Resource extraction and the Green Paradox: Accounting for political economy issues and climate policies in a heterogeneous world

Ryszka, K.A.

2016

Appendix B

Appendices to Chapter 3

B.1 Equilibria in the Two-Region Model

B.1.1 No Speculators

We assume that $0 < T_1 < T_2 < T_3 < T_4$. Let us first derive an expression for $T_4 - T_3$. Equations (3.20b) and (3.22) imply:

$$\lambda_2 e^{rT_4} = b - k. \tag{B.1}$$

Since $p(t) > \hat{b}$ for $T_3 \leqslant t \leqslant T_4$, we have $\mu_{22}(t) = 0$ for $T_3 \leqslant t \leqslant T_4$. Moreover, it follows from (3.20b) together with (B.1) that:

$$(b - k)(e^{rT_3 - rT_4}) = b - k + p_2'(\tilde{q}_B)(\tilde{q}_B - \mu_{21}(T_3^+)). \tag{B.2}$$

We show that $\mu_{21}(T_3^+) = 0$. Since $\mu_{21}(T_3^-) = 0$ because $p(t) < b$, there would have to be an upward jump in μ_{21} if $\mu_{21}(T_3^+)$ were positive. But this contradicts the imperative of everything else being continuous at T_3. The length of the interval $T_4 - T_3$ hence equals:

$$T_4 - T_3 = \frac{1}{r} \ln \left(\frac{b - k}{[(1 - \eta_2(\tilde{q}_B))]b - k} \right). \tag{B.3}$$

Next we consider $T_3 - T_2$. By assumption it holds that $T_3 - T_2 > 0$. Then we have $\mu_{21}(T_2^+) = \mu_{21}(T_3^-) = 0$ since $p_2(T_2^+) < b$, and $p_2(T_3^-) < b$. Moreover, $p(T_3) = b, q(T_3) = \tilde{q}_B$ and $\mu_{22}(T_3^-) = 0$ since $p_2(T_3^-) > \hat{b}$. We use this in (3.28b) to get:

$$T_3 - T_2 = \frac{1}{r} \ln \left(\frac{(1-\eta_2(\tilde{q}_B))\, b - k}{\left(1-\eta_2[q(T_2^+)]\right) p_2[q(T_2^+)] - k} \right). \tag{B.4}$$

The value of $q(T_2^+)$ is still unknown. Using the results from Lemma 4, especially equation (3.28a), and combining it with $\mu_{21}(T_2^+) = 0$, we obtain the following expression for $q(T_2^+)$:

$$\frac{p_1[q(T_2^-)] - p_2[q(T_2^+)]}{p_2'[q(T_2^+)]} = \left[q(T_2^-) - q(T_2^+)\right] \frac{q(T_2^+)}{q(T_2^-)}. \tag{B.5}$$

Note that we have $p_1[q(T_2^-)] = \hat{b}$ and $q(T_2^-) = \hat{q}_A + \hat{q}_B$. Note also that $T_3 = T_2$ iff $q(T_2^+) = \hat{q}_B$.

Finally, we consider $T_2 - T_1$. By assumption it holds that $T_2 - T_1 > 0$. Then $p(T_1^+) = p(T_2^-) = \hat{b}$, $q(T_1^+) = q(T_2^-) = \hat{q}_A + \hat{q}_B = \hat{q}$. Also $\mu_{21}(t) = 0$ for all $T_2 \geqslant t \geqslant T_1$ since $p(t) < b$ for all $T_1 \leqslant t \leqslant T_2$. Moreover, $\mu_{11}(T_1^+) = 0$, because the price is continuous at T_1. Combining three equations, namely (3.20a) for $t = T_1$, (3.20a) for $t = T_2$ and (3.20b) for $t = T_2$ with $\lambda_1 = \lambda_2$ in (3.27), we obtain:

$$e^{rT_2 - rT_1} = \frac{p'(\hat{q})(\hat{q} - \mu_{11}(T_2^+)) + \hat{b} - k}{p'(\hat{q})(\hat{q}) + \hat{b} - k}. \tag{B.6}$$

Clearly, $T_1 = T_2 \Leftrightarrow \mu_{11}(T_2^+) = 0$. So, we still need to solve for $\mu_{11}(T_2^+)$. As explained above, we solve for $q(T_2^+)$ from equation (B.5). Then we use (3.28b) to find $\mu_{11}(T_2^+)$. We can hence write $T_2 - T_1$ as follows:

$$T_2 - T_1 = \frac{1}{r} \ln \left(\frac{\left(1-\eta_2[q(T_2^+)]\right) p_2[q(T_2^+)] - k}{\left(1-\eta_1[\hat{q}]\left[1 - \frac{\mu_{11}(T_2^-)}{\hat{q}}\right]\right)(\hat{b}) - k} \right). \tag{B.7}$$

In the phases with $\mu_{11} = 0$ or $\mu_{21} = 0$, (3.20a)-(3.20b) give q as implicit functions of $\lambda_i e^{rt}$, i.e., $q(t) = f_1(\lambda_1 e^{rt})$ from (3.20a) and $q(t) = f_2(\lambda_2 e^{rt})$ from (3.20b). During phases with $\mu_{11} > 0$ or $\mu_{12} > 0$, the resource quantities are at the limit-pricing levels $q = \hat{q}$ and $q = \tilde{q}_B$, respectively. We can therefore characterize the stock threshold levels as follows:

$$S_{01} = (T_4 - T_3)\tilde{q}_B, \tag{B.8a}$$

$$S_{02} = \int_{T_2}^{T_3} f_2(\lambda_2 e^{rt})dt + (T_4 - T_3)\tilde{q}_B, \tag{B.8b}$$

$$S_{03} = (T_2 - T_1)\hat{q} + \int_{T_2}^{T_3} f_2(\lambda_2 e^{rt})dt + (T_4 - T_3)\tilde{q}_B. \tag{B.8c}$$

If $S_0 > S_{03}$, equating cumulative supply of and demand for the resource implies:

$$S_0 = \int_0^{T_1} f_1(\lambda_1 e^{rt})dt + (T_2 - T_1)\hat{q} + \int_{T_2}^{T_3} f_2(\lambda_2 e^{rt})dt + (T_4 - T_3)\tilde{q}_B. \tag{B.9}$$

By using (3.19a), (3.19b), (3.27), (3.28a)-(3.28b), (B.7), (B.4), (B.3) and (B.1) and (B.9) we have 13 equations to solve for λ_1, λ_2, $\mu_{11}(T_2^-)$, $\mu_{21}(T_2^+)$, $q(T_2^-)$, $q(T_2^+)$, T_1, T_2, T_3, T_4, S_{01}, S_{02}, and S_{03}.

In the generic case, where all four phases exist, we can determine the equilibrium outcome depending on the initial resource stock:

> If $S_{02} < S_0 < S_{03}$, we get $T_1 = 0$ while the endogenous variable S_{03} and the equations (B.7) and (B.8c) are dropped.
>
> If $S_{01} < S_0 < S_{02}$, we get $T_1 = T_2 = \mu_{21}(T_2^+) = 0$ while the endogenous variables λ_1, $q(T_2^-)$, $q(T_2^+)$, $\mu_{11}(T_2^-)$, S_{02}, S_{03} and the equations (3.19a), (3.19b), (3.27), (3.28a)-(3.28b), (B.7) and (B.4), and (B.8b)-(B.8c) are dropped.
>
> Finally, if $S_0 < S_{01}$, we get $T_1 = T_2 = T_3 = 0$ while the endogenous variables λ_1, $q(T_2^-)$, $q(T_2^+)$, $\mu_{21}(T_2^+) = 0$, $\mu_{11}(T_2^-)$, S_{02}, S_{03} and the equations (3.19a), (3.19b), (3.27), (3.28a)-(3.28b), (B.7), (B.4) and(B.3), and (B.8a)-(B.8c) are dropped.

B.1.2 Speculators

By making use of (3.22) and (3.20b), we get $\mu_{21}(T_4) = q(T_4) > 0$, implying a limit-pricing phase at the end with $q = \tilde{q}_B$ and length

$$T_4 - T_3 = \frac{1}{r}\ln\left(\frac{b-k}{(1-\eta_2(\tilde{q}_B))\,b-k}\right). \tag{B.10}$$

By using $\mu_{12}(T_2^+) = \mu_{22}(T_2^+) = \mu_{12}(T_3^-) = \mu_{22}(T_3^-) = 0$ in (3.20b) we obtain

$$T_3 - T_2 = \frac{1}{r} \ln \left(\frac{(1 - \eta_2(\tilde{q}_B))\, b - k}{(1 - \eta_2(\hat{q}_B))\, \hat{b} - k} \right). \tag{B.11}$$

In the phase with $\mu_{12} = 0$ and $\mu_{22} = 0$ holding between T_2 and T_3 when the price is between $\hat{b} < p < b$, (3.20b) gives q as implicit functions of $\lambda_2 e^{rt}$, i.e., $q = g_2(\lambda_2 e^{rt})$. During the phase with $\mu_{12} > 0$, holding between T_3 and T_4, we have $q = \tilde{q}_B$. Integrating resource use over time, we find:

$$S_{01} = (T_4 - T_3)\tilde{q}_B, \tag{B.12a}$$

$$S_{02} = \int_{T_2}^{T_3} g_2(\lambda_2 e^{rT_2} e^{r(t-T_2)})dt + (T_4 - T_3)\tilde{q}_B, \tag{B.12b}$$

where $\lambda_2 e^{rT_2}$ is obtained from (3.29b) and (3.20b) with $\mu_{21} = \mu_{22} = 0$. Accordingly, we have determined the thresholds S_{01} and S_{02} as well as the duration of the two phases and the shadow price of the resource stock during the second stage of the optimal control problem.[1]

By using (3.30a) in (3.20a) to substitute for $\mu_1(T_2^-)$, we find the duration of the limit price phase at the end of the first stage:

$$T_2 - T_1 = \frac{1}{r} \left(\frac{\hat{b}\left(1 - \eta_2(\tilde{q}_B)\frac{\tilde{q}_B}{\hat{q}}\right) - k}{\hat{b}\,(1 - \eta_1(\hat{q})) - k} \right). \tag{B.13}$$

In the phases with $\mu_{11} = 0$ in the interval $[0, T_1]$, (3.20a) gives q as an implicit function of $\lambda_1 e^{rt}$, i.e., $q(t) = g_1(\lambda_1 e^{rt})$. During the phase with $\mu_{11} > 0$ we have $q = \hat{q}$. Equilibration of cumulative supply of and demand for the resource then implies:

$$\begin{aligned} S_{03} &= (T_2 - T_1)\hat{q} + \int_{T_2}^{T_3} g_2(\lambda_2 e^{rT_2} e^{r(t-T_2)})dt + (T_4 - T_3)\tilde{q}_B \\ &= (T_2 - T_1)\hat{q} + S_{02}, \end{aligned} \tag{B.14a}$$

$$S_0 = \int_0^{T_1} g_1(\lambda_1 e^{-rt})dt + (T_2 - T_1)\hat{q} + \int_{T_2}^{T_3} g_2(\lambda_2 e^{rT_2} e^{r(t-T_2)})dt + (T_4 - T_3)\tilde{q}_B. \tag{B.14b}$$

[1] We did not yet investigate the case with $\mu_{22} > 0$. Suppose $\mu_{22} > 0$ at the beginning of the second stage of the problem, during an interval from T_2' until T_2, implying that the second stage would start with a limit-pricing phase during which $p_2(q) = \hat{b}$ and $q = \hat{q}_B$. Cumulative extraction in this phase would be given by $(T_2 - T_2')\hat{q}_B \geqslant 0$, where T_2' denotes the starting time of the concerning phase. Therefore, we have $S(\hat{T}_2') \geqslant S_{02}$. However, from (3.30a) we have $\lambda_1(\hat{T}_1^-) > \lambda_2(\hat{T}_1^+)$ implying that $S(\hat{T}_2') = S^*$, $T_2' = T_2$, and $\mu_2 2(T_2) = 0$. Hence, we get a contradiction.

Finally,

$$S_0 = \int_0^{T_1} g_1(\lambda_1 e^{-rt})dt + S_{03}. \tag{B.15}$$

Equations (3.30a), (B.13) and (B.14b) can be used to solve for T_1, T_2, and λ_1 if $S_0 > S_{03}$. Note that, if $S_{02} < S_0 < S_{03}$, we get $T_1 = 0$ while the endogenous variable S_{03} and the equations (B.13) and (B.14a) are dropped. When $S_{01} < S_0 < S_{02}$, we get $T_1 = T_2 = 0$ while the endogenous variables λ_1, S_{02}, S_{03} and the equations (3.29b), (B.11), (B.13), (B.12b), and (B.14a) are dropped. Finally, when $S_0 < S_{01}$, we get $T_1 = T_2 = T_3 = 0$ while the endogenous variables λ_1, S_{01}, S_{02}, S_{03} and the equations (3.29b), (B.10)-(B.11), (B.13), (B.12a)-(B.12b), and (B.14a) are dropped.

B.2 Numerical Effects of Changes in b, σ, τ

Table B.1: Effects of policy changes in the continuous price case

	Baseline	$b = 7.8$	$\sigma = 0.75$	$\tau = 0.75$
$p(0)$	6.5567	6.57	6.57	6.45
$p(T_2^+)$	7	6.822	6.77	6.77
$q(0)$	1.7295	1.718	1.71	1.705
$q(T_2^+)$	0.7328	0.856	0.886	0.886
T_1	19.43	10.7	8.35	13.74
$T_2 - T_1$	4.06	6.6	7.24	2.4
$T_3 - T_2$	35.02	37.7	46.1	46.1
$T_4 - T_3$	3.34	6.5	3.34	3.34
T	61.85	61.5	65	65.6
W_g^A	-469.23	-492.35	-450.32	-461.63
W_r^A	14	17.84	11.07	12.48
W^A	-455.23	-474.5	-439.24	-449.14
W^B	24.98	29.1	26.4	29.24

Table B.2: Effects of policy changes in the discontinuous price case

	Baseline	$b = 7.8$	$\sigma = 0.75$	$\tau = 0.75$
$p(0)$	6.55	6.553	6.55	6.44
$p(T_2^+)$	7.49	7.38	7.35	7.32
$q(0)$	1.7374	1.73	1.7325	1.73
$q(T_2^+)$	0.45	0.52	0.53	0.55
T_1	19.75	11.3	9.1	14.3
$T_2 - T_1$	12.75	16.9	18.1	14.1
$T_3 - T_2$	17.16	15.2	22.3	25.4
$T_4 - T_3$	3.34	6.5	3.34	3.34
T	53	50	52.8	55.33
W_g^A	-506.02	-493.42	-492.76	-481.96
W_r^A	14.69	18.3	11.12	14.61
W^A	-491.33	-475.28	-481.64	-467.35
W^B	22.39	26.5	24.62	27.35