Resource extraction and the Green Paradox: Accounting for political economy issues and climate policies in a heterogeneous world

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Chapter 3

Double Limit Pricing *

3.1 Introduction

Most of globally traded fossil fuels, in particular oil, is exported by a small group of countries. Most of those countries, in turn, constitute a monopolistic cartel called OPEC. Another group of countries does not have a substantial oil production on its own and depends hence on fossil fuels imports. This group, which includes most of the OECD, has already implemented measures to reduce carbon emissions or it strives to do so in the near future. A third group of countries, also fossil fuel importers, does not employ climate policies and does not take part in any international agreements to limit carbon emissions from fossil fuel consumption.

Given the existence of the unregulated importer group and the monopolistic fossil fuel supply, what are the effects of a *unilateral* implementation of climate policies in the regulated importer group? Can climate policies be effective in such a setting at all? Can a Green Paradox occur? And does the regulated importer group lose welfare by installing or tightening its climate policies?

In this paper we tackle these questions and contribute to the Green Paradox literature by combining the idea of heterogeneity in climate policies with monopolistic resource production.

The literature on the so called "Green Paradox" has been growing incessantly since Sinn's (Sinn, 2008) seminal paper. Sinn (2008) has redirected the theoretical analysis of climate

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policy effects to the supply side of the market and highlighted unintended and adverse responses of resource producers to policy changes. The step away from focussing on the demand side only was a step towards a more complete understanding of market reactions to climate policies and their economic and environmental effects. Yet, most of the research that has emerged so far, including Sinn (2008), has studied Green Paradox effects in a competitive resource production environment. Fossil fuel markets, especially the oil market, however, are far from being competitive: the world's largest oil producer, OPEC, is a major player and can be best described as a monopolistic cartel, which uses resource quantities as its instrument. Nevertheless, also a monopolist needs to account for the potential existence of backstops, i.e., close substitutes for fossil fuel, the supply of which is not constrained by a finite resource stock. Examples are renewables such as wind and solar energy, although substitution possibilities are restricted for some uses (see Michielsen, 2014a). According to Salant (1977), Solow (1974) might be considered to be the first to pay attention to the existence of a backstop technology and its consequences on price formation in competitive resource markets. Stiglitz and Dasgupta (1981) have extended the theory to the case of monopolistic fossil fuel supply. However, they have ignored or overlooked the fact that in such a setting limit pricing is part of the market outcome. The first full analysis of limit pricing was given by Hoel (1978), followed by Salant (1977).[1] Hoel (1978) compares three cases of which only two are relevant in our framework: first, monopolistic supply in the absence of a backstop technology, and, second, monopolistic supply of fossil fuel in the presence of a backstop, which is supplied competitively by other firms than the monopolist. Hoel shows that, in the presence of a backstop and given linear fossil fuel extraction and linear backstop production costs, there always exists a phase with limit pricing. Using an example with isoelastic demand and zero extraction costs, Hoel (1978) also shows that the initial fossil fuel price is higher in the presence of a backstop than without it. The backstop reduces the future market price of fossil fuel and the monopolist seeks to compensate his consequent profit loss by charging a higher price initially. Salant (1977) examines the case of strictly convex extraction costs and concludes that the optimal pricing strategy includes a limit-pricing phase in this setting as well. Gilbert and Goldman (1978) set up a model where the monopolist extracts at constant marginal costs which are smaller than the minimum average cost of the backstop, and where the price elasticity of demand exceeds unity. They prove that the initial price is higher in the presence of a

[1] Hoel's paper was submitted in November 1976, the date of Salant's working paper version was August 1977.

perfect substitute than without the backstop being in place. Moreover, they show that the existence of a backstop technology may result in a welfare loss as compared to the case of an unconstrained monopoly. Hoel (1983) generalizes the analysis of Gilbert and Goldman (1978) and considers the effects of changing future conditions on present extraction.

The authors mentioned have neither considered the effects of giving a backstop subsidy, nor the effects of imposing a carbon tax. The reason is most likely that climate policy was not a focus of attention in those years. The main concern was with the modelling of the oil market. The situation is different nowadays with the climate change issues at hand. The Green Paradox literature, for instance, revolves around suboptimal policies in conjunction with inter- and intratemporal carbon leakage. Van der Ploeg and Withagen (2015) provide a comprehensive literature survey. Although the literature on Green Paradoxes is abundant, there are still substantial gaps.

An assumption usually made is that the resource consumers act together and introduce *global* policies or *unified* strategies. Needless to say, this is a simplifying assumption and not very realistic in view of the decades of international struggles concerning not only global climate policies but also binding agreements on emission goals. Furthermore, theoretical and empirical literature on carbon leakage (Copeland and Taylor, 2000; Aichele and Felbermayr, 2011) tells us that unilateral environmental policies might not have the desired global outcomes. Hence, the analysis of the effects of *global* climate policies is not only unrealistic, but the researcher might also assess the effectiveness of climate policies very differently if she focuses on *global* policies instead of accounting for possible interactions in the presence of *heterogeneous* or *unilateral* measures. Hoel (2011) addresses this point by analyzing the effect of climate policies in a two-region model where the regions differ in the levels of their carbon taxes and backstop subsidies. In his settting, he finds differences in the effects of changes in climate policies on the emission paths as compared to those in a common model with identical countries. Fischer and Salant (2014), building on Hoel's paper, introduce heterogeneous extraction costs and exogenously decreasing backstop production costs into Hoel's framework, and analyze the effects of different climate policies in a regulated region (as opposed to an unregulated region with no climate policies) on cumulative carbon emissions. Ryszka and Withagen (2014) also extend Hoel's framework by accounting for heterogeneous extraction costs, and study the effects of different climate policies on the extraction path and on the welfare of the different regions. They calibrate the model and find that forming a 'climate coalition' and introducing carbon taxation is beneficial for the largest fossil fuel-using regions, both regarding climate costs and with respect to their terms of trade. Eichner and Pethig (2011) investigate the effects of unilateral climate policies in a two-period general equilibrium model with one resource exporter and two resource importers. They find that the Green Paradox may be weakened or even reversed due to general equilibrium effects and intertemporal carbon leakage.

The aforementioned studies model a competitive world. Some papers address the strategic interaction between suppliers of fossil fuel and fossil fuel users. The idea of policy heterogeneity can be found in game theoretic literature of resource extraction: Strand (2013) and Karp et al. (2015) employ a game theoretical setting in which a resource importer bloc and a resource importing fringe face a group of resource exporters. Strand (2013) compares a carbon tax and a cap-and-trade scheme in order to identify the optimal policy strategies of both players in a static environment, whereas Karp et al. (2015) study a dynamic game where the players use either taxes or quotas to exercise market power in the presence of a group of non-strategic developing countries. Accounting for climate damages stemming from carbon emission accumulation, they examine a Markov perfect equilibrium outcome under the four combinations of trade policies. Yet, they disregard the finiteness of the natural resource and make use of specific linear demand functions. They lack the focus on monopolistic limit pricing and do not examine policy-induced Green Paradox effects. Kagan et al. (2015), in contrast, investigate oil extraction and carbon accumulation for various production function specifications for both open and closed loop Nash equilibria, and compare these with the efficient and competitive outcomes. Their paper is based on Liski and Tahvonen (2004) who find Markov perfect strategies for coalitions of resource importing and exporting countries. Similar to us, they distinguish between Pigouvian and trade-policy components of a carbon tax and find that the policy-implementing countries benefit at the expense of the exporting cartel. However, they do not take into account the existence of a backstop technology.

The current paper combines the idea of heterogeneity in climate policies with the fact of monopolistic resource production and the existence of a backstop technology, which gives rise to the existence of limit pricing. To the best of our knowledge, it is the first to unravel the effects of unilateral climate policies in a two-region setting and in the presence of monopolistic limit pricing. Literature on the effects climate policy in a limit pricing framework is scarce. Recently, De Sa and Daubanes (2014) consider inelastic demand for oil, implying that the monopolist will choose for limit pricing throughout. As a result, carbon taxes are ineffective and backstop subsidies increase resource extraction. Our analysis is complementary to theirs. We show that, even without imposing inelastic demand for oil, limit pricing is more important in our framework than in existing models with monopolistic resource supply, due to heterogeneity of climate policies. Moreover, in contrast to De Sa and Daubanes (2014), we account for climate costs and conduct a welfare analysis.

We consider a monopolistic resource producer facing constant unit extraction costs and the presence of a costly backstop with constant production costs. Resource demand comes from region A, which employs both a carbon tax and a backstop subsidy, and from region B, which does not have any policies in place. Two frameworks are considered: one with speculators on the market, where the monopolist is constrained to a continuous price, and one without speculators, where the monopolist is free to choose a discontinuous price path. A first finding is that in the latter case, it is optimal for the monopolist to let the price jump upwards when demand from the regulated region drops to zero. Second, we find that in the cases with and without speculators on the market, the resource extraction paths may contain two limit-pricing phases: one just before the demand from region A vanishes due to climate policies, and one just before the depletion of the resource. Accordingly, in a world with heterogeneous climate policies, it becomes even more important to take the effects of limit pricing into account. Third, we show that the presence of speculators is beneficial for the climate: initial extraction is lower and the overall resource extraction phase is longer than in the case without speculators, reducing the present value of climate costs. Fourth, a tightening of climate policies does not result in a Weak Green Paradox: initial resource consumption falls in both regimes. Climate costs might still rise as intermediate extraction goes up and, in the absence of speculators, as the overall resource extraction phase is shortened. Finally, our numerical welfare analysis shows that the different climate policy changes that we investigate have varying effects on region A's non-green welfare. In the presence of speculators, however, region A is consistently worse off regarding its non-green welfare because the resource producer sells more resources to the unregulated region B than in the absence of speculators. Furthermore, climate policy tightening decreases region A's non-green welfare as the monopolist shifts more of its resource supply to the unregulated region, an effect that we refer to as (intertemporal) carbon leakage.

The remainder of the paper is structured as follows: in order to facilitate a better grasp of the paper's idea and analysis, we first present a single-market model with monopolistic limit pricing in Section 3.2, characterize the durations of the limit-pricing and no limit-pricing phases, and analyze the Green Paradox and welfare effects of policy changes using HARA utility functions. Section 3.3 extends the single-market model to a two-region model in which one region employs climate policies, whereas the other stays idle. Sections 3.3.3 and 3.3.4 give the characterizations for the continuous and discontinuous price equilibria respectively, whereas Section 3.3.5 compares the particular equilibrium outcomes. Comparative statics and welfare analyses are conducted in Section 3.3.6 and Section 3.3.7 respectively. We summarize our findings and conclude in Section 3.4.

3.2 A Single Market

3.2.1 Equilibrium in the Single-Market Case

We first consider limit pricing in case of a single market for energy. There are two energy sources, non-polluting renewables and fossil fuel. They are assumed to be perfect substitutes in consumption. Renewable energy is produced by a carbon-free backstop technology that is not hampered by stock limitations. It can be competitively produced at a constant cost $b > 0$, so that also its producer price equals b. Fossil fuel is supplied by a monopolist at a rate $q(t)$ at instant of time t. Consumption of renewables is subsidized at rate $\sigma \geqslant 0$, whereas consumption of fossil fuel is taxed at rate $\tau \geqslant 0$. We assume that the subsidy and the tax rate are constant over time. The monopolist faces a unit extraction cost $k \geqslant 0$, which is not prohibitively high: $k < \hat{b}$ with $\hat{b} \equiv b - \sigma - \tau$. It is convenient to consider fossil fuel supply as the monopolist's policy instrument. The corresponding producer price is denoted by p. We define $\hat{q}$ as total energy demand if the consumer price is $b - \sigma$.[2] If the monopolist supplies more than $\hat{q}$, the consumer price $p + \tau$ is smaller than $b - \sigma$, and only fossil fuel is demanded. If it sets $q \leqslant \hat{q}$, the consumer price is $b - \sigma$. If $q = \hat{q}$, the monopolist serves the entire market, whereas, if $q < \hat{q}$, a part of the demand for energy is met by renewables. Clearly, the monopolist will never supply an amount $q < \hat{q}$ over a nondegenerate period of time, because profits in such an interval of time can be increased by concentrating supply at a level $\hat{q}$ at the beginning of the interval and leave the supply during the later part of the interval to renewables. Such a policy is feasible as the monopolist is able to undercut the suppliers of renewables and keep them out of the market. This is what is called limit pricing. It occurs if $p(t) = \hat{b}$ and $q(t) > 0$.

[2]We assume stationary demand in our model.

3.2.2 The Monopolist's Problem

The monopolist is therefore facing the problem of designing an extraction path over an endogenous period of time $[0, T]$, such that along this interval the resource stock is exhausted and the extraction rate is large enough:[3]

$$\Lambda(S_0, b, \sigma, \tau) = \max_{q(t),T} \int_0^T e^{-rt}(p(q(t)) - k)q(t)dt, \tag{3.1}$$

subject to $\dot{S}(t) = -q(t)$, $S(0) = S_0$, $S(T) = 0$, and $q(t) \geqslant \hat{q}$, where $p(q)$ is the producer price at supply q. Note that the maximization also takes place with respect to T: the time horizon is endogenous. We assume the net revenue $(p(q) - k)q$ to be strictly concave in q if $q > \hat{q}$, implying $2p'(q) + qp''(q) < 0$ if $q > \hat{q}$. The Hamiltonian and the Lagrangian of the problem read:

$$\mathcal{H}(q, \mu, t) = e^{-rt}(p(q) - k)q - \lambda q,$$
$$\mathcal{L}(q, \lambda, \mu, t) = e^{-rt}(p(q) - k)q - \lambda q + \mu(q - \hat{q}),$$

where λ is the shadow price of the resource stock and μ is the Lagrange multiplier corresponding with the constraint on the extraction rate. The shadow price of the resource stock is a constant, since the stock itself does not enter the Lagrangian. Necessary conditions for the maximization of the Lagrangian with respect to the extraction rate read:

$$e^{-rt}[p'(q(t))q(t) + p(q(t)) - k] + \mu(t) = \lambda, \tag{3.2a}$$
$$\mu(t) \geqslant 0, \quad \mu(t)(q(t) - \hat{q}) = 0. \tag{3.2b}$$

Moreover, the Hamiltonian vanishes at T. Since $q(T) \geqslant \hat{q} > 0$, we have at the end of the extraction phase:

$$e^{-rT}(p(q(T)) - k) = \lambda. \tag{3.3}$$

The following lemma characterizes the optimum. It states that, for a large enough initial resource stock, the consumer price of fossil fuel is initially below the consumer price of renewables. Then a final phase with limit pricing follows. The first interval of time is degenerate if the initial resource stock is small. In order to specify when the resource stock

[3]The analysis in this section is similar to the second monopoly case in Hoel (1978).

is small or large, we define $\hat{T}$ and $\hat{S}_0$ by:

$$p'(\hat{q})\hat{q} + p(\hat{q}) - k = (p(\hat{q}) - k)e^{-r\hat{T}}, \tag{3.4a}$$

$$\hat{T}\hat{q} = \hat{S}_0. \tag{3.4b}$$

Here $\hat{S}_0$ is the initial resource stock which induces the monopolist to limit price from the start.

Lemma 1 *Suppose $S_0 \leqslant \hat{S}_0$. Define $T = S_0/\hat{q} \leqslant \hat{T}$. Then it is optimal for the monopolist to set $q(t) = \hat{q}$ for all $T \geqslant t \geqslant 0$. Suppose $S_0 > \hat{S}_0$. Then there exist $T_2 > T_1 > 0$ such that it is optimal for the monopolist to set $q(t) > \hat{q}$ for all $T_1 > t \geqslant 0$ and $q(t) = \hat{q}$ for all $T_2 \geqslant t \geqslant T_1$ such that $S(T_2) = 0$.*

Proof. Evaluate equation (3.2a) at the final instant of extraction T. Then it follows from (3.3) that $\mu(T) > 0$ so that (3.2b) requires $q(T) = \hat{q}$. Hence, there is always a final phase with limit pricing. Suppose $S_0 = \hat{S}_0$. Take $\lambda = (\hat{b} - k)e^{-r\hat{T}}$ and $q(t) = \hat{q}$ for all $\hat{T} \geqslant t \geqslant 0$. Then it follows from (3.2a) that $\mu(0) = 0$ and $\mu(t) > 0$ for all $\hat{T} \geqslant t \geqslant 0$. Hence, all necessary conditions are satisfied along the proposed program, so that the optimum has been identified. If $S_0 < \hat{S}_0$ we take $\lambda = (\hat{b} - k)e^{-rT}$ and $q(t) = \hat{q}$ for all $T \geqslant t \geqslant 0$ with $T = S_0/\hat{q}$. Then, $\mu(0) > 0$ so that we should start with limit pricing as well. Finally, if $S_0 > \hat{S}_0$ then it cannot be optimal to start with limit pricing. This can be seen as follows. If $q(0) = \hat{q}$, it follows from (3.2a), (3.3) and (3.4a)-(3.4b) that:

$$e^{-r\hat{T}}(\hat{b} - k) + \mu(0) = e^{-rT}(\hat{b} - k), \tag{3.5}$$

where we have used the fact that at the final instant of time $q(T) = \hat{q}$. By definition, $\hat{T}$ is the fastest depletion time in the presence of a small resource stock, so that $T > \hat{T}$. This yields a contradiction since μ is bound to be nonnegative. □

In general, the optimum consists of three phases: phase 1, from time 0 till T_1, is characterized by a producer price below $\hat{b}$, phase 2, from T_1 till T_2, exhibits limit pricing, and phase 3, after T_2, exhibits backstop use only. As stated in Lemma 1, the first phase may be degenerate. It is also clear now that we can express the shadow price λ as:

$$\lambda = (\hat{b} - k)e^{-rT_2}. \tag{3.6}$$

The interpretation is straightforward. The shadow price λ is the present shadow value of a marginal increase of the initial stock. The revenues of selling an additional amount of fossil fuel at the instant of time T_2 are $\hat{b} - k$, which is $(\hat{b} - k)e^{-rT_2}$ in present value terms. Also note that for $t \uparrow T_1$ we can use (3.2a) and (3.6) to write:

$$e^{-rT_1}[p'(\hat{q})\hat{q} + p(\hat{q}) - k] = (\hat{b} - k)e^{-rT_2}. \tag{3.7}$$

Hence the length of the limit pricing interval is independent of the initial resource stock, as long as there is an initial interval of time with a price below the limit price, i.e., if $S_0 > \hat{S}_0$.

3.2.3 Comparative Statics

The next two propositions state some comparative statics results. They deal with the effects of changes in the tax rate, the subsidy, or the renewables production cost. More importantly, they show that the Weak Green Paradox does not occur as a consequence of climate policy tightening. Define the inverse of the price elasticity of demand $\eta(q) \equiv -p'(q)q/p(q)$.[4] This implies:

$$T_2 - T_1 = \frac{1}{r} \ln \left(\frac{\hat{b} - k}{(1 - \eta(\hat{q}))\hat{b} - k} \right). \tag{3.8}$$

Define also $\Lambda(S_0, b, \sigma, \tau)$ as the maximum value of profits given the parameters (S_0, b, σ, τ). With respect to the changes in the backstop cost b we abstract from modelling important processes such as R&D costs. A direct decrease in b might hence seem to be a poor approximation of the processes of technological innovation which result in lower backstop costs. Yet, other factors such as an (incremental) increase in the overall technological level or a (random) technological breakthrough might result in a drop in b without direct R&D investment costs. The first proposition hence deals with a decrease of the cost of renewables and an increase of the subsidy.

Proposition 3 *Provided that $S_0 > \hat{S}_0$, a decrease in b or an increase in the subsidy σ, keeping $b - \sigma > k$,*

(i) decreases the producer price $p(T_1)$ and the consumer price $b - \sigma$ at T_1;

(ii) increases the extraction rate during the limit pricing phase $\hat{q}$;

[4] In our analysis we assume $\eta(q) < 1$, i.e., the price elasticity of demand to be larger than one, which ensures that $T_2 - T_1 > 0$. If $\eta(q) \geqslant 1$, we obtain a regime with only limit pricing, as described in De Sa and Daubanes (2014)

(iii) increases (decreases) the duration of the limit pricing phase $T_2 - T_1$ *if and only if* $\eta(\hat{q})k > (<)(\hat{b} - k)\hat{b}\eta'(\hat{q})/p'(\hat{q})$;

(iv) decreases initial resource extraction $q(0)$;

(v) decreases the time of exhaustion T_2.

Proof. The consumer price in the limit pricing phase equals $b - \sigma$ and goes down. Hence, equilibrium extraction during the limit pricing phase goes up. The producer price $\hat{b}$ falls. This proves parts (i) and (ii). The derivative of expression (3.8) with respect to $\hat{b}$ reads:

$$\frac{d(T_2 - T_1)}{d\hat{b}} = \frac{1}{r}\frac{(1-\eta(\hat{q}))\hat{b} - k}{\hat{b} - k}\frac{(\hat{b} - k)\hat{b}\eta'(\hat{q})/p'(\hat{q}) - \eta(\hat{q})k}{\left((1-\eta(\hat{q}))\hat{b} - k\right)^2}, \tag{3.9}$$

from which the result in part (iii) follows since in order to have $T_2 - T_1 > 0$ we need $\hat{b} - k > (1-\eta(\hat{q}))\hat{b} - k > 0$. To prove (iv) note that the time derivative of the Hamiltonian is given by

$$\dot{\mathcal{H}} = \mathcal{H}_S\dot{S} + \mathcal{H}_\mu\dot{\lambda} + \mathcal{H}_q\dot{q} + \frac{\partial\mathcal{H}}{\partial t} = \mu(t)p'(q(t))\dot{q} + \frac{\partial\mathcal{H}}{\partial t} = \frac{\partial\mathcal{H}}{\partial t}, \tag{3.10}$$

where the second and third equality use $\dot{\lambda} = 0$ and (3.2a), and (3.2b), respectively. Integration of (3.10) over time gives

$$\int_0^T e^{-rt}(p(q) - k)q\,dt = \frac{\mathcal{H}(0) - \mathcal{H}(T_2)}{r}. \tag{3.11}$$

By imposing transversality condition $\mathcal{H}(T_2) = 0$ in (3.11) and by using (3.1), we find

$$\Lambda(S_0, b, \sigma, \tau) = \frac{\mathcal{H}(0)}{r}, \tag{3.12}$$

where $\mathcal{H}(0)$ is the short-hand notation for the Hamiltonian evaluated at time 0. We have $\mu(0) = 0$ if $S_0 > \hat{S}_0$. Hence, we substitute (3.2a) into the Hamiltonian to get

$$\mathcal{H}(0) = -p'(q(0))q^2(0). \tag{3.13}$$

An increase in b, or a decrease in σ lowers $\hat{q}$ and thus relaxes the constraint that the monopolist faces in problem (3.1) (which is always binding at some point in time). Hence $d\Lambda(S_0, b, \sigma, \tau)/db > 0$ and $d\Lambda(S_0, b, \sigma, \tau)/d\sigma < 0$, so that (3.12) gives $d\mathcal{H}(0)/db > 0$ and

$d\mathcal{H}(0)/d\sigma < 0$. Moreover, from the strict concavity of $(p(q) - k)q$ in q, (3.13) implies $d\mathcal{H}(0)/dq(0) > 0$. Therefore, we get $dq(0)/db > 0$ and $dq(0)/d\sigma < 0$. Finally, (3.2a) with $\mu(0) = 0$ gives $d\lambda/dq(0) = [2p'(q(0)) + q(0)p''(q(0))] - d\tau/dq(0)$, the first term of which is negative due to strict concavity of $(p(q)-k)q$ in q. Substituting this result in (3.3), keeping $d\tau = 0$ we find $dT_2/db > 0$ and $dT_2/d\sigma < 0$, which proves part (v). □

The second proposition characterizes the effects of an increase in the carbon tax, which, in fact, is a fossil fuel tax, since we assume that the backstop technology is carbon-free.[5]

Proposition 4 *Provided that $S_0 > \hat{S}_0$, an increase in the fossil fuel tax τ*

(i) decreases the producer price at $p(T_1)$, but does not affect the consumer price at T_1, $p(T_1) + \tau = b - \sigma$;

(ii) does not affect the extraction rate during the limit pricing regime $\hat{q}$;

(iii) increases the duration of the limit pricing regime $T_2 - T_1$;

(iv) decreases initial resource extraction $q(0)$.

Proof. The consumer price at T_1 equals $p(T_1) + \tau = b - \sigma$. Since the consumer price of renewables does not change, supply in the limit pricing phase also does not change. Moreover, the producer price drops at T_1 by the same amount as the increase of the tax. The partial derivative of (3.8) with respect to $\hat{b}$ reads

$$\frac{\partial(T_2 - T_1)}{\partial\hat{b}} = -\frac{1}{r}\frac{(1-\eta(\hat{q}))\hat{b} - k}{\hat{b} - k}\frac{\eta(\hat{q})k}{\left((1-\eta(\hat{q}))\hat{b} - k\right)^2}, \tag{3.14}$$

from which the result in part (iii) follows since $\hat{q}$ does not depend on τ and because in order to have $T_2 - T_1 > 0$ we need $\hat{b} - k > (1 - \eta(\hat{q}))\hat{b} - k > 0$. The proof of part (iv) is similar to the proof of part (iv) of Proposition 3. □

The fourth result in Proposition 3, that the initial extration rate decreases as the renewables cost increases, has been found before. However, analyzing the effect with regard to the

[5]In reality renewable energy sources may cause (less, more or equal) carbon emissions than fossil fuels and hence also be subject to carbon taxation. In the numerical exercises we find that taxes levied on the backstop tend to increase initial resource extraction and to prolong T_1 while shortening the limit pricing phase.

subsidy and the tax is novel. These results are of relevance for the incidence of the so called Green Paradox (Sinn, 2008, 2012). A *Weak* Green Paradox is said to occur if the initial emissions of carbon dioxide increase as a result of climate policies (e.g., the introduction of a subsidy for renewable energy), whereas a *Strong* Green Paradox materializes if the present discounted value of climate damages increases (Gerlagh, 2011). The Green Paradox has been predominantly studied in the perfect competition case in the literature. In that framework, a higher subsidy for renewables results in more initial supply and consumption of fossil fuel, implying the occurrence of a Weak Green Paradox. In contrast, we have shown that a higher renewables subsidy leads to lower short-term fossil fuel supply and consumption levels in the case of a resource market monopoly. Consequently, a Weak Green Paradox does not materialize. The intuition is simple: More resources will be demanded during the future limit pricing phase, as a result of a combination of effects (ii) and (iii) of Proposition 3. Therefore, fewer fossil fuels are available for extraction during the first phase, causing the initial supply of fossil fuels to fall.

In the perfect competition framework, and in the presence of constant extraction costs, a higher backstop subsidy typically also causes a *Strong* Green Paradox, because extraction is increased at any point in time until exhaustion of the stock of fossil fuels. The same amount of fossil fuels is hence extracted over a smaller time horizon. Indeed, we have shown that the resource stock will be depleted faster (i.e., T_2 goes down) upon a higher renewables subsidy in the monopolistic production framework as well. Yet, the occurrence of a Strong Green Paradox depends on the specification of the climate damage function, as fossil fuel use decreases initially.

Several of the results derived above can conveniently be illustrated for the class of HARA utility functions, given by

$$U(q+x) = \frac{1-\varphi}{\varphi}\left[\left(\frac{\psi(q+x)}{1-\varphi}+\chi\right)^{\varphi} - \chi^{\varphi}\right], \tag{3.15}$$

where x denotes consumption of renewables, $\varphi > 0$, $\psi > 0$, $\chi \geqslant 0$, and $\zeta \geqslant 0$. The corresponding inverse demand function is:

$$p(q+x) = \psi\left(\frac{\psi(q+x)}{1-\varphi}+\chi\right)^{\varphi-1} - \tau.$$

We have $p(\hat{q}) = \hat{b}$ and we take $x = 0$. Moreover,

$$T_2 - T_1 = \frac{1}{r} \ln \left(\frac{\hat{b} - k}{\hat{b} - k - (1-\varphi)(b - \sigma - \psi\chi(\frac{b-\sigma}{\psi})^{\frac{\varphi-2}{\varphi-1}})} \right).$$

The sign of the derivative of the right-hand side with respect to $b - \sigma$ equals the sign of:

$$-\psi\chi \left(\frac{b-\sigma}{\psi} \right)^{\frac{\varphi-2}{\varphi-1}} - (\tau + k) \left(1 - \varphi + (\varphi - 2)\chi(\frac{b-\sigma}{\psi})^{\frac{-1}{\varphi-1}} \right).$$

The first term is negative. If $(1 - \varphi + (\varphi - 2)\chi \left(\frac{b-\sigma}{\psi}\right)^{\frac{-1}{\varphi-1}}) \geqslant 0$ the entire expression is negative. Otherwise, the second term is positive for $\tau + k = b - \sigma$. But then the expression boils down to:

$$(1-\varphi)\psi \left(\frac{b-\sigma}{\psi} \right) \left\{ \chi \left(\frac{b-\sigma}{\psi} \right)^{\frac{-1}{\varphi-1}} - 1 \right\},$$

which is definitely negative since $\hat{q} = \frac{1-\varphi}{\psi} \left\{ \left(\frac{b-\sigma}{\psi} \right)^{\frac{1}{\varphi-1}} - \chi \right\} > 0$. Hence, for HARA utility functions the limit pricing phase becomes longer upon a decrease in the cost of renewables b or an increase in the subsidy σ.

An example of a non-HARA utility function is:

$$U(q) = \chi q + \frac{q^{1-\varphi}}{1-\varphi}.$$

Following the same approach as before we can show that the sign of the derivative of the length of the limit pricing phase is given by the sign of:

$$\varphi(\chi - \tau - k),$$

which is positive for small enough $\tau + k$. Hence, the effect of a larger subsidy and a lower production cost on the limit pricing phase is ambiguous, as suggested by part (iii) of Proposition 3.

3.2.4 Green Paradox and Welfare Effects

Effects of policy changes on the consumers' welfare are quite complex. To begin with, we have to distinguish between a *green* and a *non-green* welfare component: green welfare encompasses the climate costs, whereas non-green welfare denotes the consumers' utility from resource consumption. With separability, *overall* welfare denotes the sum of both the green and non-green welfare components. For our numerical exercises instantaneous social utility from resource consumption is given by equation (3.15). It is assumed that taxes are refunded to consumers in a lump sum way and that the backstop subsidy is also financed by imposing a lump sum tax on the consumers. Another welfare component is climate damages which depend on the accumulated CO_2 stock E. We abstract from decay of atmospheric CO_2, so that the emission stock $E(t)$ corresponds to total emissions up to the instant of time t. Total damages from climate change are then:

$$\int_0^\infty e^{-rt} D(E(t)) dt.$$

Assuming that the representative household has separable utility and incorporating all this into a social welfare function yields for the importing region:

$$W = \int_0^\infty e^{-\rho t} \left(U(q(t), x(t)) - D(E) - bx(t) - p(q)q(t) \right) dt.$$

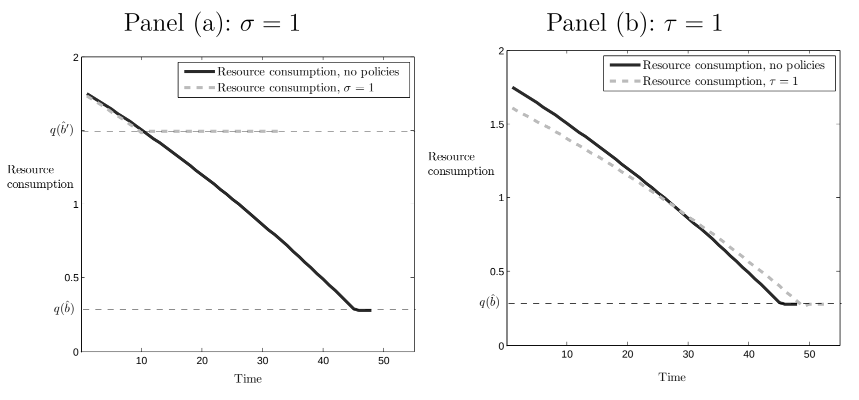
Accordingly, we assume that all of the fossil fuel used by this region is imported. Our world comprises hence only one fossil fuel importing economy and one fossil fuel exporting region; no other regions exist or import or export fossil fuels.

Table 3.1: Parameter values - single-market model

Parameter values		
HARA parameter	φ	2
HARA parameter	χ	9.072
HARA parameter	ψ	0.9072
Renewables production cost	b	8
Initial renewables subsidy	σ	0
Initial carbon tax	τ	0
Extraction cost	k	1
Discount rate	r	0.01
Initial resource stock	S_0	50
Climate damage parameter	δ	0.005

In Section 3.2.3 we have shown the occurrence of a Weak Green Orthodox, in the case of both a backstop subsidy and a tax increase. Proposition 3, however, states that a rise in the backstop subsidy and a fall in the backstop production cost shorten the time of exhaustion T_2. Hence, the same amount of fossil fuels is extracted over a shorter time horizon. Therefore, total climate costs might increase, which refutes the environmental rationale of the policy tightening. Overall welfare might still increase if a policy change improves the country's terms of trade. We give a numerical example of the country's welfare incentives to implement or tighten its climate policies, and analyze both climate damages and overall welfare effects of policy changes.[6] We use a quadratic climate damage function $D(E(t)) = \delta \frac{E^2(t)}{2}$, with $\delta > 0$. The parameter values used for the numerical examples are displayed in Table 3.1. By choosing $\varphi = 2$ we are considering the case with quadratic utility and linear demand. We examine the effects of a unit decrease in the backstop cost parameter b and those of a unit increase in the backstop subsidy σ and the carbon tax τ. We do not study *optimal* unilateral taxation or subsidization. This is left for future research. For an analysis of optimal taxation in the case without renewables we refer the reader to Liski and Tahvonen (2004) and Kagan et al. (2015).

Figure 3.1: Consumption paths - change in σ and τ

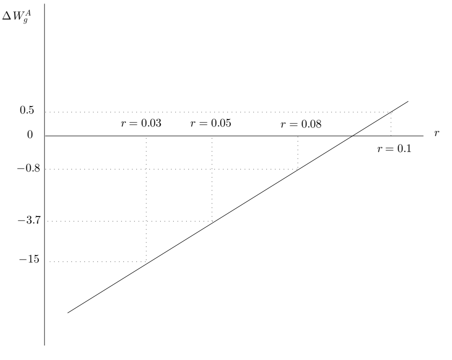


A decrease in the backstop production price and an increase in the backstop subsidy have the same effects regarding the resource price and quantity changes: the initial price

[6] Our welfare analysis differs from the analysis given by Gilbert and Goldman (1978) who compare the cases of an unconstrained monopoly and a monopoly facing a backstop. Our focus lies on assessing the shifts in both the green and non-green welfare components in view of climate policy changes.

increases to compensate the monopolist for future losses, thereby decreasing initial resource consumption. This is displayed in panel (a) of Figure 3.1.[7] A higher equilibrium resource consumption during the limit pricing phase and a shorter extraction period might increase climate costs, which indeed is the case in our numerical exercise. The results are very sensitive to the choice of parameter values: Climate costs are more likely to increase if the discount rate is low, because the reduction in initial emissions has a lower welfare impact than increased emissions in the later periods and during the limit pricing phase. We illustrate this in Figure 3.2 which shows the changes in green welfare as a consequence of a unit increase in the subsidy rate σ for different discount rates. For our set of parameter values an increase in climate costs occurs for reasonably low discount rates.

Figure 3.2: Effect of a unit increase in the backstop subsidy on green welfare for different discount rates



Note: W_g^A denotes green welfare of region A. The subsidy rate σ is increased from 0 to 1.

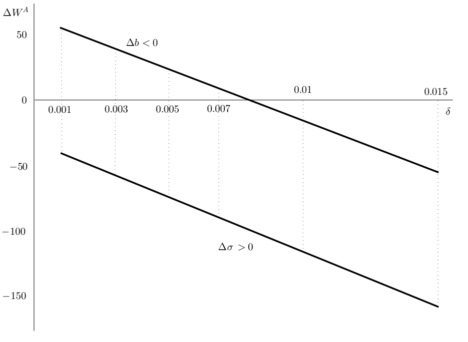
Although for green welfare an increase in the subsidy rate has the same effect as a decrease in the backstop cost parameter of equal size, the effect on overall welfare differs between the two. An increase in the backstop subsidy lowers non-green welfare, especially after the switch to the backstop, as the subsidy burden is assumed to stay in place forever.[8] The resource consumers, however, profit from a unit reduction in the cost of backstop production b: an increase in climate costs, which is just as high as in the case of a higher subsidy, is dominated by a rise in non-green welfare. The country benefits from a lower

[7]In the figure the resource consumption effects of a subsidy increase to $\sigma = 1$ are equivalent to the effects of a decrease of the backstop production cost to $b = 7$.

[8]We disregard commitment issues here and assume that the subsidy, once in place, cannot be altered after the switch to renewable energy, even after exhaustion of the fossil fuel.

backstop price forever after its switch to using the renewable energy source, without having to shoulder the subsidy burden. Yet, this positive effect on overall welfare depends on the chosen parameter values and can easily be reversed for higher climate damage parameters. Figure 3.3 illustrates the change in welfare upon a unit increase in the subsidy and a unit decrease in the backstop price for different values of the damage parameter δ.

Figure 3.3: Effect of higher σ or lower b on overall welfare for different damage parameter values



Note: ΔW^A denotes the change in region A's overall welfare as a consequence of a subsidy increase from 0 to 1 and a backstop production cost decrease from 8 to 7.

The effects of an increase in the carbon tax differ considerably from those of an increase the subsidy or a decrease in the backstop cost parameter. In our numerical example, the initial resource price goes down upon a unit increase in the carbon tax. Initial demand decreases nevertheless since resource demand is dampened by the higher tax, as can be seen in panel (b) of Figure 3.1. No Weak Green Paradox occurs and we find no aggravation of climate damages as the overall extraction period is prolonged. Overall welfare increases: climate damages decrease due to the prolonged extraction period and lower initial consumption, and non-green welfare is improved as the country is able to reap a part of the monopolistic resource rent for itself.

Our analysis has shown that climate policy tightening can have adverse climate effects implying a *Strong* Green Paradox, despite the occurrence of a Weak Green Orthodox.[9]

[9]A Strong Green Paradox is defined as an increase in the *overall climate costs*, which does not necessarily

Lower backstop costs and higher backstop subsidies result in a shorter time horizon for resource extraction, making an increase in climate damages likely and affecting overall welfare in a negative way. In our numerical example carbon taxation improves both green and non-green welfare by extending the resource extraction phase and by enabling the resource consumer to acquire a part of the monopolist's resource rent.

3.3 Two-Region Model

3.3.1 Equilibria in the Two-Region Model

In this section we consider a monopolistic non-renewable resource supplier that faces resource demand from two regions. A non-tradable perfect substitute for the resource is available at unit cost b in both regions. One region, region A, levies a unit tax τ on the resource use of its citizens and provides a per unit subsidy σ on the use of renewables. The other region, region B, does not have a tax nor a subsidy in place.[10] The monopolist supplies fossil fuel to a global market which is characterized by a global single resource price. Hence, the monopolist cannot earmark fossil fuel for each individual market, he has no power to discriminate between the regions. We define $\hat{q}_A$ as total energy demand in region A if the consumer price approaches $b - \sigma$ from below, whereas we denote demand for energy in region B if the consumer price is $b - \sigma - \tau$ as $\hat{q}_B$, and write $\tilde{q}_B$ for total energy demand in region B if the consumer price approaches b from below.

The graphs of Figure 3.4 display the consumer price on the vertical axis. Panel (a) depicts demand q_A in region A, panel (b) shows demand q_B in region B and panel (c) presents total demand. To ensure that demand is a function of the price, we again assume that demand in region B equals $\tilde{q}_B$ at a producer price b, and that aggregate demand equals $\hat{q}_A + \hat{q}_B$ at a producer price $\hat{b} \equiv b - \sigma - \tau$. The aggregate demand function is discontinuous: there is a jump from zero demand to $\tilde{q}_B$ as the price goes from $p > b$ to $p = b$, and a jump from $\hat{q}_B$ to $\hat{q}_A + \hat{q}_B$ as the price goes from $p > \hat{b}$ to $p = \hat{b}$. We use multi-stage optimal control theory to cope with these discontinuities.[11]

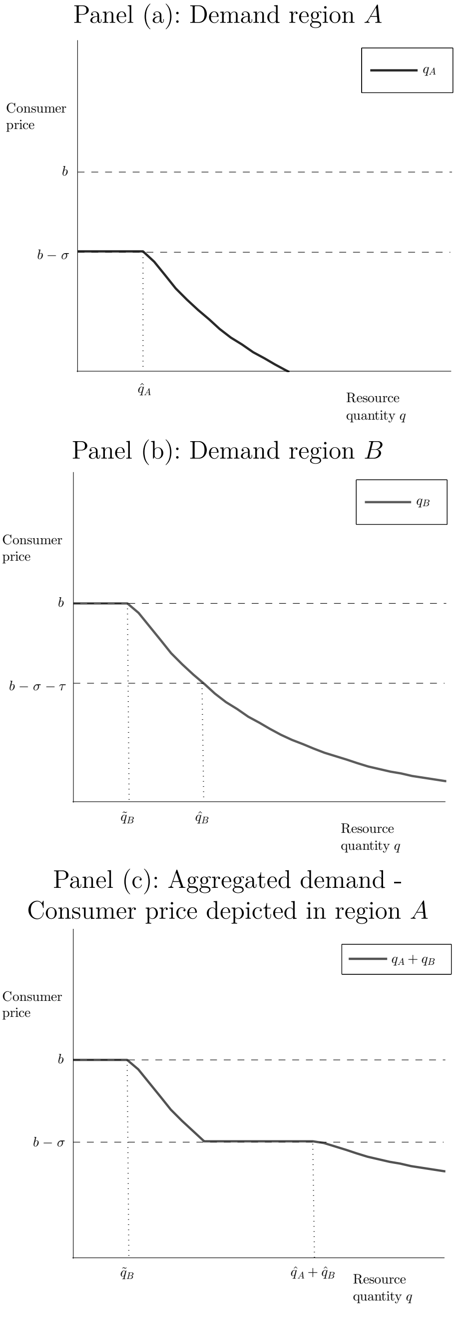
The discontinuities in the aggregate demand function that the monopolist faces may give rise to discontinuities in the profit-maximizing time profile of the resource price as well.

imply higher initial climate costs, i.e., increased initial emissions.

[10]This is, of course, an extreme case. What matters ultimately are the differences in policies between the two regions.

[11]In standard optimal control theory the integrand is required to be a continuously differentiable function of all state and control variables (cf. Seierstad and Sydsæter, 1987, p. 73).

Figure 3.4: Inverse demand curves



One could argue, though, that price jumps must be ruled out due to arbitrage behavior by speculators: if resource stocks can be built up and stored without costs by speculators, upward jumps in the price will be arbitraged away.[12] However, if the cost of storing oil by speculators is prohibitively high, there is no mechanism preventing the monopolist from choosing a discontinuous price path. Therefore, after describing the monopolist's problem, we will discuss both the equilibrium without and with speculators on the market in the remainder of this section.

3.3.2 The Monopolist's Problem

Let us define aggregate resource demand as $q = q_A + q_B$. We split up the problem in two stages. Stage 1 starts at time zero and lasts until time T_2. Stage 2 starts at time T_2 and lasts until time T_4. Both the switching time T_2 and the exhaustion time T_4 can be chosen by the monopolist. We denote the producer price when $t < T_2$ by $p_1(q)$, and the producer price when $t \geqslant T_2$ by $p_2(q)$. Accordingly, the maximization problem of the monopolist can be specified as

$$\Lambda(S_0, b, \sigma, \tau) = \max_{q(t), T_2, T_4} \left\{ \int_0^{T_2} e^{-rt} \left(p_1(q(t)) - k\right) q(t) dt + \int_{T_2}^{T_4} e^{-rt} \left(p_2(q(t)) - k\right) q(t) dt \right\}, \tag{3.16a}$$

$$\text{subject to} \quad \begin{array}{l} \dot{S}(t) = -q(t), \quad S(t) \geqslant 0, \quad S(0) = S_0, \quad S(T_4) = 0, \\ p_1(q(t)) \leqslant \hat{b}, \quad \hat{b} \leqslant p_2(q(t)) \leqslant b, \quad q(t) \geqslant 0. \end{array} \tag{3.16b}$$

Stated in this way, the monopolist's problem can be solved by using two-stage optimal control theory (cf. Tomiyama, 1985; Makris, 2001; Valente, 2011). Intuitively, we first solve the problems in the two stages separately for given T_2 and $S(T_2)$. Second, two additional matching conditions are used to determine the optimal T_2 and $S(T_2)$. To simplify the exposition, we start by taking T_4 as given and use a transversality condition later on to determine its optimal value. We assume net revenue $(p_1(q) - k)q$ to be strictly concave in q if $p < \hat{b}$, and $(p_2(q) - k)q$ to be strictly concave in q if $\hat{b} < p < b$, implying $2p_1'(q) + qp_1''(q) < 0$ if $p < \hat{b}$ and $2p_2'(q) + qp_2''(q) < 0$ if $\hat{b} < p < b$.

The Hamiltonians associated with the first and second stage of the optimal control problem

[12] Downward jumps in the price cannot be arbitraged away as the monopolist controls the amount of resource extraction.

are given by:

$$\mathcal{H}_i = (p_i(q) - k)e^{-rt}q - \lambda_i q, \quad i = 1, 2, \tag{3.17}$$

where λ_i denotes the shadow price of the resource stock in stage i. The corresponding Lagrangians read

$$\mathcal{L}_1 = e^{-rt}(p_1(q) - k)q - \lambda_1 q - \mu_{11}e^{-rt}(p_1(q) - (b - \tau - \sigma)), \tag{3.18a}$$

$$\mathcal{L}_2 = e^{-rt}(p_2(q) - k)q - \lambda_2 q - \mu_{21}e^{-rt}(p_2(q) - b) + \mu_{22}e^{-rt}(p_2(q) - (b - \tau - \sigma)), \tag{3.18b}$$

where the μ_{ij}'s are Lagrange multipliers associated with the inequality constraints on the resource price. The complementary slackness conditions are given by

$$\mu_{11}(p_1(q) - (b - \tau - \sigma)) = 0, \quad \mu_{11} \geqslant 0, \tag{3.19a}$$

$$\mu_{22}(p_2(q) - (b - \tau - \sigma)) = 0, \quad \mu_{22} \geqslant 0, \tag{3.19b}$$

$$\mu_{21}(p_2(q) - b) = 0, \quad \mu_{21} \geqslant 0, \tag{3.19c}$$

which require that the μ_{ij}'s equal zero as long as the relevant restrictions on the price are non-binding. The necessary first-order conditions with respect to resource extraction imply:

$$\lambda_1 e^{rt} = p_1'(q)q + p_1(q) - k - \mu_{11}p_1'(q), \tag{3.20a}$$

$$\lambda_2 e^{rt} = p_2'(q)q + p_2(q) - k - \mu_{21}p_2'(q) + \mu_{22}p_2'(q). \tag{3.20b}$$

Conditions (3.20a)-(3.20b) state that, at an interior solution where the price constraints are not binding so that the μ_{ij}'s equal zero, the marginal instantaneous profit equals the shadow price of the resource. The first-order conditions with respect to the resource stock require the shadow price of the resource to be constant over time: $\dot{\lambda}_1 = \dot{\lambda}_2 = 0$, which is a manifestation of the Hotelling rule.

The optimality conditions (3.19a)-(3.20b) can be used to find the solution to problem (3.16a)-(3.16b) for given T_2, T_4, and $S(T_2)$. The optimal value functions associated with the maximization problems in the two stages are given by, respectively[13]

$$V_1(0, T_2, S_0, S(T_2)) \equiv \int_0^{T_2} e^{-rt} \left(p_1(q^*(t)) - k\right) q^*(t) dt, \tag{3.21a}$$
$$V_2(T_2, T_4, S(T_2), 0) \equiv \int_{T_2}^{T_4} e^{-rt} \left(p_2(q^*(t)) - k\right) q^*(t) dt, \tag{3.21b}$$

where $q^*(t)$ denotes the associated optimal extraction path. Furthermore, the following transversality condition holds since the optimal stopping time T_4 is endogenous:

$$\mathcal{H}_2(T_4) = 0, \tag{3.22}$$

where $\mathcal{H}_2(T_4)$ is shorthand for the Hamiltonian evaluated at the instant of time T_4. The intuition behind this transversality condition follows from the result that the Hamiltonian evaluated at the terminal time equals the partial derivative of the optimal value function with respect to the terminal time (e.g., Seierstad and Sydsæter, 1987, p. 213): $\mathcal{H}_2(T_4) = \partial V_2/\partial T_4$, implying that it does not pay off to change the stopping time once condition (3.22) is satisfied. Finally, since the optimal switching time T_2 is endogenous as well, we use two-stage optimal control theory to get the following matching condition (Tomiyama, 1985):[14]

$$\mathcal{H}_1(T_2^-) = \mathcal{H}_2(T_2^+). \tag{3.23}$$

To understand this matching condition, note that the Hamiltonian evaluated at the terminal (initial) time equals (minus) the partial derivative of the optimal value function with respect to the terminal (initial) time (e.g., Seierstad and Sydsæter, 1987, p. 213): $\partial V_1/\partial T_2 + \partial V_2/\partial T_2 = \mathcal{H}_1(T_2) - \mathcal{H}_2(T_2)$. Hence, if (3.23) is satisfied, it does not pay off to reallocate time between the two stages. Lemma 2 uses matching condition (3.23) to derive a relationship between the Hamiltonian at time zero and discounted profits, which turns out to be useful for the comparative statics later on.[15]

Lemma 2 *In equilibrium, the relationship between the Hamiltonian at time zero and dis-*

[13]As in Seierstad and Sydsæter (1987), the optimal value function is defined as the supremum of the integral in (3.16a) associated with each stage, for all admissible pairs $(S(t), q(t))$.

[14]Throughout, we use the standard notation $x(T^-) \equiv \lim_{t\uparrow T} x(t)$, $x(T^+) \equiv \lim_{t\downarrow T} x(t)$.

[15]Whereas the result of Lemma 2 is well known in control theory for one-stage optimal control problems (Seierstad and Sydsæter, 1987), we extend its validity for two-stage optimal control problems.

counted profits of the monopolist is given by:

$$\frac{\mathcal{H}_1(0)}{r} = \Lambda(S_0, b, \sigma, \tau). \tag{3.24}$$

Proof. The time derivative of the Hamiltonian $\mathcal{H}_1$ in (3.17) is given by

$$\dot{\mathcal{H}}_1 = \frac{\partial \mathcal{H}_1}{\partial S}\dot{S} + \frac{\partial \mathcal{H}_1}{\partial \lambda_1}\dot{\lambda}_1 + \frac{\partial \mathcal{H}_1}{\partial q}\dot{q} + \frac{\partial \mathcal{H}_1}{\partial t} = \mu_{11}(t)p_1'(q(t))\dot{q} + \frac{\partial \mathcal{H}_1}{\partial t} = \frac{\partial \mathcal{H}_1}{\partial t}, \tag{3.25a}$$

where the second and third equality use $\dot{\lambda}_1 = 0$ and (3.20a), and (3.19a) (with either $\dot{q} = 0$ or $\mu_{11} = 0$), respectively. Similarly, by using $\dot{\lambda}_2 = 0$, (3.20b), and (3.19b)-(3.19c) we obtain

$$\dot{\mathcal{H}}_2 = (\mu_{22}(t) - \mu_{21}(t))\, p_2'(q(t))\dot{q} + \frac{\partial \mathcal{H}_2}{\partial t} = \frac{\partial \mathcal{H}_2}{\partial t}. \tag{3.25b}$$

Integration of (3.25a)-(3.25b) over t gives

$$\int_0^{T_2} e^{-rt}(p_1(q) - k)q dt = \frac{\mathcal{H}_1(0) - \mathcal{H}_1(T_2)}{r}, \tag{3.26a}$$

$$\int_{T_2}^{T_4} e^{-rt}(p_2(q) - k)q dt = \frac{\mathcal{H}_2(T_2) - \mathcal{H}_2(T_4)}{r}. \tag{3.26b}$$

Combining (3.26a)-(3.26b) while using (3.22), (3.23), and (3.16a) gives the expression in the lemma. □

In order to complete the characterization of the equilibrium, we still need to find the optimal value for $S(T_2)$. This value, however, will depend on whether the monopolist is able to freely choose all extraction and price paths that satisfy the restrictions (3.16b), which is only the case if there are no speculators and the price path may be discontinuous. If there are speculators on the resource market, additional restrictions apply.[16]

The distinction between a price path in the presence and in the absence of speculators is quite intuitive: Suppose it were optimal for the monopolist to have a continuous price path, characterized by limit pricing at a producer price $\hat{b}$ initially, followed by a phase where $\hat{b} < p(t) < b$, and finally a phase with limit pricing at b. In the first phase the resource producer serves both markets and sells at rate $\hat{q}_A + \hat{q}_B$. In the second phase he

[16]An equilibrium price path in the presence of speculators is continuous, whereas it is most likely discontinuous in the absence of speculators. We use the terms "continous and discontinuous price case" and "in the presence and absence of speculators" interchangeably.

supplies only to region B and can sell $\hat{q}_B$ at most. The monopolist hence faces a downward jump in revenues. If there was no restriction on the price path, it would be profitable to lengthen the first limit pricing period and let the price jump upward after this phase. Consequently, in the absence of speculators, the monopolist can do better by letting the price be discontinuous.
We will discuss both price regimes in turn.

3.3.3 Equilibrium without Speculators

Without speculators on the market, the monopolist is free to choose the resource stock $S(T_2)$ which solves the maximization problem (3.16a)-(3.16b). In addition to the optimality conditions (3.19a)-(3.20b), transversality condition (3.22), and matching condition (3.23), the two-stage optimal control theory requires the following additional matching condition to be satisfied at the optimum (Tomiyama, 1985):

$$\lambda_1 = \lambda_2, \tag{3.27}$$

Intuitively, if (3.27) would not hold, the monopolist could increase its profits by reallocating cumulative extraction from the stage with the relatively lower to the stage with the relatively higher shadow price.
For the time being, let us assume that the optimal T_2 is positive.[17] We can then use the matching conditions (3.23) and (3.27) to show what happens to the resource price at T_2, when the economy switches from stage 1 to stage 2:

Lemma 3 *Provided that $T_2 > 0$, the resource price jumps up at T_2.*

Proof. Suppose, on the contrary, that the price is continuous at T_2. Then $p(T_2^-) = p(T_2^+) = \hat{b}$. Since $q(T_2^-) = \hat{q}_A + \hat{q}_B$ and $q(T_2^+) = \hat{q}_B > 0$, it follows from (3.22) and (3.27) in (3.23) that $\mathcal{H}_1(T_2) = \mathcal{H}_2(T_2) = 0$. However, substitution of (3.20b) into (3.17) gives

$$\mathcal{H}_2(T_2) = -\left\{p_2'(q(T_2^+))q(T_2^+) + \mu_{22}(T_2^+)p_2'(q(T_2^+))\right\} q(T_2^+) > 0,$$

where we have used $p_2(q(T_2^+)) = \hat{b} < b$, implying that $\mu_{21}(T_2^+) = 0$. So, we have reached a contradiction. Hence, the price must jump up at $t = T_2$. □

[17]The optimal T_2 will be positive if the initial resource stock S_0 is large enough, as will be discussed below.

To provide further intuition for the result in Lemma 3, consider Figure 3.5, which is an extended version of Figure 2 in Hoel (1984). The figure shows a discontinuous line for the marginal profit $\pi'(q) = p'(q)q + p(q) - k$, corresponding to the aggregate demand function in panel (c) of Figure 3.4, and a flat line for the present value of the scarcity rent at $t = T_2$, λe^{rT_2}.[18] According to (3.20a)-(3.20b), at an interior equilibrium where the μ_{ij}'s are zero, marginal profit equals the present value of the scarcity rent λe^{rT_2}.

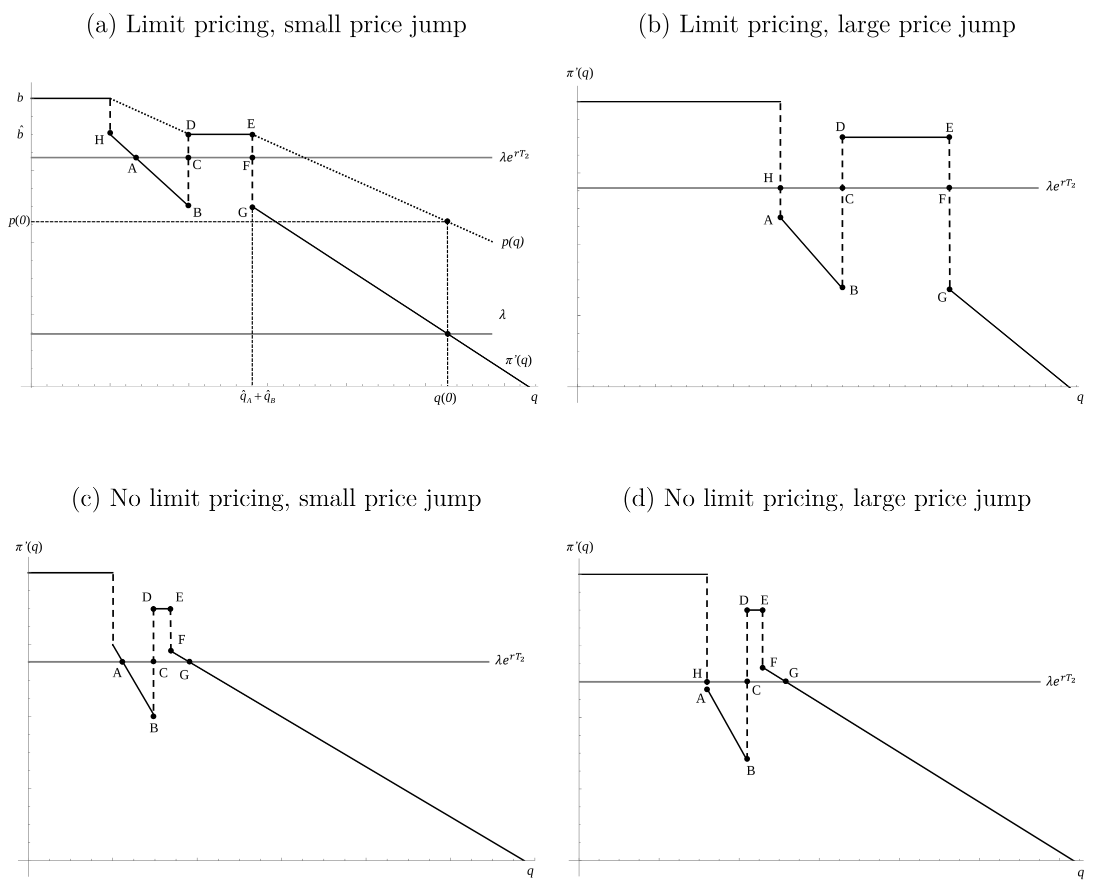
Consider first panel (a), which additionally contains a line for the inverse aggregate demand function $p(q)$ (solid flat parts, dotted downward sloping parts) and the initial scarcity rent λ. We have assumed that the initial resource stock is large enough to get an initial scarcity rent below point G in the figure. Initially, the economy is at an equilibrium where resource extraction is given by $q(0)$ and the resource price by $p(0)$. The Hamiltonian has a unique maximum at $q(0)$. Over time, the scarcity rent λe^{rt} gradually increases and resource extraction goes down, giving rise to an increasing resource price. At point G, there is a discontinuity in the marginal revenue function.

When the scarcity rent reaches a level corresponding with this point, μ_{11} will become positive (see (3.20a)) and extraction and the resource price will continue to equal $\hat{q}_A + \hat{q}_B$ and $\hat{b}$, respectively, for a while: limit pricing. However, whenever the scarcity rent is at a level in between points B and D, there exists a second intersection point of the scarcity rent and the marginal profit line, e.g. at point A, implying that the Hamiltonian has another local maximum. The change in profits when the monopolist would move from point A to point F in the figure is given by rectangle CDEF (where marginal revenue is above marginal costs), minus triangle ABC (where marginal revenue is below marginal costs). Hence, as long as the surface given by the triangle ABC is smaller than the surface within the rectangle CDEF, the global maximum is still located at $\hat{q}_A + \hat{q}_B$. At time $t = T_2$ both areas have exactly the same size. Given that the scarcity rent keeps on rising, the optimal point will jump from F to A at T_2: extraction jumps down and the resource price jumps up. After the switch, resource extraction will gradually decline while the flat scarcity rent line increases until point H is reached, when another limit pricing phase takes place until the stock is exhausted.

In panel (b), the global maximum is still located at $\hat{q}_A + \hat{q}_B$ when the scarcity rent reaches the level corresponding with point A. At $t = T_2$, the area ABCH equals CDEF, implying that the price jumps immediately from one limit pricing regime with $p = \hat{b}$ to another limit pricing regime with $p = b$, which will last until the stock is exhausted. In panel (c), the

[18]We have used (3.27) to write $\lambda \equiv \lambda_1 = \lambda_2$. Note that λ is still to be determined.

Figure 3.5: Stage switch - four scenarios



Notes: The discontinuous line represents marginal profits $\pi'(q) = p'(q)q + p(q) - k$ and the flat line gives the present value of the scarcity rent at $t = T_2$. Panel (a) shows the case with limit pricing and $p_2(T_2^+) < b$, panel (b) presents the case with limit pricing and $p_2(T_2^+) = b$, panel (c) depicts the scenario without limit pricing and with $p_2(T_2^+) < b$, panel (d) illustrates the case without limit pricing and with $p_2(T_2^+) = b$.

regime switch will take place when area ABC equals CDEFG, which occurs before point F is reached. Hence, there will be no intermediate regime of limit pricing: the price jumps from $p < \hat{b}$ towards $p \in (\hat{b}, b)$. Finally, in panel (d) the price will jump from $p < \hat{b}$ towards $p = b$ at $t = T_2$, when area ABCH equals CDEFG.

The figure has hence shown all the possible stage switch scenarios. All of them feature an upward jump in the resource price, as claimed in Lemma 3. The upper two panels are characterized by an intermediate limit pricing regime with $p = \hat{b}$. Moreover, the two left panels feature an increasing resource price after the stage switch, whereas the two right panels show situations in which the economy jumps to a limit pricing regime with $p = b$. The size of the jump and the conditions under which an intermediate limit pricing regime before the jump takes place can also be determined analytically by combining the first-order conditions (3.20a)-(3.20b) with the two matching conditions (3.23) and (3.27), yielding the result in the following lemma.

Lemma 4 *Provided that $T_2 > 0$, the jumps in resource extraction and the resource price, and the values of the Lagrange multipliers at the stage switch satisfy:*[19]

$$p_1[q(T_2^-)] - p_2[q(T_2^+)] = p_2'[q(T_2^+)] \left[q(T_2^-) - q(T_2^+)\right] \frac{q(T_2^+) - \mu_{21}(T_2^+)}{q(T_2^-)}, \tag{3.28a}$$

$$\begin{aligned} \mu_{11}(T_2^-)p_1'[q(T_2^-)] &= p_1[q(T_2^-)] + p_1'[q(T_2^-)]q(T_2^-) \\ &\quad - \left(p_2[q(T_2^+)] + p_2'[q(T_2^+)]q(T_2^+)\right) + \mu_{21}(T_2^+)p_2'[q(T_2^+)]. \end{aligned} \tag{3.28b}$$

Proof. First, note that $\mu_{22}(T_2^+) = 0$ due to the result in Lemma 3. Second, substitution of (3.20b) into (3.17) and subsequently using (3.23) gives (3.28a). Third, combining (3.20a)-(3.20b) gives (3.28b). □

Condition (3.28a) relates the jump in the resource price to the jump in demand, whereas condition (3.28b) determines the value of the Lagrange multiplier μ_{11} at the end of the first regime. The existence of a limit pricing regime before T_2 requires a positive $\mu_{11}(T_2^-)$ (from the complementary slackness condition (3.19a)) and therefore a negative right-hand side. Although the second row of (3.28b) is always negative, the marginal revenue showing up in the first row should not be too high in order for the right-hand side to be negative on balance. In terms of panels (a) and (b) of Figure 3.5, point G should not be located too

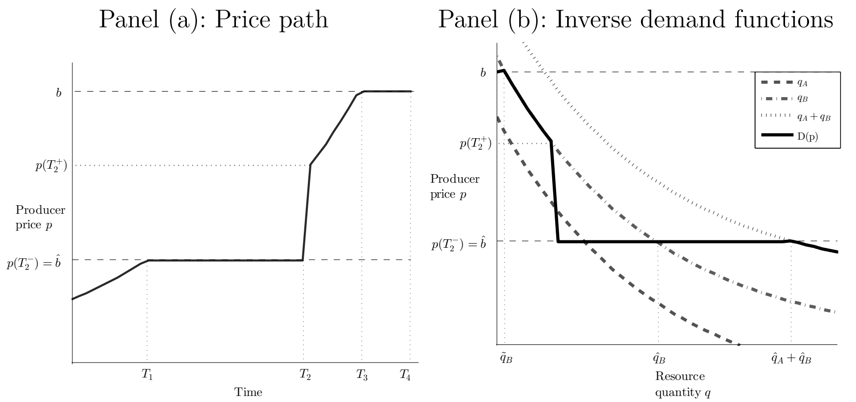
[19] Together with the complementary slackness conditions (3.19a) and (3.19c), equations (3.28a)-(3.28b) can be used to solve for, $q(T_2^-)$, $\mu_{11}(T_2^-)$, $q(T_2^+)$, and $\mu_{21}(T_2^+)$.

high (like points F are in panels (c) and (d)).

We are now able to fully characterize the equilibrium. Note first that, by combining (3.17), (3.20b), and (3.22), we get $\mu_{21}(T_4) = q(T_4) > 0$, implying that there always should be a limit pricing phase at the end. It follows from (3.17), (3.22) and (3.18b) that $0 = q(T_4) - \mu_{21}(T_4) + \mu_{22}(T_4)$. We have $q(T_4) > 0$, since $p(T_4) \leqslant b$. Hence $\mu_{21}(T_4) > 0$ and $p(T_4) = b$. We conclude thus that there potentially exists an intermediate and definitely a final limit pricing phase. The discussion of Figure 3.5 has shown that an initial phase during which the resource price is increasing and smaller than $\hat{b}$ will exist if the initial resource stock is large enough. Moreover, there may exist a phase in between the two limit pricing phases in which the price is growing and lies in between $\hat{b}$ and b.

Accordingly, the typical price path in the equilibrium without speculators on the market consists of four phases as shown in panel (a) of Figure 3.6, where panel (b) depicts the corresponding regional and aggregate inverse demand functions.[20] During the first phase,

Figure 3.6: Price path and inverse demand functions without speculators



the price is increasing over time and smaller than $\hat{b}$, implying that resource demand from both regions is positive. The second phase is a limit-pricing phase, with a constant price at $p = \hat{b}$ and positive demand from both regions. During the third phase, the price is increasing again and lies in between $\hat{b}$ and b, implying that resource demand from region A has dropped to zero, whereas demand form region B is still positive. The fourth phase

[20]Some of these phases might be degenerate.

is another limit pricing phase with $p = b$ and positive demand from region B which lasts until the resource stock is depleted.
The existence of the first three phases is dependent on the initial resource stock. Theorem 1 formally characterizes the equilibrium and explicitly states how the existence of the different regimes depends on the initial resource stock.

Theorem 1 *There exist* $0 \leqslant T_1 \leqslant T_2 \leqslant T_3 < T_4$ *such that*

(i) $p(t) \leqslant \hat{b}$ *for* $0 \leqslant t \leqslant T_1$ *(phase 1),*

(ii) $p(t) = \hat{b},\ q(t) = \hat{q}_A + \hat{q}_A$ *for* $T_1 \leqslant t \leqslant T_2$ *(phase 2),*

(iii) $\hat{b} < p(t) \leqslant b$ *for* $T_2 \leqslant t \leqslant T_3$ *(phase 3),*

(iv) $p(t) = b, q(t) = \tilde{q}_B$ *for* $T_3 \leqslant t < T_4$ *(phase 4),*

(v) $S(T_4) = 0,\ S(t) > 0$ *for* $0 \leqslant t < T_4$.

Furthermore, there exist $S_{03} > S_{02} > S_{01}$ *such that*

(i) $0 = T_1 = T_2 = T_3 < T_4$ *if* $S_0 \leqslant S_{01}$,

(ii) $0 = T_1 = T_2 < T_3 < T_4$ *or* $0 = T_1 < T_2 = T_3 < T_4$ *if* $S_{01} < S_0 \leqslant S_{02}$,

(iii) $0 = T_1 < T_2 \leqslant T_3 < T_4$ *if* $S_{02} < S_0 \leqslant S_{03}$,

(iv) $0 < T_1 \leqslant T_2 \leqslant T_3 < T_4$ *if* $S_{03} < S_0$.

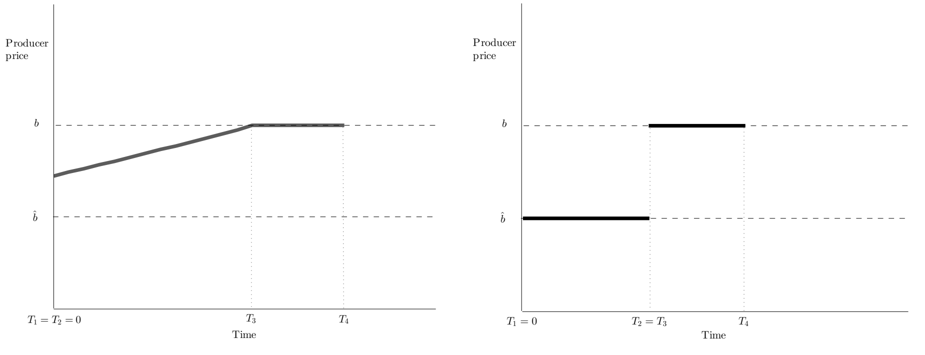
Proof. The proof can be found in Appendix B.1.1. □

Panel (a) of Figure 3.5 shows the scenario in which all phases exist: $0 < T_1 < T_2 < T_3 < T_4$. In panel (b), phase 3 is degenerate: $T_2 = T_3 < T_4$. The monopolist switches from the first limit pricing immediately to the second limit pricing phase, without an intermediate phase of price increase. Panel (c) shows the case in which phase 2 is degenerate: $T_1 = T_2 < T_3 < T_4$. The first limit pricing phase drops out and the price increases from $p < \hat{b}$ to a price above the limit price, $p > \hat{b}$. A larger jump from $p < \hat{b}$ to the second limit price is also possible, as in panel (d), where both the second and the third phase are degenerate: $T_1 = T_2 = T_3 < T_4$. The occurrence of these 'degenerate' equilibria depends on the functional form of the marginal profit functions and therefore on the aggregate demand functions. Hence, even large initial resource stocks do not guarantee a 'non-degenerate' equilibrium comparable to the one depicted in panel (a).

The following example illustrates how we obtain different (degenerate) outcomes with the same resource stock. We consider two parameter sets, κ_1 and κ_2, with S_0 yet unspecified, which yield a different resource demand $\hat{q}_A$ in region A at the limit price $\hat{b}$: for the first parameter set κ_1, $\hat{q}_A$ is close to zero, whereas $\hat{q}_A$ is large for the second parameter set κ_2. There exist initial resource stocks $S_{\kappa_1 0}$ and $S_{\kappa_2 0}$ for which it is optimal to start immediately with the second limit pricing phase, i.e., $T_{\kappa_1 3} = 0$ and $T_{\kappa_2 3} = 0$. We can assume that both stocks are equal and yield limit pricing phases of equal length. Let us now consider marginally larger $S_{\kappa_1 0}$ and $S_{\kappa_2 0}$. The resulting equilibrium outcomes for the marginally larger stocks might be very different for the two sets of parameter values, depending on the monopolist's profitability of supplying to region A. Due to a low $\hat{q}_A$, a larger resource stock prolongs the supply phase to region B in panel (a) of Figure 3.7, implying that $T_{\kappa_1 2} < T_{\kappa_1 3}$, whereas the monopolist starts limit pricing in the $A + B$ market in panel (b), where $\hat{q}_A$ is assumed to be large, such that $T_{\kappa_2 2} = T_{\kappa_2 3}$.

Figure 3.7: Price paths for the parameter sets κ_1 and κ_2

Panel (a): Price path for κ_1, small $\hat{q}_A$ Panel (b): Price path for κ_2, large $\hat{q}_A$



3.3.4 Equilibrium with Speculators

If there are speculators on the market who can buy the resource, store it, and sell it at a later moment, upward jumps in the resource price will be arbitraged away. Therefore, the solution derived in the preceding section can no longer be an equilibrium. Instead, the resource price should be continuous and thus equal to $\hat{b}$ at the moment of the switch from stage 1 to stage 2 in problem (3.16a)-(3.16b). Formally, two extra restrictions need to be satisfied at the equilibrium, in addition to the continuity of the Hamiltonian in (3.23):

$$p_1[q(T_2^-)] = \hat{b}, \tag{3.29a}$$

$$p_2[q(T_2^+)] = \hat{b}. \tag{3.29b}$$

The additional restriction (3.29b) can only be satisfied together with the other optimality conditions if $S(T_2)$ takes a specific value, which we label S^*. Intuitively, in a standard problem like the one discussed in Section 3.2, the size of the initial resource stock determines its shadow price. Here, however, the shadow price in the second stage, $e^{rT_2}\lambda_2$, is pinned down by (3.29b) and (3.20b) (with $\mu_{21} = \mu_{22} = 0$). A resource stock size $S(T_2)$ lower than S^* corresponds to larger values for $e^{rT_2}\lambda_2$ and $p_2[q(T_2^+)]$, and therefore to an upward jump in the resource price at $t = T_2$. Given that the resource stock at the switching instant is fixed at S^*, the matching condition (3.27), which determines the allocation of resource use between the two stages, is no longer relevant and can be dropped. Matching condition (3.23), corresponding to matching condition (3.23) in the case without speculators, still needs to be satisfied, though, because the monopolist is free to choose the duration of the two stages.

Hence, the solution to the monopolist's problem (3.16a)-(3.16b) in the presence of speculators needs to satisfy the optimality conditions (3.19a)-(3.20b), the transversality condition (3.22), and the matching condition (3.23). Furthermore, as discussed, the restriction $S(T_2) = S^*$ is implied by the requirement of a continuous resource price path due to the existence of speculators in the market. Then we can direcly compute the jump in the shadow price of the resource and the values of the Lagrange multipliers.

Lemma 5 *Provided that $T_2 > 0$, the jump in the shadow price of the resource and the values of the Lagrange multipliers at the stage switch satisfy:*

$$\lambda_1 - \lambda_2 = \left[\lambda_2 - (\hat{b} - k)e^{-rT_2}\right] \frac{q(\hat{T}_2^+) - q(\hat{T}_2^-)}{q(\hat{T}_2^-)}, \tag{3.30a}$$

$$\mu_{21}(T_2^+) = \mu_{22}(T_2^+) = 0, \tag{3.30b}$$

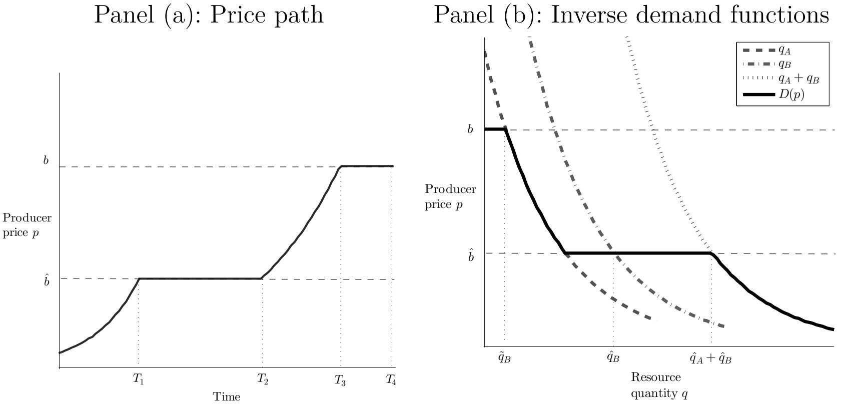
$$\mu_{11}(T_2^-) = q(T_2^-) - \frac{p_2'[q(T_2^+)]q(T_2^+)}{p_1'[q(T_2^-)]q(T_2^-)} q(T_2^+). \tag{3.30c}$$

Proof. Condition (3.30a) is obtained by substitution of (3.29a)-(3.29b) into (3.23). Conditions (3.19c) and (3.29b) together imply $\mu_{21} = 0$. By using (3.20b) with $\mu_{21} = 0$, it follows that the term between brackets in (3.30a) is negative, implying that $\lambda_1 - \lambda_2 > 0$ (as $q(\hat{T}_2^+) - q(\hat{T}_2^-) < 0$). As a result, there will be no regime with $p_2(q) = \hat{b}$ at the beginning of stage 2, implying $\mu_{22} = 0$. Finally, combining (3.23) with (3.20a)-(3.20b) and using (3.30b) gives (3.30c). □

Note that, due to (3.29a)-(3.29b), $q(T_2^-)$ and $q(T_2^+)$ are fixed. Condition (3.30a) implies that the shadow price of the resource stock is lower in stage 2 than in stage 1. The reason is that, although the monopolist would like to shift some extraction towards stage 1, doing so would necessarily imply a jump in the resource price after T_2 or a violation of the first-order condition (3.20b). Accordingly, the existence of speculators on the market forces the monopolist to sell more oil to the region without climate policy. The second equality in (3.30b) implies that the monopolist will never choose for an intermediate limit-pricing regime during which he only sells to the country without climate policy. Intuitively, discounted profits would increase by selling to both countries without lowering the price instead. The last condition in Lemma 5 implies that there will be an intermediate regime of limit pricing, as long as the downward jump in resource extraction at the switching time is not too small. Again the reason is that it is more profitable for the monopolist to sell $\hat{q}_A + \hat{q}_B$ at a price $\hat{b}$ than only $\hat{q}_B$, as long as the price continuity restriction is not violated.
We are now able to fully characterize the equilibrium for the case with speculators. Recall that, by combining (3.17), (3.20b), and (3.22), we get $\mu_{21}(T_4) = q(T_4) > 0$, implying that there always is a limit pricing phase at the end. Moreover, in the equilibrium with speculators the third regime with a growing price in between $\hat{b}$ and b can no longer be degenerate, if $T_2 > 0$. If the initial resource stock is large enough, an initial phase will exist during which the resource price is increasing and smaller than $\hat{b}$. Similarly to the case without speculators, the typical price path in the equilibrium with speculators on the market consists of four (potentially degenerate) phases as shown in panel (a) of Figure 3.8, where panel (b) depicts the corresponding regional and aggregate inverse demand functions.
As in the case without speculators, the existence of the first three phases depends on the initial resource stock. Lemma 2 formally characterizes the equilibrium and explicitly states how the existence of the different phases depends on the initial resource stock.

Theorem 2 *There exist* $0 \leqslant T_1 \leqslant T_2 \leqslant T_3 < T_4$ *such that*

Figure 3.8: Price path and inverse demand functions with speculators



(i) $p(t) \leqslant \hat{b}$ *for* $0 \leqslant t \leqslant T_1$ *(phase 1),*

(ii) $p(t) = \hat{b},\ q(t) = \hat{q}_A +\ \hat{q}_A$ *for* $T_1 \leqslant t \leqslant T_2$ *(phase 2),*

(iii) $\hat{b} < p(t) \leqslant b$ *for* $T_2 \leqslant t \leqslant T_3$ *(phase 3),*

(iv) $p(t) = b, q(t) = \tilde{q}_B$ *for* $T_3 \leqslant t < T_4$ *(phase 4),*

(v) $S(T_4) = 0,\ S(t) > 0$ *for* $0 \leqslant t < T_4$.

Furthermore, there exist $S_{03} > S_{02} > S_{01}$ *such that*

(i) $0 = T_1 = T_2 = T_3 < T_4$ *if* $S_0 \leqslant S_{01}$,

(ii) $0 = T_1 = T_2 < T_3 < T_4$ *if* $S_{01} < S_0 \leqslant S_{02}$,

(iii) $0 = T_1 < T_2 < T_3 < T_4$ *if* $S_{02} < S_0 \leqslant S_{03}$,

(iv) $0 < T_1 \leqslant T_2 < T_3 < T_4$ *if* $S_{03} < S_0$.

Proof. The proof can be found in Appendix B.1.2. □

If $T_2 > 0$, the only possible collapse of an interval occurs if $T_1 = T_2$: the first limit-pricing phase is nonexistent, and the monopolist switches to supplying market B immediately at the instant of time when the price path reaches $\hat{b}$.

3.3.5 Comparison of the Equilibria with and without Speculators

The presence of speculators affects the resource extraction path considerably. Proposition 5 deals with the effect on initial extraction and on the overall resource extraction length.

Proposition 5 *Provided that $T_1 > 0$, initial resource extraction is lower in the presence of speculators than without speculators. Furthermore, it takes longer to deplete the non-renewable resource in the presence of speculators.*

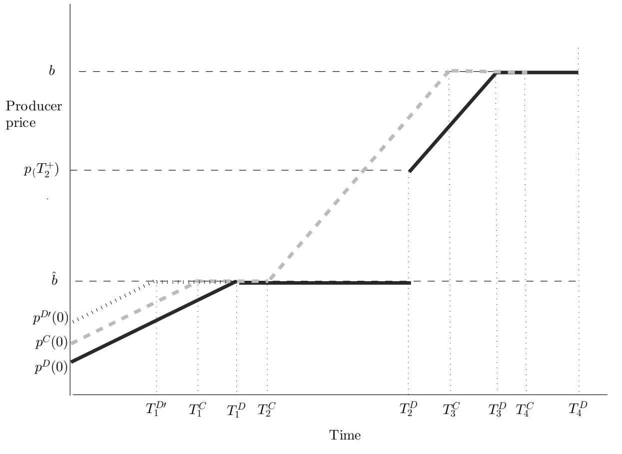
Proof. The presence of speculators implies that the monopolist cannot choose the optimal price path, which implies a positive jump in the price at T_2, due to the result in Lemma 3. Therefore, the presence of speculators has a negative effect on $\Lambda(S_0, b, \sigma, \tau)$ and thus on $\mathcal{H}_1(0)$, according to Lemma 2. We have $\mu_{11}(0) = 0$ if $T_1 > 0$. Hence, substitution of (3.20a) into the Hamiltonian (3.17) gives

$$\mathcal{H}_1(0) = -p_1'(q(0))q^2(0). \tag{3.31}$$

From the strict concavity of $(p_1(q) - k)q$ in q, (3.31) implies $d\mathcal{H}_1(0)/dq(0) > 0$. Therefore, the decline in $\mathcal{H}_1(0)$ implies a fall $q(0)$.
In order to prove the second part of the proposition, let us assume that the overall extraction period were longer in the discontinuous price case, as depicted in Figure 3.9, where the solid line (dashed line) represents the resource price in the discontinuous (continuous) price case and the superscript D (C) pertains to variables in the discontinuous (continuous) price case. Then, however, the price path in the discontinuous price case is always lower or equal to the price path in the continuous price case. This implies that more than S_0 would be extracted in the absence of speculators, assuming that S_0 is extracted in the presence of speculators. The only possibility to comply with the resource constraint in the discontinuous case is then to increase the initial resource price $p^D(0)$ to $p^{D\prime}(0)$, implying that $q^{D\prime}(0) < q^C(0)$, and shorten the first interval to $[0, T_1^{D\prime}]$, as depicted by the dotted line in Figure 3.9. This, however, would violate the first part of the proposition. We hence conclude that the extraction phase in the continuous price case must be longer than in the discontinuous price case. □

We illustrate the effect of the presence of speculators on the entire time paths of resource prices and extraction with a numerical example. Figure 3.10 shows the resource consumption and price paths in the case of both discontinuous and continuous prices,

Figure 3.9: Price paths in the discontinuous and the continuous price case



where superscript C (D) again pertains to variables in the continuous (discontinuous) price case. The parameters underlying the numerical example are displayed in Table 3.2. In our numerical exercises we assume that both regions are of equal size. Again, the HARA parameters used imply linear demand functions. Table 3.3 contrasts the corresponding equilibrium values.

Table 3.2: Parameter values - two-region model

Parameter values		
HARA parameter	φ	2
HARA parameter	χ	9.072
HARA parameter	ψ	0.9072
Backstop production cost	b	8
Initial backstop subsidy	σ	0.5
Initial carbon tax	τ	0.5
Extraction cost	k	1
Discount rate	r	0.01
Initial resource stock	S_0	50
Climate damage parameter	δ	0.005

Figure 3.10: Consumption and price paths - discontinuous and continuous case

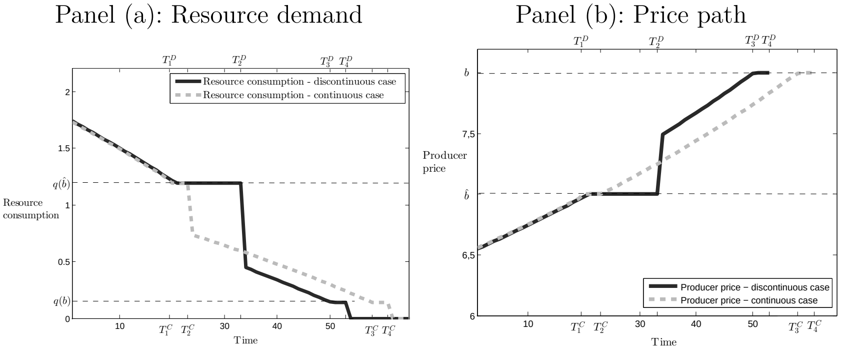


Table 3.3: Comparison of equilibrium results in the continuous and discontinuous price case

	Continuous price	Discontinuous price
$p(0)$	6.56	6.55
$p(T_2^+)$	7	7.49
$q(0)$	1.74	1.73
$q(T_2^+)$	0.73	0.45
T_1	19.42	19.75
$T_2 - T_1$	4.06	12.75
$T_3 - T_2$	35.02	17.16
$T_4 - T_3$	3.34	3.34
T_4	61.85	53

From Figure 3.10 we observe that in the continuous price case, the first limit pricing phase is shorter and the monopolist switches earlier to supplying region B only, starting at a lower price $p_2(T_2^+) = \hat{b}$. This can be explained by noting from the figure that the resource extraction patterns are similar in both price regimes until T_1 is reached. The differences in initial resource consumption related to Proposition 5 are small in our numerical example, as can be seen in Table 3.3. The resource extraction paths then begin to diverge only in the course of the first limit pricing phase $[T_1, T_2]$. The monopolist needs to supply exclusively to market B earlier in the case with speculators in order to ensure the continuity of the price path, otherwise too much of the resource is depleted to guarantee such a price path.

The differences in the overall length of resource extraction and in the length of exclusively supplying to market B have consequences for the comparative statics and welfare analysis which we perform below.

3.3.6 Comparative Statics

We seek to shed some light on the following questions by looking into the comparative statics and numerical analyses:

> What is the effect of stricter climate policies on the monopolist's incentives to sell to the different regions, i.e. to shift supply between the markets B and $A+B$?
>
> Given our focus on the Green Paradox we are particularly interested in the consequences of policy tightening on climate damages via the effects on the overall duration of resource extraction and on the extraction path.

We limit our analysis to the generic scenario where none of the phases is degenerate and the initial resource stock is large enough for all four phases to exist.

Let us start with the second phase when the monopolist supplies to market B.

Proposition 6 *Consider the two-region model, where region A employs a carbon tax and a backstop subsidy, i.e., $(\tau, \sigma > 0)$, whereas region B does not have any climate policies in place. Provided that $0 < T_1 < T_2 < T_3 < T_4$, it holds that*

(i) if $\eta(\tilde{q}_B)k > (b-k)b\eta'(\tilde{q}_B)/p'(\tilde{q}_B)$, a decrease in b extends the second limit pricing phase $[T_3, T_4]$ in both price regimes;

(ii) neither an increase in σ nor an increase in τ changes the length of the second limit pricing phase $[T_3, T_4]$ in both price regimes;

(iii) if $\eta(\tilde{q}_B)k > (b-k)b\eta'(\tilde{q}_B)/p'(\tilde{q}_B)$, a lower b extends the phase of supply to market B, i.e., $[T_2, T_4]$ and increases S^* (i.e., cumulative extraction during this phase) in the continuous price case;

(iv) an increase in σ or τ prolongs the phase of supply to market B, i.e., $[T_2, T_4]$ and increases S^* in the continuous price case.

Proof. For the proof of part (i), see equation (3.9) with b instead of $\hat{b}$, $\tilde{q}_B$ instead of $\hat{q}$, and T_4-T_3 instead of T_2-T_1. Part (ii) follows immediately from (B.3) and (B.10), which give the expressions for the final intervals with limit pricing. The parameters σ and τ do not appear in these expressions. To prove part (iii), note that if a decrease in b prolongs the second limit pricing phase $[T_3, T_4]$, the interval $[T_2, T_4]$ is prolonged and hence also S^* increases. Part (iv). An increase in σ or τ changes neither the length of $[T_3, T_4]$, nor the extraction rate during this limit pricing phase, but extends the interval $[T_2, T_3]$ since $\hat{b}$, the price at which supply to market B begins in the continuous price case, decreases. This results in a longer interval $[T_2, T_4]$ and a higher cumulative extraction S^* during this phase. □

Similar to the single market case, $[T_3, T_4]$ is prolonged under the condition given in part (i) of the proposition if b decreases, both in the absence and in the presence of speculators, whereas changes in σ or τ do not affect the interval's length. Furthermore, if the same condition is satisfied, a lower limit price $\hat{b}$, which is a consequence of a lower b, a higher σ, or a higher τ, prolongs the overall phase when market B is supplied in the continuous price case. Both panels (a) and (b) in Figures 3.11, 3.12 and 3.13 display the resulting longer supply phase to market B. A prolonged phase $[T_2, T_4]$ also signifies that S^*, i.e., the amount of resources that is supplied to market B only, increases. This means that climate policy tightening in region A has a carbon leakage effect which, however, occurs in the future. Furthermore, this shift in resource supply between region A and region B will have implications welfare ass well.

We proceed by examining the effects of climate policies in the two-region model. Similar to the single market case, we show that a subsidy for clean energy nor a constant carbon tax lead to a Weak Green Paradox in the two-region model.

Proposition 7 *Consider the two-region model, where region A employs a carbon tax and a backstop subsidy, i.e., $(\tau, \sigma > 0)$, whereas region B does not have any climate policies in place. Provided that $T_1 > 0$, an increase in the backstop subsidy, an increase in the carbon tax, or a decrease in the backstop cost lower initial resource extraction, i.e., no Weak Green Paradox occurs.*

Proof. As in parts (iv) of Propositions 3-4, a decrease in b, or an increase in σ or τ makes the constraints that the monopolist faces more stringent. Hence $d\Lambda(S_0, b, \sigma, \tau)/db > 0$, $d\Lambda(S_0, b, \sigma, \tau)/d\sigma < 0$, and $d\Lambda(S_0, b, \sigma, \tau)/d\tau < 0$, which from the result in Lemma 2 gives $d\mathcal{H}_1(0)/db > 0$, $d\mathcal{H}_1(0)/d\sigma < 0$, and $d\mathcal{H}_1(0)/d\tau < 0$. Moreover, from the strict concavity of $(p_1(q) - k)q$ in q, (3.31) implies $d\mathcal{H}_1(0)/dq(0) > 0$. Therefore, we get $dq(0)/db > 0$, $dq(0)/d\sigma < 0$, and $dq(0)/d\tau < 0$. $\square$

Regarding the first supply phase to both regions $A + B$, we note that the initial resource producer price increases if b falls, or if σ or τ increases. The mechanism is the same as in the single market case: there will be an increase in resource demand during the intermediate limit pricing phase as a result of an increase in σ or τ and an increase in resource demand during both limit pricing phases as a result of a fall in b. Hence, fewer fossil fuels are available for extraction during the initial phase, implying that $q(0)$ will go down. This finding of a Weak Green Orthodox as the outcome of climate policy tightening generalizes our result from the single market case.

The last proposition in our comparative statics section deals with the entire resource extraction and consumption period.

Proposition 8 *Consider the two-region model, where region A employs a carbon tax and a backstop subsidy, i.e., $(\tau, \sigma > 0)$, whereas region B does not have any climate policies in place. Provided that $T_1 > 0$, an increase in the backstop subsidy and a decrease in the backstop cost lower the time of exhaustion T_4 in the discontinuous price case.*

Proof. First-order condition (3.20a) with $\mu_{11}(0) = 0$ gives $d\lambda/dq(0) = [2p_1'(q(0)) + q(0)p_1''(q(0))] - d\tau/dq(0)$, the first term of which is negative due to strict concavity of $(p_1(q) - k)q$ in q. Using this result together with $\lambda = (b - k)e^{-rT_4}$ from (3.20b) and (3.22), and $dq(0)/db > 0$ and $dq(0)/d\sigma < 0$ from Proposition 7, while keeping $d\tau = 0$, we find $dT_4/db > 0$ and $dT_4/d\sigma < 0$. $\square$

In the next section, we will discuss the welfare implications of climate policies and of the differences between the equilibria with and without speculators that we have discussed in the current and former section, respectively.[21]

3.3.7 Green Paradox and Welfare Effects

While we could exclude the occurrence of a Weak Green Paradox in the previous analysis, the environmental effectiveness of policy tightening in this framework is not yet guaranteed. An increase in resource consumption during the first limit-pricing phase, which can be seen in Figures 3.11 to 3.13, implies that climate benefits of reduced initial resource consumption might be outweighed by higher climate costs in later periods. This means that the concept of a "Weak Green Paradox" is not as illuminating in our monopolistic framework as it is under perfect competition: the resource price paths before and after the policy change cross multiple times, implying several changes and shifts in the extraction pattern. Similar to the single-market case, the result of Proposition 8 shows that in the case without speculators the resource stock is depleted sooner as a consequence of a lower backstop production cost or a higher renewables subsidy, which is illustrated by panels (b) in Figures 3.11 and 3.12. The resulting shorter time horizon of resource extraction in combination with higher intermediate resource consumption might result in higher overall climate costs.

Proposition 6 in the comparative statics analysis gives the condition under which climate policy tightening increases both the duration of the stage with exclusive supply to region B and cumulative extraction during this stage in the case with speculators on the market. This effect can be seen in panels (a) of Figures 3.11, 3.12 and 3.13. Panels (b) show that in our numerical example the same effects on duration of and extraction in the second stage apply to the case without speculators. This (future) carbon leakage effect also has welfare consequences for both regions, which we analyze in this section.

Similar to the single market case, we can distinguish between a green and a non-green welfare component in region A: green welfare encompasses the climate costs, whereas non-green welfare denotes the region's utility from resource consumption.[22] Overall welfare denotes the sum of green and non-green welfare for region A. Region B, on the other

[21] The numerical effects of policy changes in the continuous and the discontinuous price case are summarized in Appendix B.2 in Tables B.1 and B.2, respectively.

[22] Note that region A's green welfare depends on total emissions and hence also on region B's resource consumption.

Figure 3.11: Consumption path - change in b

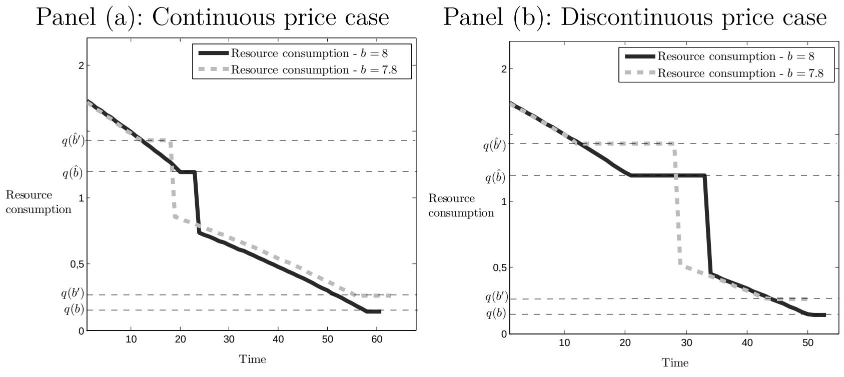


Figure 3.12: Consumption path - change in σ

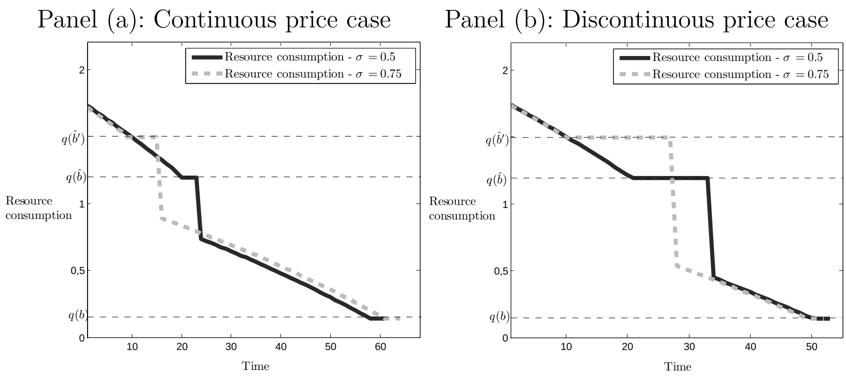
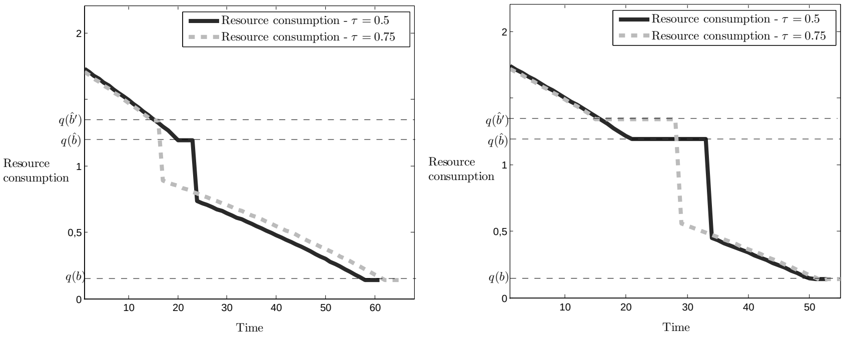


Figure 3.13: Consumption path - change in τ

Panel (a): Continuous price case Panel (b): Discontinuous price case



hand, only cares about non-green welfare. In Table 3.4 we give an overview of region B's non-green welfare W^B, region A's non-green and green welfare components (W_r^A and W_g^A, respectively), and region A's overall welfare W^A in the baseline case and after the climate policy changes for both price regimes.[23]

Table 3.4: Effects of policy changes on welfare

	Continuous price				Discontinuous price			
	Base	$b = 7.8$	$\sigma = 0.75$	$\tau = 0.75$	Base	$b = 7.8$	$\sigma = 0.75$	$\tau = 0.75$
W_g^A	-469.23	-492.35	-450.32	-461.63	-506.02	-493.42	-492.76	-481.96
W_r^A	14	17.84	11.07	12.48	14.69	18.3	11.12	14.61
W^A	-455.23	-474.5	-439.24	-449.14	-491.33	-475.28	-481.64	-467.35
W^B	24.98	29.1	26.4	29.24	22.39	26.5	24.62	27.35

A comparison of the welfare components in the baseline cases in Table 3.4 reveals that the qualitative and quantitative differences between the frameworks with and without speculators, which we have identified in Section 3.3.5, indeed have welfare consequences. On the one hand, non-green welfare is consistently lower (higher) for region A (B) in the presence of speculators. On the other hand, green welfare is lower in the baseline case in the absence of speculators: this is a consequence of the shorter overall resource extraction

[23]Table 3.2 in Section 3.3.5 provides the parameter values for the baseline case.

time.

Based on the previous propositions and the numerical results summarized in Table 3.4, let us have a closer look at the welfare effects of the different policy measures.

For both price regimes a lower b improves A's non-green welfare because region A can use more renewables at a lower price forever after the switch. Also region B profits from the lower backstop price and lower non-renewable resource prices as we see $p_2(T_2^+)$ decreasing in our numerical example. The effects on green welfare, however, might be negative and differ between the price regimes. Whereas initial resource consumption and therefore initial emissions fall, they increase during the limit-pricing phases. Additionally, the overall resource extraction phase is shortened in the discontinuous price case, as shown in Proposition 8. The change in A's *overall* welfare as a consequence of a lower backstop production cost depends on the respective parameter values, which are arbitrary in our case. We conclude that the positive effect of a lower backstop production cost on region A's non-green welfare might be offset by an increase in climate costs, which is made more probable by a major rise in resource consumption during the (first) limit-pricing phase and a shorter overall extraction period, especially in the discontinuous price case.[24]

Compared to a decrease in b, the picture is reversed regarding the effect of a rise in σ on region A's non-green welfare. Although a higher backstop subsidy increases region A's renewables consumption, its non-green welfare decreases because of the subsidy burden, which the region has to bear after its switch to renewables.[25] This pattern is also observed for both price regimes. Region B benefits indirectly as the monopolist shifts its supply to the phase when only the unregulated region B is on the market. The effects on green welfare display the same pattern as in the case of a lower backstop price: initial resource consumption decreases, but later emissions increase in both price regimes, and the overall extraction period shortens in the discontinuous price case. Given these findings, overall welfare is likely to be affected negatively by a higher backstop subsidy.

Similarly, a unilateral increase in the carbon tax worsens A's non-green welfare as it lowers the region's non-renewable resource consumption in both price regimes. This negative effect of a tax increase on A's non-green welfare contrasts our findings in the single-market case, where the regulated country is able to reap a part of the monopolistic scarcity rent for

[24]In our numerical example green welfare improves (worsens) in the case with (without) speculators. The results are sensitive to the parameter values used: a lower discount rate r, for instance, makes higher climate costs more probable.

[25]As noted earlier, we do not tackle commitment issues here.

itself. In the two-region framework, the regulated region A loses the resource rent gains to region B. Region B profits from the resulting lower producer price of the resource, its consequently higher resource consumption and the monopolist's supply shift to market B. Regarding green welfare, a higher τ decreases initial emissions and postpones the moment of depletion of the resource. Just as in the case of b and σ it is difficult to make definite statements about the effects of a higher carbon tax on overall welfare. We note, however, that green welfare improvement is likely in both price regimes, although region A does not benefit regarding its non-green welfare.

3.4 Conclusion

In this paper we have investigated Green Paradox effects in a resource extraction framework characterized by monopolistic production and multiple heterogeneous regions with unilateral climate policies.

In the first part of the paper we deal with a monopolist which supplies to a single market only. Our analysis shows that climate policy tightening can have adverse climate effects implying a *Strong* Green Paradox, despite the occurrence of a Weak Green Orthodox. Lower backstop costs and a higher backstop subsidy result in a shorter time horizon for resource extraction, making an increase in climate damages likely and affecting overall welfare in a negative way. In our numerical example, carbon taxation improves both green and non-green welfare by extending the resource extraction phase and by enabling the resource consumer to acquire a part of the monopolist's resource rent.

We expand our single-market model in the second part of the paper. Resource demand comes from two regions, one of which employs a tax on the imported resource and a subsidy on the available backstop technology. Two frameworks are considered: one with speculators on the market, where the monopolist is constrained to a continuous price, and one without speculators, where it is optimal for the monopolist to let the price jump upward when demand from the regulated region drops to zero. In both cases the resulting resource extraction paths possibly contain two limit pricing phases: one just before resource demand from the regulated region vanishes and one just before depletion of the resource.

Whereas we can exclude the occurrence of a Weak Green Paradox as a consequence of climate policy implementation or tightening, this concept is not so helpful when judging the desirability of climate policies in our framework. The reason is that due to the existence of the limit-pricing phases, the resource extraction and price paths before and after the policy changes cross several times. As a result, the effect of carbon taxes and backstop subsidies on the present value of climate damages depends on the discount rate and the overall length of the resource extraction phase. Upon a climate policy tightening, the monopolist shifts its supply to the unregulated region such that the regulated region switches earlier to backstop use. This (intertemporal) carbon leakage effect lowers non-green welfare in the regulated region.

Our results are complementary to those of De Sa and Daubanes (2014). They argue that in case of inelastic demand, oil suppliers choose for limit pricing throughout, which limits the effectiveness of climate policies such as carbon taxation and renewables subsidies. We show that, also in the case of elastic demand, limit pricing may be more important than suggested by conventional analyses of climate policy effects. The reason is that heterogeneous climate policies may cause an additional, intermediate limit-pricing phase. Nevertheless, climate policies in the form of a carbon tax or a renewables subsidy can still be effective in our setting as initial resource extraction and hence initial climate costs decrease. Moreover, climate policies may postpone the time at which the resource stock is depleted. The effect on the present discounted value of climate damages depends on the policy instrument, the absence or presence of speculators, on the functional form of climate damages and the value of discount rates.

Our study exhibits some limitations. Due to the complexity of our welfare representation, we do not derive optimal policies. Yet, we hope to provide the reader with an idea about welfare effects of climate policies. Furthermore, we assume that the monopolist is not able to use price discrimination. This is a valid assumption for the oil market, for instance, since oil can be easily shipped and is traded globally. Yet, the assumption might not hold in the case of gas, which is traded mostly regionally or by bilateral trading agreements. Furthermore, we do not consider strategic behavior on the part of the importing and exporting regions. This is an interesting and promising direction to extend the paper.