Resource extraction and the Green Paradox: Accounting for political economy issues and climate policies in a heterogeneous world

Ryszka, K.A.

2016

document version
Publisher's PDF, also known as Version of record

Appendix A

Appendices to Chapter 2

A.1 Analysis

A.1.1 Carbon Taxation

In order to obtain analytical results, we make some simplifying assumptions. In particular, we work with a demand function $D(p_i)$, satisfying $U'(D(p_i)) = p_i$, $i = y, x$, with p_i being the consumer price of the resource or the substitute. Note that the demand functions D_i and D_i^* , $i = x, y$, can vary between the two regions. We also suppose that $r = \rho$, i.e., the world market interest rate equals the rate of time preference, as in Section 2.2.3.
With the possibility of different taxes τ_y, τ_x, τ_y^* and τ_x^* the first-best equilibrium is characterized by six equations, comprising two stock conditions and four transition equations:

$$\int_0^{T_1} D_y^*(\lambda_y^* e^{rt} + c_y^* + \tau_y^*)dt + \int_0^{T_1} D_y(\lambda_y^* e^{rt} + c_y^* + \tau_y)dt = Y_0^*, \tag{A.1}$$

$$\lambda_y e^{rT_1} + c_y^* + \tau_y^* = \lambda_x e^{rT_1} + c_x + \tau_x^*, \tag{A.2}$$

$$\lambda_y^* e^{rT_1} + c_y^* + \tau_y = \lambda_x e^{rT_1} + c_x + \tau_x \tag{A.3}$$

$$\int_{T_1}^{T_2^*} D_x^*(\lambda_x e^{rt} + c_x + \tau_x^*)dt + \int_{T_1}^{T_2} D_x(\lambda_x e^{rt} + c_x + \tau_x)dt = X_0, \tag{A.4}$$

$$\lambda_x e^{rT_2^*} + c_x + \tau_x^* = b^* - \sigma^*, \tag{A.5}$$

$$\lambda_x e^{rT_2} + c_x + \tau_x = b - \sigma, \tag{A.6}$$

Equations (A.2) and (A.3) are equivalent, assuming that $\tau_t^* = \tau_x^*$ and $\tau_y = \tau_x$. One of them can thus be dropped. Following Hoel (2011), we differentiate the equation system (A.1) to (A.6) which gives:

$$M \begin{pmatrix} dT_1 \\ d\lambda_y^* \\ dT_2^* \\ dT_2 \\ d\lambda_x \end{pmatrix} = \begin{pmatrix} -B_y^* \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} d\tau_y^* + \begin{pmatrix} -B_y \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} d\tau_y + \begin{pmatrix} 0 \\ 1 \\ -B_x^* \\ -1 \\ 0 \end{pmatrix} d\tau_x^* + \begin{pmatrix} 0 \\ 1 \\ -B_x \\ 0 \\ -1 \end{pmatrix} d\tau_x$$
$$+ \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} d\sigma^* + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} d\sigma, \tag{A.7}$$

where

$$B_y^* = \int_0^{T_1} D_y^{*\prime}(\lambda_y^* e^{rt} + c_y^* + \tau_y^*)dt, \quad B_y = \int_0^{T_1} D_y'(\lambda_y^* e^{rt} + c_y^* + \tau_y)dt$$

and

$$B_x^* = \int_{T_1}^{T_2^*} D_x^{*\prime}(\lambda_x e^{rt} + c_x + \tau_x^*)dt, \quad B_x = \int_{T_1}^{T_2} D_x'(\lambda_x e^{rt} + c_x + \tau_x)dt,$$

and:

$$M = \left(\begin{matrix} \frac{D_y(\lambda_y^* e^{T_1} + c_y^* + \tau_y) + D_y^*(\lambda_y^* e^{T_1} + c_y^* + \tau_y^*)}{(\lambda_y^* - \lambda_x) r e^{rT_1}} & \frac{\int_0^{T_1} e^{rt} D_y^{*\prime}(\lambda_y^* e^{rt} + c_y^* + \tau_y^*)dt + \int_0^{T_1} e^{rt} D_y^{\prime}(\lambda_y^* e^{rt} + c_y^* + \tau_y)dt}{e^{rT_1}} \\ -D_x(\lambda_x e^{rT_1} + c_x + \tau_x) - D_x^*(\lambda_x e^{rT_1} + c_x + \tau_x^*) & \\ 0 & 0 \\ 0 & 0 \end{matrix} \right. \cdots$$

$$\cdots \left. \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -e^{rT_1} \\ D_x^*(\lambda_x e^{rT_2^*} + c_x + \tau_x^*) & D_x(\lambda_x e^{rT_2} + c_x + \tau_x) & \int_{T_1}^{T_2^*} e^{rt} D_x^{*\prime}(\lambda_x e^{rt} + c_x + \tau_x^*)dt + \int_{T_1}^{T_2} e^{rt} D_x^{\prime}(\lambda_x e^{rt} + c_x + \tau_x)dt \\ r\lambda_x e^{rT_1} & 0 & e^{rT_2^*} \\ 0 & r\lambda_x e^{rT_2} & e^{rT_2} \end{matrix} \right) \tag{A.8}$$

Adjusting this general framework to our baseline case of no taxes and no subsidy in the foreign country ($\tau_x^* = \tau_y^* = \sigma^* = 0$), a uniform tax in the home country ($\tau_x = \tau_y = \tau > 0$) and a positive subsidy ($\sigma > 0$), we get

$$\int_0^{T_1} D^*(\lambda_y^* e^{rt} + c_y^*)dt + \int_0^{T_1} D(\lambda_y^* e^{rt} + c_y^* + \tau_y)dt = Y_0^*, \tag{A.9}$$

(total demand for cheap oil equals supply)

$$\lambda_y^* e^{rT_1} + c_y^* = \lambda_x e^{rT_1} + c_x, \tag{A.10}$$

(continuous world market price at transition)

$$\int_{T_1}^{T_2^*} D^*(\lambda_x e^{rt} + c_x)dt + \int_{T_1}^{T_2} D(\lambda_x e^{rt} + c_x + \tau)dt = X_0, \tag{A.11}$$

(total demand for expensive oil equals supply)

$$\lambda_x e^{rT_2^*} + c_x = b^*, \tag{A.12}$$

(continuous consumer price at transition in foreign)

$$\lambda_x e^{rT_2} + c_x + \tau = b - \sigma. \tag{A.13}$$

(continuous world market price at transition in home)

We have then 5 equations in 5 unknowns ($\lambda_y^*, \lambda_x, T_1, T_2^*, T_2$). From these equations (A.9) to (A.13) we derive the direction of change of our Green Paradox variables of interest.

The effect of an increased tax τ on the resource rents is:

$$\frac{d\lambda_y^*}{d\tau} = \frac{-B_x - B_y + \frac{D_x + D_x^*}{\lambda_x}(\lambda_y^* - \lambda_x)\frac{-B_y}{D_y + D_y^*} + \frac{D_x}{r\lambda_x e^{rT_2}} + Q(\lambda_y^* - \lambda_x)r\frac{B_y}{D_y + D_y^*}}{A + \frac{D_x^* + D_x}{\lambda_x}\left((\lambda_y^* - \lambda_x)\frac{A}{D_y + D_y^*} + 1\right) + Q\left((\lambda_y^* - \lambda_x)r\frac{-A}{D_y + D_y^*} + 1\right)}, \tag{A.14}$$

$$\frac{d\lambda_x}{d\tau} = \frac{-B_x + B_y\frac{D_y + D_y^*}{L} + \frac{D_x}{\lambda_x e^{rT_2}}}{-(D_y + D_y^*)\frac{A}{L} - \frac{D_x + D_x^*}{\lambda_x} + Q}, \tag{A.15}$$

where:

$B_y = \int_0^{T_1} D_y'(\lambda_y^* e^{rt} + c_y^* + \tau)dt < 0$ (derivative of the demand function with respect to τ),

$B_x = \int_{T_1}^{T_2} D_x'(\lambda_x e^{rt} + c_x + \tau)dt < 0$ (derivative of the demand function with respect to τ),

$$A = \int_0^{T_1} e^{rt} D'_y(\lambda^*_y e^{rt} + c^*_y + \tau)dt + \int_0^{T_1^*} e^{rt} D^{*\prime}_y(\lambda^*_y e^{rt} + c^*_y + \tau^*)dt < 0$$

(derivative of the demand function with respect to λ^*_y),

$$Q = \int_{T_1}^{T_2} e^{rt} D'_x(\lambda_x e^{rt} + c_x + \tau)dt + \int_{T_1^*}^{T_2^*} e^{rt} D^{*\prime}_x(\lambda_x e^{rt} + c_x + \tau^*)dt < 0$$

(derivative of the demand function with respect to λ_x),

$$L = A(\lambda^*_y - \lambda_x)r - (D_y + D^*_y) < 0,$$

and abbreviating $D_y(\lambda^*_y e^{rT_1} + c^*_y + \tau)$ with D_y, $D^*_y(\lambda^*_y e^{rT_1^*} + c^*_y + \tau^*)$ with D^*_y, $D_x(\lambda_x e^{rT_2} + c_x + \tau)$ with D_x and $D_x(\lambda_x e^{rT_2^*} + c_x + \tau^*)$ with D^*_x.
Equations (A.14) and (A.15) imply that $\frac{d\lambda^*_y}{d\tau} < 0$ and $\frac{d\lambda_x}{d\tau} < 0$.

The effect of an increase in τ on the first switching point T_1 equals:

$$\frac{dT_1}{d\tau} = \frac{-B_x - \frac{D_x + D^*_x}{r\lambda_x}\frac{B_y}{A} + \frac{D_x}{r\lambda_x e^{rT_2}} + \frac{QB_y}{A}}{-(D_y + D^*_y) - \frac{D_x + D^*_x}{r\lambda_x}\left((\lambda^*_y - \lambda_x)r - \frac{D_y + D^*_y}{A}\right) + Q\left(\lambda^*_y - \lambda_x)r - \frac{D_y + D^*_y}{A}\right)}, \quad \text{(A.16)}$$

where:

$$M = D_y + D^*_y.$$

Whereas the denominator is unambigously negative, the nominator is also negative but for the first and second terms, i.e., $\left(-B_x + \frac{D_x}{r\lambda_x e^{rT_2}}\right) > 0$. In order to have $\frac{dT_1}{d\tau} < 0$, it needs to hold that:

$$\left|-B_x + \frac{D_x}{e^{rT_2} r\lambda_x}\right| > \left|-\frac{(D_x + D^*_x)B_y}{Ar\lambda_x} + \frac{QB_y}{A}\right|. \quad \text{(A.17)}$$

The effect of an increase in τ on the second switching point of the home region is:

$$\frac{dT_2}{d\tau} = \frac{-B_x + (D_y + D^*_y)\left(\frac{-B_y + e^{-rT_2}A}{K}\right) - \frac{D^*_x}{e^{rT_2}} + \frac{Q}{e^{rT_2}}}{(D_y + D^*_y)\frac{r\lambda_x A}{K} + D_x + D^*_x - Qr\lambda_x}, \quad \text{(A.18)}$$

with

$$K = D_y + D^*_y - r(\lambda^*_y - \lambda_x)A > 0.$$

Whereas the denominator is unambigously positive, the sign of the nominator depends on the parameter values.

For $\frac{dT_2}{d\tau} > 0$ it must hold that:

$$\left| -B_x + (D_y + D_y^*) \left(\frac{-B_y}{K} \right) \right| > \left| -(D_y + D_y) \left(\frac{A}{Ke^{rT_2}} \right) - \frac{D_x^*}{e^{rT_2}} + \frac{Q}{e^{rT_2}} \right|.$$

The effect of an increase in τ on the second switching point of the foreign region is:

$$\frac{dT_2^*}{d\tau} = \frac{-B_x - B_y \frac{D_y + D_y^*}{K} + \frac{D_y^*}{r\lambda_x e^{rT_2}}}{-Ar\lambda_x \frac{D_y + D_y^*}{K} + D_x + D_x^* - Qr\lambda_x} > 0. \tag{A.19}$$

This expression is unambigously positive.

A.1.2 Proofs of Propositions 1 and 2

Proof of Proposition 1.

The home country increases its subsidy σ. Suppose that the scarcity rent of the low cost resource increased in response, i.e., $\frac{d\lambda_y^*}{d\sigma} > 0$. Then the low cost resource would be depleted later as less resources are consumed at each point in time. The switching point to the high cost resource T_1 would have to occur later. If both T_1 and λ_y^* increase, also λ_x needs to increase according to:

$$(\lambda_y^* - \lambda_x)e^{rT_1} = c_x - c_y^* + \tau. \tag{A.20}$$

An higher λ_x results in a shortening of the switching times to the backstop resource, because for T_2^* and T_2 the following needs to hold:

$$\lambda_x e^{rT_2} = b - \sigma - c_x - \tau, \tag{A.21}$$

$$\lambda_x e^{rT_2^*} = b^* - c_x. \tag{A.22}$$

This, however, is impossible since there would not be enough demand for the high cost resource. Since also the high cost resource needs to be depleted, both T_1 and λ_y^* need to decrease. Since $(\lambda_y^* - \lambda_x)$ must rise, λ_x decreases, too. In this case we know that T_2^* has to increase (see equation (A.22)). The higher demand for the high cost resource from the part of the foreign region is matched by lower demand in the home region. The home region's non-renewable resource usage period is shortened and T_2 goes down. □

Proof of Proposition 2. The home country increases its tax τ. Let us suppose first that the scarcity rent of the high cost resources increased. From (A.21) and (A.22) we know this would accelerate the switch to using the backstop resource, i.e., T_2 and T_2^* would occur earlier.

Now, λ_y^* could increase or decrease. Let us suppose that it decreases. We see that the low cost resource phase needs to be prolonged from equation (A.20), i.e., T_1 increases. An increase in T_1 and a decrease in T_2 and T_2^* result in the high cost resource not being depleted completely. An increase in λ_x and a decrease in λ_y^* is hence not possible in equilibrium.

Suppose now that λ_y^* increases. Then two things can happen: $(\lambda_y^* - \lambda_x)$ can increase or decrease. Firstly, let us suppose it decreases, i.e., the increase of λ_x is bigger than the increase of λ_y^*. From (A.20) we know that T_1 needs to increase. But, as we have already argued before, this cannot be an equilibrium as the high cost resource would not be depleted.

Let us hence examine the case when $(\lambda_y^* - \lambda_x)$ increases, which happens when the increase of λ_x is smaller than the increase of λ_y^*. In this case the extraction time of the low cost resource T_1 shortens. This, however, cannot be an equilibrium since then a part of the low cost resource would be left in the ground.

We conclude hence that an increase in λ_x together with both an increase and a decreas in λ_y^* cannot occur in equilibrium.

Let us suppose now that the high cost resource rent decreases. From (A.22) we know that T_2^* is prolonged, whereas T_2 might be prolonged or shortened. Since λ_x decreases, also λ_y^* needs to fall, otherwise the low cost resource is not fully extracted.

For an equilibrium to exist both scarcity rents have to decrease in response to a tax increase. However, T_1 can increase or decrease: in case the high cost resource rent decreases more than the low cost rent, i.e., $\frac{d\lambda_x}{d\tau} > \frac{d\lambda_y^*}{d\tau}$, T_1 goes down. $\square$

A.2 Change in T_1 in Response to a Tax Increase

A.2.1 Analytical Example with Linear Demand

In this appendix we demonstrate the ambiguity of $\frac{dT_1}{d\tau}$. To that end we assume linear demand, identical across the two countries:

$$y_c(p) = x_c(p) = \alpha - \beta p,$$
$$y_c^*(p^*) = x_c^*(p^*) = \alpha - \beta p^*,$$

with $\alpha > b^* > b$. In an equilibrium we must have that:

$$\lambda_x e^{rT_1} + c_x = \lambda_y^* e^{rT_1} + c_y^*, \tag{A.23}$$
$$\lambda_x e^{rT_2} = b - c_x - \tau, \tag{A.24}$$
$$\lambda_x e^{rT_2^*} = b^* - c_x, \tag{A.25}$$
$$\int_0^{T_1} (y_c(t) + y_c^*(t))\, dt = Y_0^*, \tag{A.26}$$
$$\int_{T_1}^{T_2} x_c(t)dt + \int_{T_1}^{T_2^*} x_c^*(t)dt = X_0. \tag{A.27}$$

Let us, for the sake of notation, define:

$$v = rT_2 - rT_1, \quad w = rT_2^* - rT_1, \quad z = rT_1.$$

Then, inserting the linear demand functions into equations (A.26) and (A.27) and solving the integrals, (A.23) to (A.27) boil down to:

$$(2\alpha - 2\beta c_y^* - \beta\tau)z - 2\beta\left\{(b - c_x - \tau)e^{-v} + c_x - c_y^*\right\}(1 - e^{-z}) = rY_0, \tag{A.28}$$
$$(\alpha - \beta c_x - \beta\tau)v - \beta(b - c_x - \tau)(1 - e^{-v}) + (\alpha - \beta c_x)w - \beta(b^* - c_x)(1 - e^{-w}) = rX_0, \tag{A.29}$$
$$(b - c_x - \tau)e^{-v} = (b^* - c_x)e^{-w}. \tag{A.30}$$

For a given τ, the unknowns in this system of three equations are v, w and z.

Next, we compare the outcome under a no-tax regime with a tax regime where the home country never uses its own (expensive) resource.

For $\tau = 0$ we find $\tilde{v}$ and $\tilde{w}$ from (A.29) and (A.30) by inserting $\tau = 0$. Then $\tilde{z}$ follows from (A.28) upon insertion of $\tilde{v}$.

In the second regime we impose $\hat{v} = 0$. Then $\hat{w}$ follows from (A.29), with $v = \hat{v} = 0$. The corresponding tax $\hat{\tau}$ follows from (A.30). Finally, $\hat{z}$ is found from (A.28) with $\hat{\tau}$ and $\hat{v} = 0$

inserted.

We now need to compare $\tilde{z}$ and $\hat{z}$. In the zero tax regime we have, using (A.30):

$$(2\alpha - 2\beta c_y^*)\tilde{z} - 2\beta \left\{(b^* - c_x)e^{-\tilde{w}} + c_x - c_y^*\right\}(1 - e^{-\tilde{z}}) = rY_0. \tag{A.31}$$

In the tax regime we have, using (A.30):

$$(2\alpha - 2\beta c_y^* - \beta\hat{\tau})\hat{z} - 2\beta \left\{(b^* - c_x)e^{-\hat{w}} + c_x - c_y^*\right\}(1 - e^{-\hat{z}}) = rY_0. \tag{A.32}$$

Evidently, it holds that:

$$2\alpha - 2\beta c_y^* > 2\alpha - 2\beta c_y^* - \beta\hat{\tau}.$$

It also holds that:

$$(b^* - c_x)e^{-\tilde{w}} + c_x - c_y^* > (b^* - c_x)e^{-\hat{w}} + c_x - c_y^*,$$

because $\hat{\tau} > 0$ and $\hat{w} > \tilde{w}$. Moreover, $\hat{\tau}, \hat{w}$ and $\tilde{w}$ are determined by (A.29) and (A.30) and do not depend on Y_0. Note also that $1 - e^{-z}$ is increasing in z. Hence, $\hat{z} < \tilde{z}$ is only possible for small Y_0^*, which results in small z and consequently makes $(1 - e^{-z})$ small and the right hand side of (A.31) and (A.32) less important.
For high Y_0 is must be the case that $\hat{z} > \tilde{z}$. Hence, for high Y_0, T_1 increases with a higher tax. However, for low Y_0 the effect may be reversed due to (A.32).

The intuition is straightforward: The high cost producers are faced with an immense drop in demand as the home country basically ceases to demand the expensive resource due to $\hat{\tau}$. They have to adjust their scarcity resource more than the low cost resource producers, who still face demand from both regions, even though the home region's demand is reduced. Hence, $\frac{d\lambda_x}{d\tau} > \frac{d\lambda_y^*}{d\tau}$ and the switching point T_1 occurs earlier.

A.2.2 Numerical Example with Linear Demand

We illustrate the analytical possibility of a weak Green Paradox as a consequence of a tax increase by a numerical example. For the linear demand case presented in the previous Appendix A.2.1, we employ the parameter values in Table A.2.
For the first scenario of Appendix A.2.1, when $\tau = 0$, the resulting switching points amount

Table A.1: Parameter values for the numerical exercise regarding $\frac{dT_1}{d\tau} < 0$

Model's parameters		
Maxiumum demand	α	60
Demand slope	β	1.1
World interest rate	r	0.001
Extraction costs	c_y^*	0.005
	c_x	0.01
Backstop production costs	$b(b^*)$	45(55)
Resource stocks	$X_0(Y_0^*)$	10000 (0.5)

to $T_1 = 0.01126$, $T_2 = 269.68938$ and $T_2^* = 470.40049$ (i.e., $\tilde{w} = 0.470389$, $\tilde{v} = 0.269678$ and $\tilde{z} = 0.00001126$).

Setting $v = 0$ results in a positive $\hat{\tau} = 16.19972$. Whereas the foreign region switches now much later, namely at $T_2^* = 647.11838$, the first switching point occurs much earlier at $T_1 = 0.004894$ (in this case $\hat{w} = 0.647113486$ and $\hat{z} = 0.000004894$).

A Green Paradox is likely to occur if Y_0^* is small and X_0 relatively large. Furthermore, the world interest rate has to be relatively low, since it lowers z and hence the importance of the term $(1 - e^{-z})$ in equations (A.31) and (A.32). Also the parameters of the linear demand function play a role: a Green Paradox is likely to occur if α is relatively small, but β relatively large. Extraction costs, as long as they are not equal to zero, do not seem to play a major role: for $c_y^* = 0.5$ and $c_x = 1$, the switching points do not change much, only $\hat{\tau}$ decreases significantly.

A.3 Climate Damage and Social Cost of Carbon

We define $h(t) = \psi(x_c + x_c^*) + \psi^*(y_c + y_c^*)$. The total damage from atmospheric CO_2 in both regions and constant marginal damages ν equals:

$$\int_0^\infty e^{-\rho t} 2\nu(S_1(s) + S_2(s))ds \tag{A.33}$$

$$= \int_0^\infty 2\nu(S_1(s) + S_2(s))d\left(-\frac{1}{\rho}e^{-\rho s}\right) \tag{A.34}$$

$$= \frac{2\nu}{-\rho}(S_1(t) + S_2(t))e^{-\rho t}\,|_0^\infty + \int_0^\infty \frac{2\nu}{\rho}e^{-\rho s}\left(\dot{S}_1(s) + \dot{S}_2(t)\right) \tag{A.35}$$

$$= \frac{2\nu}{\rho}(S_1(0) + S_2(0)) + \int_0^\infty \frac{2\nu}{\rho}e^{-\rho s}\left(\alpha h(s) + (1-\alpha)h(s) - \delta S_2(s)\right) \tag{A.36}$$

$$= \frac{2\nu}{\rho}(S_{10} + S_{20}) + \int_0^\infty \frac{2\nu}{\rho}e^{-\rho s}h(s)ds - \int_0^\infty \frac{2\nu\delta}{\rho}e^{-\rho s}S_2(s)ds. \tag{A.37}$$

We need to solve for the last term in equation (A.37):

$$\int_0^\infty e^{-\rho s}S_2(s)ds = \int_0^\infty S_2(s)d\left(-\frac{1}{\rho}e^{-\rho s}\right) \tag{A.38}$$

$$= -\frac{1}{\rho}S_2(s)e^{-\rho s}\,|_0^\infty + \int_0^\infty \frac{1}{\rho}\dot{S}_s(s)e^{-\rho s}ds \tag{A.39}$$

$$= \frac{1}{\rho}S_2(0) + \int_0^\infty \frac{1}{\rho}e^{-\rho s}((1-\alpha)h(s) - \delta S_2(s))ds \tag{A.40}$$

$$= \frac{1}{\rho}S_{20} + \int_0^\infty \frac{1}{\rho}e^{-\rho s}(1-\alpha)h(s)ds - \frac{\delta}{\rho}\int_0^\infty e^{-\rho s}S_2(s)ds. \tag{A.41}$$

Bringing the right term of equation (A.41) to the left of equation (A.39), we solve for $\int_0^\infty e^{-\rho s}S_2(s)ds$:

$$\left(1 + \frac{\delta}{\rho}\right)\int_0^\infty e^{-\rho s}S_2(s)ds = \frac{1}{\rho}S_2(0) + \frac{1}{\rho}\int_0^\infty e^{-\rho s}(1-\alpha)h(s)ds, \tag{A.42}$$

$$\int_0^\infty e^{-\rho s}S_2(s)ds = \frac{1}{\rho+\delta}S_2(0) + \frac{1}{\rho+\delta}\int_0^\infty e^{-\rho s}(1-\alpha)h(s)ds. \tag{A.43}$$

Inserting this into (A.37), total damages equal:

$$\frac{2\nu}{\rho}(S_{10}+S_{20})+\frac{2\nu}{\rho}\int_0^\infty e^{-\rho s}h(s)ds-\frac{2\nu\delta}{\rho}\left\{\frac{1}{\rho+\delta}S_2(0)+\frac{1}{\rho+\delta}\int_0^\infty e^{-\rho s}(1-\alpha)h(s)ds.\right\} \tag{A.44}$$

$$=\frac{2\nu}{\rho}(S_{10}+S_{20})-\frac{2\nu\delta}{\rho}\frac{1}{\rho+\delta}S_{20}+\left(\int_0^\infty e^{-\rho s}h(s)\right)\left\{\frac{2\nu}{\rho}-\frac{2\nu\delta}{\rho(\delta+\rho)}(1-\alpha)\right\} \tag{A.45}$$

Disregarding the given initial carbon stock in the atmosphere, the first two terms of equation (A.45) disappear. The rest can be rewritten as:

$$\left\{2\nu\left(\frac{\alpha}{\rho}+\frac{(1-\alpha)}{\rho+\delta}\right)\right\}\int_0^\infty e^{-\rho s}h(s)ds. \tag{A.46}$$

A.4 Numerical Analysis and Calibration

We assume the home (foreign) region's utility function to be of a standard CRRA form:

$$A\frac{(x_c+y_c+z_c)^{1-\gamma}-1}{1-\gamma},\left(A^*\frac{(x_c^*+y_c^*+z_c^*)^{1-\gamma^*}-1}{1-\gamma^*}\right), \tag{A.47}$$

where $A(A^*)$ is a scaling parameter. From the first order condition we know that the home (foreign) region's demand equals (with $i=y,x$):

$$D_i(p_i)=\left(\frac{A}{p_i+\tau}\right)^{\frac{1}{\gamma}},\left(D_i^*(p_i)=\left(\frac{A^*}{p_i+\tau^*}\right)^{\frac{1}{\gamma^*}}\right)$$

As noted above, we use logarithmic utility in most of the numerical exercises in Sections 2.2.4 and 2.2.5 and set $\gamma=\gamma^*=1$. Yet, the calibration requires us to use the flexibilities provided by the functional forms we assumed. Table A.3 summarizes the parameter values used.

Table A.2: Parameter values for the numerical exercises

Model's parameters		Section 2.2.4	Section 2.2.5
Social discount rate	ρ	0.05	0.05
Coefficient of Relative risk aversion	$\gamma(\gamma^*)$	1 (1)	5(2)
World interest rate	r	0.05	0.05
Extraction costs	c_y^*	10	10
	c_x	20	20
Backstop production costs	$b(b^*)$	45(50)	45(50)
Emission factor	$\psi(\psi^*)$	1(1)	1(1)
Tax rate EP	τ	5	-
Subsidy EP	σ	5	5
Social cost of carbon	$2\nu\left(\frac{\alpha}{\rho}+\frac{1-\alpha}{\rho+\delta}\right)$	5	5

Table A.3: Parameter values for the calibration exercise

Calibration parameters		
Social discount rate (5 years)	ρ	0.05
Coefficient of Relative risk aversion	$\gamma(\gamma^*)$	2
Scaling parameter	A^*	$(\lambda_y + c_y) * 3.8^{\gamma^*}$
	A	$(\lambda_y + c_y) * 48.028^{\gamma}$
World interest rate	r	0.05
Extraction costs (in $trillion/ppmv)	c_y^*	0.1065
	c_x	0.852
Backstop production costs	$b(b^*)$	1.278(1.6614)
Emission factor	$\psi(\psi^*)$	1(1)
Social cost of carbon	$2\nu\left(\frac{\alpha}{\rho}+\frac{1-\alpha}{\rho+\delta}\right)$	0.0639