t \left[ (c_{t+k} / c_t)^{-\sigma} (P_t / P_{t+k}) \right]$. The resulting optimal reset price is then a mark-up over current and future discounted real marginal costs, i.e.

$$P_{H,t} = M \frac{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} P_{H,t+k}^{1+\sigma} y_{t+k}^{1+\sigma} \left( P_{H,t+k} / P_{H,t} \right)}{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} P_{H,t+k} \left( P_{H,t+k} / P_{H,t} \right)^{1+\sigma} y_{t+k}^{1+\sigma}}, \quad (4.19)$$

where $M \equiv \varepsilon / (\varepsilon - 1)$ is the price mark-up and $mc_t \equiv MC_t / P_{H,t}$ denotes real marginal costs. Note that, under flexible prices, $\theta \to 0$ and $P_{H,t} = P_{H,t}$, such that this equation collapses to $mc_t = 1 / M$.

A foreign intermediate goods firm faces a similar technology and price-setting constraint and sets prices according to

$$P_{F,t} = M \frac{E_t \sum_{k=0}^{\infty} (\theta^*)^k Q_{t,t+k}^{*} \left( P_{F,t+k}^{*} / P_{F,t}^{*} \right)^{1+\sigma} y_{t+k}^{*} m_{c,t+k}^{*}}{E_t \sum_{k=0}^{\infty} (\theta^*)^k Q_{t,t+k}^{*} \left( P_{F,t+k}^{*} / P_{F,t}^{*} \right)^{1+\sigma} y_{t+k}^{*}}, \quad (4.20)$$

where $m_{c,t}^{*} \equiv MC_t^{*} / P_{F,t}^{*} = (W_t^{*} / a_t^{*}) / P_{F,t}^{*}$ denotes foreign real marginal costs.

### 4.1.4 Monetary and fiscal policy

Monetary policy is conducted at the supranational level by a single monetary authority which follows a variant of the familiar Taylor rule (Taylor, 1993). It aims to maintain macroeconomic stability at the union-wide level by relating the gross nominal policy rate, $R_t$, to deviations of union-aggregate inflation.

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6Due to symmetry across firms, the optimal reset price can be written without the $h$ index.
and output from their respective targets/steady-state levels:

\[
\frac{R_t}{R_t} = \left( \frac{R_{t-1}}{R_t} \right)^{\rho_R} \left[ \left( \frac{\pi_{W,t}}{\pi_W} \right)^{\alpha_{\pi}} \left( \frac{y_{W,t}}{y_W} \right)^{\alpha_y} \right]^{1-\rho_R},
\]

(4.21)

where \( \rho_R \in [0, 1] \) reflects the degree of interest rate smoothing (or policy inertia) and \( \alpha_{\pi} > 1 (\alpha_y \geq 0) \) measures the aggressiveness of the central bank’s inflation (output) policy. Union-wide inflation and output are weighted averages of Home and Foreign variables, where country sizes \( s \) and \( 1 - s \) are used as weights, i.e. \( \pi_{W,t} \equiv s \pi_t + (1 - s) \pi^*_t \), where \( \pi_t \equiv P_t/P_{t-1} \) and \( \pi^*_t \equiv P^*_t/P^*_{t-1} \) denote Home and Foreign CPI inflation, and \( y_{W,t} \equiv s y_t + (1 - s) y^*_t \). The target values for union-wide inflation and output, denoted by \( \pi_W \) and \( y_W \), are equal to their respective steady state values.

In each country, there is a fiscal authority which levies a proportional income tax and issues one-period nominal bonds (to domestic households only) to cover public expenditures. The stock of government debt at the beginning of the period equals current outstanding public liabilities minus the government’s primary budget surplus. For the Home country, this boils down to the following period budget constraint:

\[
B_t = R_{g,t-1} B_{t-1} - P_{H,t} (\tau_t y_t - g_t),
\]

(4.22)

where \( g_t \) denotes real public consumption. Since \( g_t \) does not affect household utility, nor the production technology of the intermediate goods firms, we assume that \( g_t \) evolves exogenously according to an AR(1) process.

Dividing (4.22) by \( P_t \) and then solving forward recursively yields the intertemporal government budget constraint:

\[
R_{g,t-1} \frac{B_{t-1}}{P_t} = \sum_{n=0}^{\infty} \left( \prod_{m=0}^{n} r_{g,t+m}^{-1} \right) \frac{P_{H,t+n}}{P_{t+n}} (\tau_{t+n} y_{t+n} - g_{t+n}),
\]

(4.23)

where we have imposed the following no-Ponzi scheme condition which prevents the government from rolling over its debt indefinitely:

\[
\lim_{k \to \infty} \left( \prod_{m=0}^{k-1} r_{g,t+m}^{-1} \right) \frac{B_{t+k}}{P_{t+k}} = 0.
\]

(4.24)

Equation (4.23) is an equilibrium condition and requires that the real value
of outstanding government debt (left-hand side) equals the discounted sum of future real primary budget surpluses (right-hand side). In order to ensure that the fiscal authority makes appropriate adjustments in its primary surplus over time to satisfy \((4.23)\), we impose the following rule for taxes:\(^7\)

\[
\tau_t = \tau + \gamma_b \left( \frac{b_{t-1}}{y_{t-1}} - \overline{b} \right) + \gamma_y (y_{t-1} - \overline{y}),
\]

\((4.25)\)

where \(b_t \equiv B_t/P_t\) denotes the level of real public debt. The target values for the debt ratio and the level of aggregate output, denoted by \(\overline{b}/\overline{y}\) and \(\overline{y}\), are assumed to equal their respective steady state values. Rules similar to equation \((4.25)\) have been used extensively in the empirical literature to test for the sustainability of public debt and estimate the cyclical fiscal stance in both advanced and emerging market economies (see e.g. Gavin and Perotti, 1997; Galí and Perotti, 2003; Greiner et al., 2007; Mendoza and Ostry, 2008; Debrun and Kapoor, 2010; Ghosh et al., 2013). In line with empirical evidence, we restrict the government to respond only to lagged variables, reflecting political constraints that impede on and delay fiscal decision making.\(^8\)

As shown by Bohn (1998), a positive feedback from debt to the primary budget surplus is sufficient to ensure the no-Ponzi scheme condition \((4.24)\) holds (and therefore also the intertemporal government budget constraint \([4.23];\) see Bohn, 2007, and Bohn, 2008). Furthermore, the slope of the tax rule needs to be higher than the steady-state real interest rate in order for government debt to be stationary in equilibrium (see also Leeper, 1991). We therefore impose the following fiscal solvency and sustainability requirement: \(\gamma_b > r_g - 1.\)

The tax rule also captures the output stabilisation objective of the government. The desire to use fiscal policy to stabilise domestic output may arise when a country enters a monetary union and faces country-specific aggregate shocks (see for instance Beetsma and Jensen, 2005, and Galí and Monacelli, 2008). Having lost autonomy over monetary policy, a union member must then rely on fiscal policy to absorb such shocks (especially when its weight in the monetary policy rule is small). If the government sets \(\gamma_y > 0\), a con-

\(^7\)The foreign government faces a similar tax rule.

\(^8\)Allowing the government respond to contemporary variables does not alter our main results in any significant way.
traction in output is met by a fall in the primary surplus and fiscal policy is said to be ‘countercyclical’; if, on the other hand, $\gamma_y < 0$, fiscal policy tends to intensify the business cycle as the government responds in a ‘pro-cyclical’ way to changes in output. Therefore, $\gamma_y$ reflects the cyclical fiscal stance of the government. The main objective in this chapter is to examine the effects of sovereign risk on the performance of countercyclical fiscal policy in terms of output stabilisation and to uncover the optimal cyclical fiscal stance.

4.1.5 Introducing sovereign risk

We introduce sovereign risk following the approach of Davig et al. (2011) and Bi (2012). In particular, we assume that sovereign risk may arise due to the presence of a so-called ‘fiscal limit’ which determines the maximum rate of taxes, say $T$, that is politically (or economically) feasible.\(^9\) When the stock of government debt reaches a level that, by the tax rule (4.25), forces the tax rate to breach the fiscal limit, the government (partially) defaults on its outstanding debt, since it will not (or cannot) raise taxes beyond $T$; on the other hand, when $\tau_t < T$, the government fully honours its debt. Although agents do not know $T$ prior to entering a sovereign bond contract ($T$ is observed only when the bond matures), they know its distribution and, since they are forward-looking, form expectations about future sovereign default probabilities. Consequently, even if the economy is not at its fiscal limit, the mere possibility of getting there can affect today’s bond price: the higher is the probability of reaching the fiscal limit, the lower is the price.

Since the fiscal limit is stochastic and dependent on the state of the economy, explicitly modelling its conditional distribution can be quite cumbersome. Also, the fiscal limit implies non-linearities within the model, which renders a linear approximation unsuitable to study the model’s dynamics. Furthermore, since demand for sovereign bonds can or cannot be sustained in equilibrium, depending on whether or not $T$ has been breached, one easily runs into problems of multiple (and possibly self-fulfilling) equilibria (see Calvo, 1988). We do not take up the task to solve these problems here, yet instead circumvent them by assuming that $T$ is determined exogenously (see

\(^9\)As discussed by Ghosh et al. (2013), countries that belong to a monetary union are more prone to reaching the fiscal limit (which they refer to as ‘fiscal fatigue’), since they are more constrained in their use of monetary policy.
also Corsetti et al., 2013c, Daniel and Shiamptanis, 2013, Locarno et al., 2013, Roeger and in ’t Veld, 2013, and Schabert and van Wijnbergen (2014), for similar treatments of the fiscal limit). This allows us to distinguish two separate equilibria: a ‘good equilibrium’ in which $\tau_t$ is far below $T$ and the risk of breaching the fiscal limit is negligible, and a ‘bad equilibrium’ in which $\tau_t$ is close to $T$ such that breaching the fiscal limit, and therefore sovereign default, is more likely. In doing so, we dismiss realisations of sovereign defaults altogether and rather focus on the risk of default, which, as evidenced by the recent sovereign debt crisis in Europe, can have significant effects on sovereign bond spreads even if default never occurs.

Characterising investor expectations in either equilibria is a ‘sovereign risk premium’, denoted by $\Xi_{g,t}$, which is monotonically increasing in the level of government debt as a share of domestic output: $^{10}$

$$
\Xi_{g,t} = \exp \left( \chi_g \frac{b_t}{y_t} \right),
$$

(4.26)

$$
R_{g,t} = \Xi_{g,t} R_t.
$$

(4.27)

The coefficient $\chi_g \geq 0$ essentially reflects the slope of the fitted lines in Figure 4.1 and identifies which equilibrium, good or bad, prevails. In the good equilibrium, $\chi_g$ is (close to) zero such that an increase in government debt does not translate into a higher sovereign risk premium. In this case, investors fully trust the government to be able to sufficiently (and acutely) raise taxes to cover outstanding debt obligations and avoid default, even if the level of debt is relatively high (as was the case for Belgium before the euro crisis erupted). In the bad equilibrium, investors believe that further debt issuance brings the economy closer to the fiscal limit, which raises the risk of sovereign default (even if the government satisfies the fiscal solvency requirement $\gamma_b > r_g - 1$). To compensate for the additional risk, investors will then demand a higher sovereign risk premium and so $\chi_g$ takes on positive values. The bad equilibrium is thus associated with a downward sloping demand curve for sovereign bonds, which limits the government’s scope to issue debt at low rates (as was the case for many euro periphery countries during the hight of the sovereign

$^{10}$We assume that the Foreign fiscal authority does not face a sovereign risk premium such that $R^*_{g,t} = R_t$. 

110
4.1 A two-country monetary union

debt crisis). The calibration of $\chi_g$ is discussed further below.\textsuperscript{11}

4.1.6 Market clearing

Public consumption is assumed to be entirely in terms of domestically produced goods (reflecting the high degree of home bias in public spending often observed in advanced economies) and is aggregated using the same CES technology as used by households. Under these assumptions, aggregate demand for the Home and Foreign intermediate good is given by

\[ y_t(h) = \left( \frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\epsilon} \left[ S_t^{1-s} (sc_t + (1 - s) c_t^*) + g_t \right], \quad (4.28) \]

\[ y_t(f) = \left( \frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\epsilon} \left[ S_t^{-s} (sc_t + (1 - s) c_t^*) + g_t^* \right], \quad (4.29) \]

where $S_t \equiv P_{F,t}/P_{H,t}$ refers to the terms of trade. Using appropriate aggregators for the intermediate goods $y_t(h)$ and $y_t(f)$, we can derive the following goods market clearing conditions for the Home and Foreign country:

\[ y_t = S_t^{1-s} (sc_t + (1 - s) c_t^*) + g_t, \quad (4.30) \]

\[ y_t^* = S_t^{-s} (sc_t + (1 - s) c_t^*) + g_t^*. \quad (4.31) \]

Using the production function of and demand schedule for intermediate goods, we obtain the following labour market clearing condition for Home:

\[ n_t = \int_0^s n_t(h) \, dh = \frac{y_t}{a_t} \int_0^s \left( \frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\epsilon} \, dh, \quad (4.32) \]

and for Foreign:

\[ n_t^* = \int_s^1 n_t^*(f) \, df = \frac{y_t^*}{a_t^*} \int_s^1 \left( \frac{P_{F,t}^*(f)}{P_{F,t}^*} \right)^{-\epsilon} \, df, \quad (4.33) \]

Finally, the balance of payments condition is derived by consolidating the household’s and public’s budget constraints and implies that national savings

\textsuperscript{11}Equation (4.26) can also be seen as a reduced form representation of the pricing rule for sovereign bonds developed by Bi (2012), who derives the fiscal limit endogenously using dynamic Laffer curves and shows that the sovereign risk premium is a non-linear and convex function of the government debt-to-output ratio.
must equal net capital outflow:

\[ P_{H,t}(y_t - g_t) - P_tc_t = \Xi^*_{h,t-1} R^*_t F^*_t - F^*_t. \] (4.34)

By Walras’ law, we do not need a similar condition for the Foreign country.

### 4.1.7 Equilibrium

A rational expectations equilibrium is given by a sequence of the model’s endogenous variables that satisfies the labour supply and savings decisions of the households, determined by conditions (4.4)-(4.6), the household’s budget constraint (4.3), demand schedules and price indices (4.11), (4.14) and (4.16), the firm’s price-setting condition (4.19), the monetary and fiscal policy rules (4.21) and (4.25), the public’s intertemporal budget constraint (4.23), the bond pricing equations (4.26) and (4.27), the labour and goods market clearing conditions (4.30) and (4.32), the balance of payments conditions (4.34), and Foreign counterparts, given exogenous processes for productivity and government spending shocks and the fiscal limit.

### 4.1.8 Calibration

To analyse the dynamics of the model, we linearise the equilibrium conditions around a deterministic steady state.\(^{12}\) The model is then calibrated based on a quarterly frequency. Unless stated otherwise, Foreign parameters are assigned the same value as Home parameters. The benchmark parameterisation is listed in Table 4.1.

For most of the model’s structural parameters, we take those values that are commonly found in the literature. In particular, we set the discount factor equal to \(\beta = 0.99\), which implies an annualised real interest rate of about 4\%, assume \(\sigma = 1\), such that utility depends on the log of consumption, and choose \(\varphi = 3\), implying a Frisch elasticity of labour supply of 1/3. Consistent with estimates of Smets and Wouters (2003), we set \(\theta = 0.9\), which implies an average price-contract of 2.5 years, and, following Corsetti et al. (2012a), we set the elasticity of substitution between intermediate goods at \(\epsilon = 11\), implying a price mark-up of 10\%. Further, following Bouakez and Eyquem

\(^{12}\)Details of the linearisation are provided in Appendix 4.A.
### 4.1 A two-country monetary union

#### Table 4.1: Benchmark calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$1/\sigma$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1</td>
</tr>
<tr>
<td>$1/\varphi$</td>
<td>Frisch elasticity of labour supply</td>
<td>1/3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of non-price adjustment</td>
<td>0.9</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution between intermediate goods</td>
<td>11</td>
</tr>
<tr>
<td>$\chi_{h,1}$</td>
<td>Private risk premium elasticity w.r.t. private debt</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\chi_{h,2}$</td>
<td>Private risk premium elasticity w.r.t. public debt</td>
<td>0 or 0.1 $\times \chi_g$</td>
</tr>
<tr>
<td>$\chi_g$</td>
<td>Sovereign risk premium elasticity w.r.t. public debt</td>
<td>0 or 0.115</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence of productivity shocks</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Persistence of government spending shocks</td>
<td>0.95</td>
</tr>
<tr>
<td>$b/y$</td>
<td>Steady-state public debt ratio (annualised)</td>
<td>0.6</td>
</tr>
<tr>
<td>$f^*/y$</td>
<td>Steady-state private debt ratio (annualised)</td>
<td>0.6</td>
</tr>
<tr>
<td>$g/y$</td>
<td>Steady-state government consumption ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>$c/y$</td>
<td>Steady-state private consumption ratio</td>
<td>0.8</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Steady-state tax rate</td>
<td>0.2</td>
</tr>
<tr>
<td>$s$</td>
<td>Relative size of the Home country</td>
<td>0.02</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Interest rate smoothing</td>
<td>0.8</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>Monetary response to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>Monetary response to output</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>Fiscal response to government debt ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>$\gamma^*_y$</td>
<td>Fiscal response to output (in Foreign)</td>
<td>0</td>
</tr>
</tbody>
</table>
(2015), who rely on estimates of Lane and Milesi-Ferretti (2002), we set the elasticity of the private risk premium with respect to changes in household net external debt to $\chi_{h,1} = 0.0017$. Also, we assume that, in times when public finances are perceived as weak, 10% of an increase in sovereign risk is passed on to private borrowing conditions, such that $\chi_{h,2} = 0.1 \times \chi_g$.\footnote{Our choice for $\chi_{h,2}$ is somewhat on the lower end of recent estimates from the empirical literature. For example, Zoli (2013) finds that of a 1% increase in Italian sovereign risk, as measured by the sovereign CDS rate, roughly 35% is transmitted to the CDS rates of Italian banks.}

Productivity and government spending shocks evolve according to the following AR(1) processes:

\begin{align*}
a_t &= \rho_a a_{t-1} + \varepsilon_{a,t}, \\
g_t &= \rho_g g_{t-1} + \varepsilon_{g,t},
\end{align*}

where $\rho_a \in [0,1]$ and $\rho_g \in [0,1]$ measure the persistence of the shocks and where $\varepsilon_{a,t} \sim \mathcal{N}(0, \sigma_a^2)$ and $\varepsilon_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$ are random i.i.d. terms. Similar expressions apply for $a^*_t$ and $g^*_t$. Importantly, we assume that shocks are uncorrelated across countries in order to highlight the role of fiscal stabilisation policy at the national level. Following estimates from King and Rebelo (1999), we set $\rho_a = 0.95$, whereas we choose $\rho_g = 0.95$ as in Smets and Wouters (2003).

The steady-state private and public debt-to-output ratio’s are assumed to be 60% (on an annual basis), while public consumption as a share of output in steady state is set to 20%, consistent with euro area averages. Using the balance of payments condition (4.34) and the government’s flow budget constraint (4.22), we then obtain a steady-state consumption-to-output ratio of about 80%, and a steady-state tax rate of around 20%. As a benchmark, we consider a ‘small member state’ in which the relative economic size of the Home country is equal to $s = 0.02$ (which is about the size of Greece’s GDP as a share of total euro area GDP).

The monetary policy parameters are given values equal to $\rho_R = 0.8$, $\alpha_\pi = 1.5$ and $\alpha_y = 0.5$, which ensures that the Taylor-principle is satisfied and equilibrium determinacy guaranteed. We consider alternative values for the fiscal policy parameter $\gamma_y$ to assess the performance of fiscal stabilisation policy, while the debt parameter is set to $\gamma_b = 0.2$, which is large enough to ensure a stationary path for government debt in equilibrium. Throughout, we assume
4.1 A two-country monetary union

Table 4.2: Estimates for $\chi_g$: the relationship between the debt ratio and the sovereign risk premium in the euro area

<table>
<thead>
<tr>
<th>Sovereign risk premium</th>
<th>Full sample</th>
<th>Only euro crisis$^1$</th>
<th>Only euro periphery countries$^2$</th>
<th>Only euro crisis and periphery countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt ratio</td>
<td>5.57***</td>
<td>10.17***</td>
<td>7.00***</td>
<td>11.49***</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.66)</td>
<td>(0.37)</td>
<td>(1.02)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.71</td>
<td>0.80</td>
<td>0.59</td>
<td>0.71</td>
</tr>
<tr>
<td>No. of observations</td>
<td>783</td>
<td>221</td>
<td>258</td>
<td>68</td>
</tr>
</tbody>
</table>

Notes: ‘Sovereign risk premium’ is defined as the long-run (10 year) government bond yield, minus the yield on a similar German bond (in basis points), and ‘debt ratio’ denotes gross government debt as a share of nominal GDP (in percentage points); constants and country fixed effects dummies have been suppressed. Standard errors are given in parentheses. *** denotes a $p$-value lower than 0.001. Data are for Austria, Belgium, Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, Slovak Republic and Spain. Source: own estimates based on data from the OECD National Accounts Statistics database. (1) The euro crisis covers the period 2008Q1-2012Q1. (2) Euro periphery countries refer to Ireland, Italy, Portugal and Spain (quarterly data for Greece were unavailable).

fiscal policy in Foreign is a-cyclical, i.e. $\gamma_y^* = 0$.

The key parameter of this chapter is $\chi_g$, which measures the sensitivity of the sovereign risk premium with respect to changes in the government debt ratio and plays an important role in characterising equilibrium properties. Empirical estimates for $\chi_g$ are somewhat inconclusive. For instance, Laubach (2009) finds that an increase in the debt ratio of 1% in the US raises long-term real interest rates by 2-4 basis points. Jaramillo and Weber (2013), on the other hand, find higher estimates for emerging economies during times of elevated risk aversion, with 10-year bond yields increasing by 6 basis points for every 1% increase in the expected debt-output ratio. Here, we provide our own estimates for $\chi_g$ by simply estimating the sovereign risk premium function (4.26) using a fixed-effects panel model and the data presented in Figure 4.1. To increase the number of observations, we use a larger time-window (from 1995Q1 to 2013Q4) and include data for additional European countries (Estonia, Finland, Germany, Luxembourg, and Slovak Republic). All data are retrieved from the OECD database.

The results are displayed in Table 4.2. As expected, an increase in the debt
ratio is associated with a higher sovereign risk premium, especially during the crisis period. Since we are particularly interested in the relationship between the sovereign risk premium and government indebtedness in the ‘bad equilibrium’, in which investors assign higher probabilities of sovereign default, we also estimate $\chi_g$ based on data from euro periphery countries only. As evidenced by Figure 4.1, these countries have experienced sharper increases in the sovereign risk premium than other European countries, especially during the sovereign debt crisis. Columns 3 and 4 of Table 4.2 indeed report larger estimates for $\chi_g$, predicting a rise in the sovereign risk premium of 11.49 basis points for every 1% increase in the debt ratio during 2008-2012. Based on these results, we set $\chi_g = 0.115$ when considering the bad equilibrium case and $\chi_g = 0$ otherwise. However, as these results should be treated with caution, we shall experiment with alternative values for $\chi_g$ to test for the robustness of our numerical results.

4.2 Fiscal stabilisation policy and sovereign risk

4.2.1 Endogenous fiscal policy

We start by assessing the performance of the output stabilisation function of fiscal policy and the implications of sovereign risk premia. We do so by assuming the Home country experiences an economic crisis in the form of a negative country-specific productivity shock. The reason that we do not consider shocks that are symmetric across the monetary union (or shocks borne from the much larger Foreign country) is because such shocks would elicit a much stronger countercyclical monetary response by the central bank which reduces the role of fiscal policy in stabilising output at the national level. Instead, small member states facing country-specific shocks receive less ‘monetary support’ (as their weight in the monetary policy rule is small) and must therefore rely more heavily on fiscal policy.

Figure 4.3 shows the responses of output, consumption, government debt (as a share of output), CPI inflation and the sovereign risk premium following a negative productivity shock in Home. In each plot, the endogenous fiscal response to the shock can either be countercyclical (indicated by the marked (blue) lines), in which case taxes are cut following a reduction in output, a-
4.2 Fiscal stabilisation policy and sovereign risk

Figure 4.3: Responses to a negative productivity shock in Home as a function of the cyclical fiscal stance ($\gamma$)

Without sovereign risk ($\chi_g = 0$)  
With sovereign risk ($\chi_g = 0.115$)

Output

Consumption

Government debt ratio

Inflation

Sovereign risk premium

Notes: The figures show the responses of selected endogenous variables, in terms of percentage deviations from steady state, following a negative, country-specific productivity shock in Home. Alternative cyclical fiscal stances are considered: dotted lines = countercyclical; dashed lines = pro-cyclical; solid lines = a-cyclical.
cyclical (solid (black) lines), in which case taxes do not respond to changes in output, or pro-cyclical (dashed (red) lines), in which case taxes are raised following a reduction in output. In the first column, we have considered the ‘good equilibrium’ such that $\chi_g = 0$ and the sovereign risk premium remains constant over time. Upon impact, output and consumption contract, whereas inflation rises due to the rise in firm’s real marginal costs. Government indebtedness rises as well, since tax revenue is proportional to income. In the absence of a time-varying sovereign risk premium, the cyclicality of fiscal policy does not seem to affect macroeconomic conditions much, although output contractions can be mitigated somewhat if the government pursues a more countercyclical fiscal stance. Particularly, when $\gamma_g$ is assigned positive values, the government automatically reduces taxes in response to output contractions. Since lowering taxes stimulates labour supply and production, the government partially offsets the initial adverse effects of the negative productivity shock.

Note, however, that a countercyclical fiscal stance also leads to a stronger accumulation of sovereign debt. In the second column of Figure 4.3, we have assumed that such a build-up of debt pushes the economy into the ‘bad equilibrium’ and induces sovereign bond holders to raise the sovereign risk premium (i.e. $\chi_g > 0$). Under such conditions, the role of the cyclical stance of fiscal policy in stabilising the economy changes markedly. Now, a countercyclical fiscal response to the crisis worsens the contraction in output and leads to stronger output fluctuations, whereas a pro-cyclical fiscal stance mitigates the output contraction and reduces fluctuations in output. Also, the reduction in consumption is much larger under a time-varying sovereign risk premium when fiscal policy is countercyclical, whereas consumption rises under pro-cyclical fiscal policy.

To understand how raising, rather than cutting, taxes in response to a contraction in output can lead to more stable economic conditions in times of sovereign risk, note first that reducing taxes generates budget deficits and, subsequently, an increase in the accumulation of government debt due to a higher sovereign risk premium. In order to satisfy the government’s intertemporal budget constraint (4.23), future tax liabilities need to rise which lowers the expected life-time disposable income of households. At the same time, an increase in the sovereign risk premium also leads to an increase in the private risk premium, as implied by (4.2), and thus worsens private borrowing condi-
4.2 Fiscal stabilisation policy and sovereign risk

Figure 4.4: Output variability under negative productivity shocks in Home as a function of shock persistence ($\rho_a$) and fiscal policy ($\gamma_y$)

Notes: The figures show the standard deviation of output in Home following a negative productivity shock as a function of $\rho_a \in [0, 1]$, which is the auto-correlation coefficient of the productivity shock $a_t$ (see [4.35]). Alternative cyclical fiscal stances are considered: dotted lines = countercyclical; dashed lines = pro-cyclical; solid lines = a-cyclical.

Households respond by increasing (precautionary) savings and reducing the level of consumption, causing output to fall by more than under a constant sovereign risk premium. Therefore, when faced with a negative productivity shock, reducing taxes can actually be destabilising and exacerbate the ensuing crisis through a negative wealth and crowding-out effect on private spending. If, on the other hand, the government were to place more emphasis on stabilising debt and raises taxes, the effects of the crisis can be mitigated as the consequent reduction in the stock of debt reduces the need for higher future taxes and also reduces the private risk premium, both of which support household consumption.\(^{14}\)

The effects of fiscal stabilisation policy on output variability as a function of the persistence of the productivity shock, $\rho_a$, are displayed in Figure 4.4. The figure shows that the standard deviation of output, following a negative productivity shock in Home, rises as the shock becomes more persistent (i.e. the larger is $\rho_a$). The figure in the left panel, in which the sovereign risk

\(^{14}\)To some degree, these results are akin to the ideas of the ‘expansionary fiscal contractions’ hypothesis, pioneered by Giavazzi and Pagano (1990), Sutherland (1997), Alesina and Ardagna (1998) and Perotti (1999).
4 Fiscal policy and sovereign risk premia under monetary union

Figure 4.5: Responses to a positive government spending shock in Home

Notes: The figures show the responses of selected endogenous variables, in terms of percentage deviations from steady state, following a positive government spending shock in Home (of 1% of output). Here, we assume $\gamma_y = 0$.

premium remains constant and $\chi_g = 0$, shows that, for any $\rho_a$, the government can reduce output variability by pursuing countercyclical fiscal policies and reducing taxes when output contracts. On the other hand, raising taxes leads to stronger output contractions and therefore intensifies the business cycle, which is reflected by overall higher output variability. In the right panel, the sovereign risk premium rises with sovereign debt and $\chi_g > 0$. Now, countercyclical fiscal policy leads to much more unstable economic conditions, especially when shocks are very persistent, whereas a pro-cyclical fiscal stance helps reduce output variability. Again, fiscal policy can become a source of instability due to the negative wealth and crowding-out effects of sovereign risk premia on household consumption. Therefore, when public finances are perceived to be critically weak, fiscal consolidation is warranted in order to ‘calm down’ the bond market and reduce sovereign bond spreads.

4.2.2 Discretionary fiscal policy

The implications of the sovereign risk premium for the performance of endogenous fiscal responses to aggregate shocks are similar when considering discretionary fiscal policy. In Figure 4.5, we consider the effects of a transit-
ory government spending shock (of 1% of output) in Home. We assume fiscal policy is a-cyclical and set $\gamma_y = 0$ so as to ignore the effects of the endogenous fiscal response to output. According to standard Keynesian theory, government spending multipliers are typically larger when monetary policy is less responsive to fiscal policy and therefore predicts larger multipliers when interest rates are pegged (see Hebous, 2011, for an overview of the literature). This may apply to our small Home country as well, since it essentially faces a pegged interest rate. According to Figure 4.5, the output response following the government spending shock is indeed large and the impact government spending multiplier even exceeds unity in the absence of sovereign risk (indicated by the solid (black) line). Also, as aggregate demand rises, and without a strong offsetting monetary response, inflation rises. Furthermore, the government expansion leads to an increase in government debt, which creates a negative wealth effect and induces households to reduce consumption and raise labour supply.

The effects of government spending on output are weakened, however, when the sovereign risk premium becomes more responsive to changes in government indebtedness (see the marked and dashed (red) lines). As the sovereign risk premium elasticity $\chi_g$ rises, output contracts faster following the initial jump and the cumulative government spending multiplier falls (output can even fall below steady state when $\chi_g$ is sufficiently large). Underlying the weaker potency of fiscal policy are again the effects of the sovereign risk premium on consumption, as evidenced by the stronger contraction in consumption following the government spending shock the larger is $\chi_g$. In a sense, the rise in the sovereign risk premium acts as an offsetting contractionary monetary response, that would have otherwise occurred if Home was a closed economy, and amplifies the crowding-out effects on consumption.

In Figure 4.6, we examine what happens to the potency of discretionary fiscal policy when the Home country approaches the closed economy case, i.e. as $s \to 1$. In line with traditional Keynesian wisdom, we find that the impact government spending multiplier falls as country size rises, as shown in the left panel of Figure 4.6: the larger is the economy, the more responsive the central bank becomes to regional shocks and the more it will raise the interest rate following a fiscal expansion. Consistent with Figure 4.5, the government spending multiplier falls in the presence of sovereign risk premia as long as the
Figure 4.6: Government spending multiplier as a function of country size \((s)\)

<table>
<thead>
<tr>
<th>Country size ((s))</th>
<th>Without sovereign risk ((\chi_g = 0))</th>
<th>With sovereign risk ((\chi_g = 0.115))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>(\rho_R = 0.8)</td>
<td>(\rho_R = 0.8)</td>
</tr>
<tr>
<td>0.4</td>
<td>(\rho_R = 0.4)</td>
<td>(\rho_R = 0.4)</td>
</tr>
<tr>
<td>0.6</td>
<td>(\rho_R = 0)</td>
<td>(\rho_R = 0)</td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The figures show the impact output response in Home following a positive government spending shock (of 1% of output) as a function of \(s\), which denotes the size of the Home country. Here, we assume \(\gamma_y = 0\).

The central bank keeps the interest smoothing parameter at \(\rho_R = 0.8\), as shown in the right panel of Figure 4.6. However, note that when the central bank pursues a more aggressive inflation policy and reduces \(\rho_R\), the multiplier falls by less and can even increase with country size if \(\rho_R\) is sufficiently low. Intuitively, the larger is the economy, the more the central bank will accommodate the fall in inflation (and consumption), caused by the rise in the sovereign risk premium, by reducing the interest rate. Hence, when approaching the closed economy case, government spending multipliers in the presence of sovereign risk can still be large, provided the central bank fully offsets the rise in the sovereign risk premium. If the central bank were unable to reduce the interest rate, for instance, because it is constrained by the zero lower bound on the nominal policy rate, the multiplier can be low and even negative (for the case where \(\rho_R \to 1\), see Figure 4.9 in Appendix 4.D).15

The results presented in this section indicate that the performance of fiscal

---

15These results are in line with Bonam and Lukkezen (2014b), who show that government spending multipliers in the presence of sovereign risk can be larger under flexible exchange rates than under fixed exchange rates. As in the present chapter, the authors show that sovereign risk can generate considerable crowding-out effects on household consumption, yet these may be less severe under flexible exchange rates when monetary policy is free to offset the effects of sovereign risk by lowering the interest rate.
stabilisation policy crucially depends on the perceived riskiness of sovereign debt. Whereas reducing taxes in the event of an economic crisis may help stabilise output in ‘normal times’, it might exacerbate economic conditions when sovereign risk premia are very responsive to changes in government indebtedness. Furthermore, the results indicate that government spending multipliers can be low or negative, even when monetary policy is effectively unresponsive to shocks, for instance under considerable interest rate smoothing or when the zero lower bound binds. It is therefore not clear whether previous results on the effects of fiscal policy under liquidity traps (e.g. Eggertsson, 2001; Christiano et al., 2009; Woodford, 2011) can be generalised to regimes characterised by critically weak public finances. In fact, our results imply that monetary unions facing interest rates close to zero and a severe sovereign debt crisis would benefit from (coordinated) fiscal consolidation policies and can overcome the zero lower bound problem by reducing effective real interest rates through a reduction in sovereign risk premia (see also Erceg and Lindé, 2010).

4.3 Optimal fiscal policy

Having examined the implications of sovereign risk premia for the stabilising effects of fiscal policy, we now turn to the implications for the design of optimal fiscal policy. For simplicity, yet without loss of generality, we assume that government spending remains constant, i.e. $g_t = g$ for all $t$.

In order to define the optimal fiscal stance for the Home government, we need to determine a suitable welfare criterion. As has become standard practice in the macroeconomics literature, we proxy welfare by the expected lifetime utility of the Home household, as given by Equation (4.1).

---

16Earlier studies on optimal fiscal policy in a currency union often assume a benevolent social planner that aims to maximise (a weighted average of) union-wide welfare (e.g. Kirsanova et al., 2007; Pappa and Vassilatos, 2007; Ferrero, 2009). However, we believe this to be somewhat unrealistic and, in line with the current institutional set-up of the European Economic and Monetary Union, consider fiscal authorities that are only concerned with domestic welfare and conduct their policies non-cooperatively.
proximation of (4.1), which is given by
\[
E_t \sum_{k=0}^{\infty} \beta^k W_{t+k} = -\left( \frac{c}{y} \right)^{-1} \sum_{k=0}^{\infty} \beta^k \left[ \left( \frac{1 + \varphi}{2} \right) \left( \hat{y}_{t+k} - \bar{y}_{t+k} \right)^2 + \frac{\kappa}{2(1 + \varphi)} + (1 + \varphi) \hat{y}_{t+k} (\hat{y}_{t+k} - \hat{a}_{t+k}) \right] + \text{t.i.p.} + \mathcal{O} (||\zeta||^3),
\]
with \( \kappa \equiv (1 - \theta)(1 - \theta\beta)/\theta \) and where a variable with a hat denotes the percentage deviation of that variable from its steady-state level and a variable with a tilde denotes the corresponding natural level. Terms that are of order strictly higher than 2 are collected in \( \mathcal{O} (||\zeta||^3) \), while terms independent of policy (such as constants and shocks not affected by changes in policy) are collected in ‘t.i.p.’ As in Benigno (2004), we have derived Equation (4.37) around the ‘efficient steady state’, in which prices are fully flexible and monopolistic distortions are eliminated through use of an appropriate tax subsidy.\(^{17}\)

According to Equation (4.37), welfare \( W_t \) is decreasing in output variability, which provides a potential role for fiscal stabilisation policy. We assume that the Home government aims to maximise the well-being of its citizens by setting fiscal policy according to the tax rule given by Equation (4.25), while taking Foreign fiscal policy and the central bank’s monetary policy as given. Specifically, the government’s objective boils down to finding the value of \( \gamma_y \) that maximises Equation (4.37); we denote this value by \( \gamma_{y,\text{opt}} \).\(^{18}\)

Assuming the Home country faces a country-specific shock to productivity, we calculate \( \gamma_{y,\text{opt}} \) under different assumptions about the sovereign risk premium elasticity. In Figure 4.7, we plot \( \gamma_{y,\text{opt}} \) as a function of the relative size of the Home country. According to the figure in the left panel, where we have assumed a constant sovereign risk premium and \( \chi_g = 0 \), optimal fiscal policy requires a high and positive value for \( \gamma_{y,\text{opt}} \) when Home is relatively

\(^{17}\)A brief derivation of the flexible-price equilibrium and the utility-based welfare criterion is provided in Appendix 4.B and 4.C.

\(^{18}\)Alternatively, the government could choose to follow the Ramsey policy, which prescribes the processes of the model’s endogenous variables that maximise the welfare criterion. However, using an optimised tax rule like Equation (4.25), rather than the Ramsey policy, offers a number of advantages. First of all, following a simple rule makes it much easier to communicate policy with the general public. Second, although policymakers can alter their policy stance, committing to a pre-specified rule makes it more difficult to deviate from agreed-upon objectives, thereby eliminating the scope for discretion. And last, the Ramsey policy might not even be feasible in practice, as it would require policymakers to respond to variables that are unobservable (such as shocks).
4.3 Optimal fiscal policy

Figure 4.7: Optimal cyclical fiscal stance as a function of country size ($s$)

![Graph showing optimal fiscal stance](image)

Notes: The figures show the values for $\gamma_{y,\text{opt}}$, which is the value of $\gamma_y$ from the tax rule (4.25) that maximises the welfare criterion (4.37), for different assumptions about country size, $s$.

small, implying that countercyclical fiscal policies can be welfare enhancing. When the economy is large, however, optimal fiscal policy takes into account the offsetting accommodating effects of monetary policy following negative shocks and thus requires a pro-cyclical stance so as not to amplify output fluctuations.

The optimal fiscal stance critically depends, not only on country size, but also on the perceived riskiness of government debt, as evidenced by the right panel of Figure 4.7. Here, we assume that increases in government debt raise the sovereign risk premium, which makes it more difficult to pursue countercyclical policies in the face of output contractions. Consequently, the optimal fiscal stance for relatively small member states is pro-cyclical, rather than countercyclical.\footnote{These results are in line with empirical estimates reported by Gavin and Perotti (1997) and Combes et al. (2014), who show that countries with high levels of public debt, or which face severe financial market constraints, tend to pursue pro-cyclical fiscal policies.} This result follows from our earlier discussion in Section 4.2: lower taxes today raise government debt and the sovereign risk premium, which leads to lower household consumption. In order to avoid the negative crowding-out effects of sovereign risk, the government must raise taxes and thereby reduce the sovereign risk premium. However, since taxes are distor-
Figure 4.8: Optimal cyclical fiscal stance as a function of price stickiness ($\theta$)

**Without sovereign risk ($\chi_g = 0$)**

- Optimal cyclical fiscal stance ($\gamma_{y,opt}$)
  - 0.2 0.4 0.6 0.8
  - 16 12 8 4

**With sovereign risk ($\chi_g = 0.115$)**

- Optimal cyclical fiscal stance ($\gamma_{y,opt}$)
  - 0.2 0.4 0.6 0.8
  - -0.16 -0.12 -0.08 -0.04

Notes: See notes under Figure 4.7. Here, we have assumed that $s = 0.02$.

Fiscal policy and sovereign risk premia under monetary union

The ability of fiscal policy to stabilise output arises from the presence of price sluggishness. Indeed, the greater is the share of firms that are unable to adjust their prices in response to aggregate shocks (i.e. the higher is $\theta$), the stronger are the effects of productivity shocks on output and thus the more aggressive should be the fiscal authority’s response to output fluctuations. In Figure 4.8, we plot $\gamma_{y,opt}$ as a function of the Calvo parameter $\theta$ for the small member case (i.e. $s = 0.02$). According to the figure in the left panel, in which sovereign risk is assumed absent, the stronger is the degree of price stickiness, the stronger should be the countercyclical tax response to changes in output (i.e. the higher should be $\gamma_y$). Conversely, when $\theta$ is close to zero and prices are more flexible, fiscal policy should be more a-cyclical so as not to generate inefficient changes in taxes. In the presence of sovereign risk, greater price stickiness also requires a more aggressive tax response to changes in output, yet fiscal policy should now be pro-cyclical, as shown in the right panel of
Monetary union requires its member states to rely more on fiscal policy at the national level to absorb country-specific shocks, especially when member states are relatively small. In this chapter, we used a New Keynesian model for a two-country monetary union in order to provide new insights into the performance of fiscal policy under alternative assumptions about the perceived strength of public finances. To do so, we have introduced a time-varying sovereign risk premium, which is a convex function of government indebtedness, and allowed for changes in this sovereign risk premium to affect private borrowing conditions.

We have shown that, in the absence of sovereign risk premia, fiscal policy can stabilise macroeconomic conditions and enhance welfare in small member states if fiscal policy is countercyclical, i.e. if the government reduces taxes in the face of economic crises and raises taxes during expansions. Larger countries within monetary union, however, receive more monetary accommodation in the face of economic crises and should therefore adopt pro-cyclical fiscal policies so as to avoid output fluctuations from becoming too large. Furthermore, government spending multipliers tend to be larger when member states are small, since fiscal shocks borne from small countries do not induce the central bank to strongly raise interest rates.

The effects of (discretionary) fiscal policy are, however, markedly different when further increases in sovereign debt lead to concerns about fiscal insolvency and higher sovereign risk premia. Under such conditions, countercyclical fiscal policy can exacerbate economic crises, whereas pro-cyclical policies help stabilise the economy. Intuitively, when cutting taxes, the budget deficit rises, which leads to a stronger build-up of sovereign debt and an increase in the sovereign risk premium, which raises debt even further. In order to ensure a sustainable path for debt, the government is required to raise taxes over time which reduces the expected life-time income of households. In addition, the higher sovereign risk premium raises interest rates on private bonds and discourages households from borrowing. Therefore, fiscal expansions can lead to strong negative wealth and crowding-out effects on consumption through
higher sovereign risk premia. Consequently, the optimal fiscal stance should be pro-cyclical and geared towards reducing the stock of government debt, but only when a country is relatively small. When countries are large, the sovereign debt crisis and ensuing fall in consumption and inflation will prompt the central bank to reduce the interest rate and thereby offset the rise in the sovereign risk premium, which calls for a more countercyclical fiscal stance. Finally, we showed how government spending multipliers are generally lower in the face of time-varying sovereign risk premia, especially if the central bank is unable to offset the rise in the sovereign risk premium (for instance, when the nominal policy rate is stuck at the zero lower bound).

Our results shed new light on previous notions about the effects (and optimal design) of fiscal policy under monetary union. In future work, we would like to extend our framework in order to examine how the effects of fiscal policy depend on other country characteristics (e.g. country openness), other frictions (e.g. wage rigidities, liquidity constraints and capital adjustment costs) and policy regimes (e.g. fiscal union and unconventional monetary policy). Endogenising the fiscal limit that gives rise to the sovereign risk premium would be helpful in quantifying some of the numerical results and exploring the role of policy uncertainty in more detail. Finally, a natural extension to the model would be to add additional fiscal instruments (e.g. fiscal transfers, consumption taxes and capital taxes) in order to examine which fiscal tool is most successful in stabilising the economy in both the presence and absence of sovereign risk premia.

4.A Linearisation

In this section, we derive a first-order Taylor approximation of the non-linear model outlined in Section 4.1 around a non-stochastic steady state. We assume that prices are fully flexible in steady state and that \( a = a^* = 1 \). Define variables with a hat as the percentage deviation of that variable from its steady-state level, and let variables without a time-index denote the steady-state level of the corresponding variable, such that \( \hat{x}_t = (x_t - x) / x \), for any generic
variable \( x_t \). The full linearised equations are then given by

\[
\begin{align*}
\varphi \hat{n}_t &= \hat{w}_t - \sigma \hat{c}_t - \mu \hat{\tau}_t, \\
\sigma \hat{c}_t &= \sigma E_t \hat{c}_{t+1} - \left( \hat{R}_{g,t} - E_t \hat{\pi}_{t+1} \right), \\
\sigma \hat{\tau}_t &= \sigma E_t \hat{\tau}_{t+1} - \left( \hat{\xi}_{h,t}^* + \hat{R}_{t}^* - E_t \hat{\pi}_{t+1}^* \right), \\
\hat{\xi}_{h,t}^* &= \chi_{h,1} \frac{f^*}{Y} (\hat{f}_t^* - \hat{y}_t) + \chi_{h,2} \frac{b}{y} (\hat{b}_t - \hat{\gamma}_t), \\
\hat{y}_t &= \left( 1 - \frac{g}{y} \right) \left( 1 - s \right) \hat{S}_t + \hat{\gamma} \hat{w}_t + \frac{g}{y} \hat{\gamma}_t, \\
\hat{n}_t &= \hat{y}_t - \hat{a}_t, \\
\hat{\pi}_{H,t} &= \beta E_t \hat{\pi}_{H,t+1} + \kappa \hat{\mu} c_t, \\
\hat{\mu} c_t &= \hat{w}_t - \hat{\alpha}_t + (1 - s) \hat{S}_t, \\
\hat{\gamma}_t - \frac{c}{y} \hat{c}_t - \frac{g}{y} \hat{\gamma}_t &= \frac{f^*}{Y} \left[ \frac{1}{\beta} \left( \hat{\xi}_{h,t-1}^* + R_{t-1}^* - \hat{\pi}_{t-1}^* + \hat{f}_{t-1}^* \right) - \hat{f}_t^* \right] + \frac{c}{y} (1 - s) \hat{S}_t, \\
\hat{b}_t &= \frac{1}{\beta} \left( \hat{R}_{g,t-1} - \hat{\pi}_t + \hat{b}_{t-1} \right) + \left( \frac{b}{y} \right)^{-1} \left[ \frac{g}{y} \hat{\gamma}_t - \tau (\hat{\pi}_t + \hat{\gamma}_t) + \left( \frac{\tau y - g}{y} \right) (1 - s) \hat{S}_t \right], \\
\hat{\pi}_t &= \frac{1}{\tau} \left[ \gamma_{h} \frac{b}{y} (\hat{b}_{t-1} - \hat{\gamma}_{t-1}) + \gamma_{y} \hat{\gamma}_{t-1} \right], \\
\hat{\gamma}_t &= \rho_g \hat{\gamma}_{t-1} + \varepsilon_{g,t}, \\
\hat{a}_t &= \rho_a \hat{a}_{t-1} + \varepsilon_{a,t}.
\end{align*}
\]
for the Home country, and

\[
\begin{align*}
\varphi^* n_t^* &= w_t^* - \sigma^* c_t^* - \mu^* \hat{\tau}_t^*, \\
\sigma^* c_t^* &= \sigma^* E_t \hat{c}_{t+1}^* - \left( \hat{R}_{g,t}^* - E_t \hat{\tau}_{t+1}^* \right), \\
\sigma^* \hat{c}_t^* &= \sigma^* E_t \hat{c}_{t+1}^* - \left( \hat{R}_t^* - E_t \hat{\tau}_{t+1}^* \right), \\
\hat{g}_t^* &= \left( 1 - \frac{g^*}{y^*} \right) \left( -s \hat{S}_t + \hat{c}_{W,t} \right) + \frac{g^*}{y^*} \hat{g}_t^*, \\
\hat{a}_t^* &= \hat{g}_t^* - \hat{a}_t^*, \\
\hat{\pi}_{F,t}^* &= \beta^* E_t \hat{\pi}_{F,t+1}^* + \kappa^* \hat{m} c_t^*, \\
\hat{m} c_t^* &= \hat{w}_t^* - \hat{a}_t^* - s \hat{S}_t, \\
\hat{b}_t^* &= \frac{1}{\beta} \left( \hat{R}_{g,t-1}^* - \hat{\pi}_t^* + \hat{b}_{t-1}^* \right) \\
&+ \left( \frac{b^*}{y^*} \right)^{-1} \left[ g^* \hat{g}_t^* - \tau^* (\hat{\tau}_t^* + \hat{g}_t^*) - \left( \frac{\tau^* y^* - g^*}{y^*} \right) s \hat{S}_t \right], \\
\hat{\tau}_t^* &= \frac{1}{\tau^*} \left[ \gamma_b \frac{b^*}{y^*} (\hat{b}_{t-1}^* - \hat{g}_{t-1}^*) + \gamma_y \hat{y}_{t-1} \right], \\
\hat{g}_t^* &= \rho_y \hat{g}_{t-1}^* + \varepsilon_{g,t}, \\
\hat{a}_t^* &= \rho_a \hat{a}_{t-1}^* + \varepsilon_{a,t},
\end{align*}
\]

for the Foreign country, where \( \kappa \equiv (1 - \theta^*) (1 - \theta^\beta) / \theta^*, \kappa^* \equiv (1 - \theta^*) (1 - \theta^* \beta^*) / \theta^*, \mu = \mu^* \equiv \tau^\prime / (1 - \tau^\prime) \) and \( \hat{c}_{W,t} = s \hat{c}_t + (1 - s) \hat{c}_t^* \). Linearisation of the monetary policy rule (4.21) yields

\[
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) (\alpha_x \hat{\pi}_{W,t} + \alpha_y \hat{y}_{W,t}),
\]

where \( \hat{\pi}_{W,t} = s \hat{\pi}_t + (1 - s) \hat{\pi}_t^* \) and \( \hat{y}_{W,t} = s \hat{y}_t + (1 - s) \hat{y}_t^* \). Finally, the bond rate of the Foreign government bond is determined by \( \hat{R}_{g,t}^* = \hat{R}_t \), whereas the government bond rate in Home is determined by \( \hat{R}_{g,t} = \hat{\Xi}_{g,t} + \hat{R}_t \), where the sovereign risk premium function \( \hat{\Xi}_{g,t} \) has linear form \( \hat{\Xi}_{g,t} = \chi_y (b/y) (\hat{b}_t - \hat{y}_t) \).

### 4.B Flexible price equilibrium

Under flexible prices, all firms set the same prices, i.e. \( P_{H,t} = P_{H,t} \). Therefore, the optimal price-setting condition collapses to \( 1/M = mc_t \). As in Benigno (2004), we assume that the fiscal authority sets the tax rate in the flexible price
4.B Flexible price equilibrium

equilibrium such that monopolistic distortions are neutralised, which can be accomplished by setting \( \tau = \bar{\tau} = 1 - \mathcal{M} \). Using these conditions, together with the labour supply condition implied by the household’s optimal intratemporal condition (4.4), and the aggregate production function \( y_t = a_t n_t \) (which holds for all firms under flexible prices), we can write:

\[
\frac{1}{1 - \bar{\tau}_t} = m \bar{c}_t = S_t^{1-s} \bar{y}_t^{(1+\varphi)} \frac{1}{\bar{c}_t^{1-\sigma}} \frac{1}{1 - \bar{\tau}_t},
\]

where a tilde over a variable indicates a variable’s flexible-price (or ‘natural’) level. Linearise this condition to obtain the Home natural output level (assume \( g_t = g \) for all \( t \)):

\[
\bar{y}_t = \left( \frac{1 + \varphi}{\varphi} \right) \hat{a}_t - \frac{\sigma}{\varphi} \bar{c}_t - \left( \frac{1 - s}{\varphi} \right) \bar{S}_t. \tag{4.63}
\]

Eliminate \( \bar{y}_t \) from the previous equation using the goods market clearing condition to obtain:

\[
\left( 1 - \frac{g}{y} \right) \left[ (1 - s) \bar{S}_t + \bar{c}_{W,t} \right] = \left( \frac{1 + \varphi}{\varphi} \right) \hat{a}_t - \frac{\sigma}{\varphi} \bar{c}_t - \left( \frac{1 - s}{\varphi} \right) \bar{S}_t.
\]

Rewrite to obtain the Home natural consumption level:

\[
-\sigma \bar{C}_t = (1 - s) \bar{S}_t + \varphi \left( 1 - \frac{g}{y} \right) \left[ (1 - s) \bar{S}_t + \bar{c}_{W,t} \right] - (1 + \varphi) \hat{a}_t. \tag{4.64}
\]

Similarly, Foreign natural output and consumption are given by

\[
\bar{y}_t^* = \left( \frac{1 + \varphi}{\varphi} \right) \hat{a}_t^* - \frac{\sigma}{\varphi} \bar{c}_t^* + \frac{s}{\varphi} \bar{S}_t, \tag{4.65}
\]

\[-\sigma \bar{c}_t^* = -s \bar{S}_t + \varphi \left( 1 - \frac{g^*}{y^*} \right) \left( -s \bar{S}_t + \bar{c}_{W,t} \right) - (1 + \varphi) \hat{a}_t^*. \tag{4.66}
\]

Take a weighted average of the expressions for \( \bar{c}_t \) and \( \bar{c}_t^* \) (with weights \( s \) and \( 1 - s \)) to obtain an expression for natural aggregate consumption (assume that \( g/y = g^*/y^* \)):

\[
\bar{c}_{W,t} = \left[ \frac{1 + \varphi}{\varphi (1 - g/y) + \sigma} \right] \hat{a}_{W,t}. \tag{4.67}
\]
where \( \hat{a}_{W,t} \equiv s\hat{a}_t + (1-s)\hat{a}^*_t \).

Finally, from the balance of payments condition, we can derive an expression for the natural terms of trade:

\[
\tilde{S}_t = \left[ \frac{1}{(1 - \frac{g}{y})(1 - s)} \right] \left( \tilde{y}_t - \frac{c}{y} \tilde{c}_t \right).
\] (4.68)

### 4.C The utility-based welfare criterion

In this section, we briefly derive the welfare criterion used in the model, which is based on the household’s period utility function denoted by

\[
W_t = \frac{c_{1-\sigma}^1 - \sigma}{1 - \sigma} - \frac{n_{1+\varphi}^1}{1 + \varphi}.
\] (4.69)

Following Woodford (2003), we take a second-order Taylor-approximation of \( W_t \):

\[
W_t \simeq W_{c_0} \left\{ \hat{c}_t + \left( \frac{1 - \sigma}{2} \right) \hat{c}_t^2 - \frac{W_{n_0 n_t}}{W_{c_0}} \left[ \hat{n}_t + \left( \frac{1 + \varphi}{2} \right) \hat{n}_t^2 \right] \right\} + O \left( \| \zeta \|^3 \right).
\] (4.70)

where terms that are of order higher than 2 are collected in \( O \left( \| \zeta \|^3 \right) \). In order to substitute out \( \hat{n}_t \), we make use of the labour market clearing condition, \( n_t = y_t/a_t D_t \), where \( D_t \equiv \int_0^s (P_{H,t}(h) / P_{H,t})^{-\epsilon} dh \) is a measure of price-dispersion. Linearising this condition yields

\[
\hat{n}_t \simeq \hat{y}_t - \hat{a}_t + \hat{D}_t + O \left( \| \zeta \|^3 \right).
\] (4.71)

As in Woodford (2003), \( D_t \simeq \epsilon/2 \text{var} \hat{P}_{h,t} + \text{t.i.p.} + O \left( \| \zeta \|^3 \right) \), where terms independent of policy (such as constants and shocks) are collected in ‘t.i.p.’. Taking the discounted sum for all \( t \geq 0 \), we obtain

\[
\sum_{k=0}^{\infty} \beta^k D_{t+k} \simeq \frac{\epsilon}{2\kappa} \sum_{k=0}^{\infty} \hat{\pi}_{H,t+k}^2 + \text{t.i.p.} + O \left( \| \zeta \|^3 \right).
\] (4.72)

We can then use (4.71) to substitute out \( \hat{N}_t \) in (4.70).²⁰

²⁰The linear terms drop out, since we linearise around the efficient steady state.
Finally, expressing in gap terms and then taking the expected discounted sum using, (4.72), we obtain Equation (4.37) in the main text (for $\sigma = 1$).

### 4.D Additional graphs

Figure 4.9: Government spending multiplier as a function of country size ($s$) for $\rho_R \rightarrow 1$ and $\gamma_y = 0.115$

*Notes: The figure shows the impact output response in Home following a positive government spending shock (of 1% of output) as a function of $s$, which denotes the size of Home. Here, we assume very rigid interest rates, i.e. $\rho_R \rightarrow 1$ (see Equation [4.21]) and an a-cyclical fiscal stance $\gamma_y = 0$.)*
4 Fiscal policy and sovereign risk premia under monetary union