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Resource extraction and the Green Paradox: Accounting for political economy issues and climate policies in a heterogeneous world

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Appendix C

Appendices to Chapter 4

C.1 Derivations

C.1.1 The Politician's Maximization Problem

For the following calculations we assume that the instantaneous utility functions from equation (4.1) for the society and the politician are of the following standard form:

$$u_S(C^S) = \frac{(C^S)^{1-\psi}}{1-\psi}, \quad \text{if} \quad \psi \neq 1, \quad u_S(C^S) = \ln(C^S) \quad \text{if} \quad \psi = 1, \quad \text{and}$$
$$u_P(C^P) = \frac{(C^P)^{1-\eta}}{1-\eta}, \quad \text{if} \quad \eta \neq 1, \quad u_P(C^P) = \ln(C^P) \quad \text{if} \quad \eta = 1, \tag{C.1}$$

where $\psi \equiv -\frac{C^S u_S''(C^S)}{u_S'(C^S)}$ is the coefficient of relative risk aversion and of relative intertemporal inequality aversion, and $\eta \equiv -\frac{C^P u_P''(C^P)}{u_P'(C^P)}$ denotes the same for the politician.

In order to solve the maximization problem (4.3) presented in Section 4.2.2, we divide the problem into two separate maximization problems. We suppose that the resource stock which the politician accroaches for himself, S_0^P, is known. Then we have to consider the following Hamiltonian:

$$H^P \equiv e^{-\delta t} u_P(C_t^P) - \mu_t (C_t^P), \tag{C.2}$$

with μ_t being the scarcity rent of the share S_0^P of the natural resource. The first order conditions for the Hamiltonian read as follows:

$$\frac{\partial H^P}{\partial C_t^P} = e^{-\delta t} u_P'(C_t^P) - \mu(t) = 0, \tag{C.3}$$

$$\frac{\partial H^P}{\partial S_t^P} = -\dot{\mu}(t) = 0. \tag{C.4}$$

Furthermore, the following transversality conditions should be satisfied:

$$\lim_{t\to\infty}[e^{-\delta t}\mu(t)S^P(t)] = 0 \text{ in the case of an infinite horizon,} \tag{C.5}$$

$$\mu(T) \geqslant 0, \mu(T)S_T^P = 0 \text{ in the case of a finite horizon.} \tag{C.6}$$

Total differentiation of (C.3) yields:

$$\dot{\mu}(t) = -\delta e^{-\delta t} u_P' + e^{-\delta t} u_P'' \dot{C}_t^P.$$

As $-\dot{\mu}(t) = 0$, we have that the politician's consumption path evolves in the following way (using the specification in (C.1)):

$$\frac{\dot{C}_t^P}{C_t^P} = -\frac{\delta}{\eta}. \tag{C.7}$$

In the case of a finite and infinite time horizon, respectively, the starting point of the optimal extraction path for the politician can consequently be expressed as:

$$C_0^P = S_0^P \left[\int_{t=0}^{T} e^{-\delta/\eta t}\right]^{-1} dt = \frac{\delta}{\eta}(1 - e^{-\delta/\eta T})^{-1} S_0^P \text{ and} \tag{C.8}$$

$$C_0^P = S_0^P \left[\int_{t=0}^{\infty} e^{-\delta/\eta t}\right]^{-1} dt = \frac{\delta}{\eta} S_0^P, \tag{C.9}$$

and the corresponding welfare in the infinite and finite cases, respectively, amounts to:

$$W^P = \int_0^\infty e^{-\delta t} \frac{\left(\frac{\delta}{\eta} S_0^P e^{-\delta/\eta t}\right)^{1-\eta}}{1-\eta} = \left(\frac{\delta}{\eta}\right)^{-\eta} (S_0^P)^{1-\eta} \text{ and} \tag{C.10}$$

$$W^P = \int_0^T e^{-\delta t} \frac{\left(\frac{\delta}{\eta}(1 - e^{-\delta/\eta T})^{-1} S_0^P e^{-\delta/\eta t}\right)^{1-\eta}}{1-\eta} = \left(\frac{\delta}{\eta}\right)^{-\eta} (1 - e^{-\delta/\eta T})^{\eta} (S_0^P)^{1-\eta}. \tag{C.11}$$

Suppose also the resource stock used for the benefits of society, S_0^S, is known. Then we have to consider the following Hamiltonian:

$$H^S \equiv e^{-\rho t} u_S(C_t^S) - \lambda_t(C_t^S), \tag{C.12}$$

with λ_t being the scarcity rent of the share S_0^S of the natural resource. The first order conditions for the Hamiltonian read as follows:

$$\frac{\partial H^S}{\partial C_t^S} = e^{-\rho t} u_S'(C_t^S) - \lambda(t) = 0, \tag{C.13}$$

$$\frac{\partial H^S}{\partial S_t^S} = -\dot{\lambda}(t) = 0. \tag{C.14}$$

Furthermore, the following transversality condition should be satisfied.

$$\lim_{t\to\infty}[e^{-\rho t}\lambda(t) S^S(t)] = 0. \tag{C.15}$$

Total differentiation of (C.14) gives:

$$\dot{\lambda}(t) = -\rho e^{-\rho t} u_S' + e^{-\rho t} u_S'' \dot{C}_t^S.$$

As $-\dot{\lambda}(t) = 0$ the society's resource consumption path evolves in the following way:

$$\frac{\dot{C}_t^S}{C_t^S} = -\frac{\rho}{\psi}. \tag{C.16}$$

Similarly to (C.8), the society's optimal resource consumption path is characterized by the following initial consumption:

$$C_0^S = S_0^S \left[\int_{t=0}^{\infty} e^{-\rho/\psi t}\right]^{-1} dt = \frac{\rho}{\psi} S_0^S, \tag{C.17}$$

where $S_0^S = S_0 - S_0^P$. Social welfare hence amounts to:

$$W^S = \int_0^{\infty} e^{-\rho t} \frac{\left(\frac{\rho}{\psi} S_0^S e^{-\rho/\psi t}\right)^{1-\psi}}{1-\psi} = \left(\frac{\rho}{\psi}\right)^{-\psi} (S_0^S)^{1-\psi}. \tag{C.18}$$

How can we now obtain an analytical solution for (4.3)? Having computed W^S and W^P

explicitly, we can translate (4.3) into the following maximization problem:

$$\max \ \gamma W^S(S_0^S) + (1-\gamma)W^P(S_0^P) \text{ s.t. } S_0 = S_0^P + S_0^S. \tag{C.19}$$

Inserting (C.10) or (C.11) and (C.18) into Equation (C.19) and differentiating with respect to S_0^P, we obtain:

$$(1-\gamma)\frac{\partial W^P}{\partial S_0^P} = -\gamma\frac{\partial W^S}{\partial S_0^P}, \tag{C.20}$$

which, if T is infinite, equals: (C.21)

$$(1-\gamma)(S_0^P)^{-\eta}\left(\frac{\delta}{\eta}\right)^{-\eta} = \gamma(S_0 - S_0^P)^{-\psi}\left(\frac{\rho}{\psi}\right)^{-\psi}, \tag{C.22}$$

and if T is finite, equals: (C.23)

$$(1-\gamma)(S_0^P)^{-\eta}\left(\frac{\delta}{\eta}\right)^{-\eta}(1-e^{-\delta/\eta T})^{\eta} = \gamma(S_0 - S_0^P)^{-\psi}\left(\frac{\rho}{\psi}\right)^{-\psi}. \tag{C.24}$$

The S_0^P solving the two equations (C.22) and (C.24) denotes the optimal S_0^{P*} and hence the solution to the maximiation problem (4.3) in case of an infinite and finite optimization horizon of the politician respectively. Then $S_0^{S*} = S_0 - S_0^{P*}$.

C.1.2 Proofs of Propositions 9, 10 and 11

Proof of Proposition 9. A decrease in the social weight γ necessitates an increase of S_0^P, as can be seen in equation (C.22). A higher S_0^P also leads to a higher initial resource consumption of the politician, C_0^P, according to (C.9), whereas a lower S_0^S implies lower initial social consumption according to (C.17). Then also initial aggregate resource consumption $C_0^S + C_0^P$ increases as $\frac{\delta}{\eta}S > \frac{\rho}{\psi}S$ for a given resource stock S.[1] With a higher S_0^P, W^P increases due to (C.10), whereas W^S drops, as implied in (C.11). □

Proof of Proposition 10. Increasing δ can only be offset by a decrease in S_0^P, as implied by equation (C.22). S_0^S increases, and hence C_0^S also rises, according to (C.17). According to (C.9), a rise in δ has an ambiguous effect on C_0^P due to the corresponding decrease in S_0^P. However, it is likely that S_0^P does not decrease as much as δ increases because the effect of the higher discount rate is diminished by a rise in the term $(S_0 - S_0^P)$ on the right

[1] In the nuemrical exercises we assume that $\psi = \eta$.

hand side of (C.22). Then also the initial resource consumption C_0^P increases according to (C.9). Aggregate resource consumption hence also increases. A higher discount rate of the politician has an unambiguously positive effect on social and a negative effect on the politician's welfare as both parts of the product of the politician's welfare in equation (C.10) decrease in size if δ rises and S_0^P falls. □

Proof of Proposition 11 From equation (C.24) we know that a fall in T has to be balanced by a fall in S_0^P. Then S_0^S increases, and so does C_0^S and W^S. Since $(1 - e^{-\delta/\eta T})$ is approaching 1 for high T, equation (C.8) tells us that the effect of a shorter time horizon on C_0^P is ambiguous. However, as the fall in T in (C.24) is compensated both by a fall in S_0^P and a rise in $(S_0 - S_0^P)$ on the right hand side, it is likely that C_0^P ultimately rises, as was confirmed by our numerical exercises. Together with a higher C_0^S, this implies higher overall initial extraction rates. The politician's welfare falls unambiguously as the terms $(1 - e^{-\delta/\eta T})^\eta (S_0^P)^{1-\eta}$ of equation (C.10) decrease. □

C.2 The Endogenous Model

C.2.1 Maximization of the Lagrangian

The present value Lagrangian for the endogenized political economy framework reads as follows:

$$\mathcal{L} = \left(\frac{1}{1+\rho}\right)^{t-1} \pi_t(C_{t-1}^S) u_P(C_t^P) - \mu_t(C_t^S + C_t^S), \tag{C.25}$$

with μ_t being the shadow price for the resource stock. We use the CRRA utility function $u_P(C_t^P) = \frac{(C_t^P)^{1-\eta}}{1-\eta}$. The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t^P} = \left(\frac{1}{1+\rho}\right)^{t-1} \pi_t(C_{t-1}^S)(C_t^P)^{-\eta} - \mu_t = 0, \tag{C.26}$$

$$\frac{\partial \mathcal{L}}{\partial C_t^S} = \left(\frac{1}{1+\rho}\right)^{t} \frac{\partial \pi_{t+1}}{\partial C_t^S} \frac{{C_{t+1}^P}^{1-\eta}}{1-\eta} - \mu_{t+1} = 0, \tag{C.27}$$

$$\frac{\partial \mathcal{L}}{\partial S_{t+1}} = \mu_{t+1} - \mu_t = 0, \tag{C.28}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \mu_t}\mu_t &= \mu_t(S_t - S_{t+1} - C_t^P - C_t^S) = 0,\\
\frac{\partial \mathcal{L}}{\partial \mu_t} &= S_t - S_{t+1} - C_t^P - C_t^S \geqslant 0,\\
\mu_t &\geqslant 0.
\end{aligned} \tag{C.29}$$

From the first order conditions we see that the present value of μ_t does not change.

The politician's resource consumption is governed by the following equation, assuming that the probability of staying in power is of the form $\pi_t = \frac{C_{t-1}^S}{1+C_{t-1}^S}$:

$$C_{t+1}^P = \left(\frac{C_t^S}{1+C_t^S}\right)^{1/\eta}\left(\frac{1}{1+\rho}\right)^{1/\eta} C_t^P. \tag{C.30}$$

Equating (C.26) and (C.27) and inserting the expression for C_{t+1}^P as above yields:

$$C_t^P = (1+C_t^S)^{1+1/\eta}(C_t^S)^{\psi-1/\eta+1}(1+\eta)^{1/\eta}(1-\eta).$$

The aggregate extraction rate equals

$$R_t = C_t^P + C_t^S,$$

and has to satisfy the resource constraint $S_0 = \sum_{t=1}^{\infty} R_t$. The politician hence has to choose μ_0 such that, given $C_0^P = \mu_0^{-1/\eta}$ and the equation governing the evolution of the politician's consumption (C.30), the intertemporal sum of $R_t = C_t^P + C_t^S$ equals S_0.

C.2.2 Proof of Proposition 12

Proof of Proposition 12. From equation (C.26) it follows that:

$$u_P'(C_t^P) = \frac{\mu_t(1+\rho)^t}{\pi_t(u_S(C_{t-1}^S))} \quad \text{and} \quad u_P'(C_{t+1}^P) = \frac{\mu_{t+1}(1+\rho)^{t+1}}{\pi_{t+1}(u_S(C_t^S))}.$$

As $\mu_{t+1} = \mu_t$, we can substitute one equation into the other. Hence, the evolution of the politician's consumption is characterized by

$$\frac{u'_P(C_t^P)}{u'_P(C_{t+1}^P)} = \left(\frac{1}{1+\rho}\right)\frac{\pi_{t+1}(u_S(C_t^S))}{\pi_t(u_S(C_{t-1}^S))}.$$

Rewriting and including $\pi_t = 1$ yields:

$$u'_P(C_{t+1}^P)\left(\frac{\pi_{t+1}(u_S(C_t^S))}{1+\rho}\right) = u'_P(C_t^P).$$

Using specific functional forms, namely the CRRA utility function $u_P(C_t^P) = \frac{(C_t^P)^{1-\eta}}{1-\eta}$, leaves us with the following expression:

$$C_{t+1}^P = \left(\frac{\pi_{t+1}(u_S(C_t^S))}{1+\rho}\right)^{1/\eta} C_t^P.$$

This is very intuitive: the marginal utility of resource consumption in the current period t needs to equal the next period's expected marginal utility of consumption. Stated differently, the consumption of the resource in the next period $t+1$ is current consumption discounted by the discount factor and the probability of staying in power. □

C.3 Numerical Method

With the approach presented Appendix C.1.1 we can obtain analytical solutions of the model. Equations (C.20) to (C.24), however, require a numerical solution which is easily obtained by a non-linear solver in Matlab. Table C.1 displays the parameter values used in the numerical examples for the baseline scenario.

In the endogenous model analytical solutions are hard to attain. We have to find a numerical solution. As a first step, we take an initial guess for C_0^P. Equating (C.26) with (C.27), and using (C.26) to obtain an expression for C_{t+1}^P, we are able to obtain C_0^S. Now we know the aggregate resource consumption of the initial period. Knowing C_0^P gives us knowledge of the next period's optimal value of the politician's consumption. Hence, we can repeat the second step of equating (C.26) with (C.27) in order to obtain the corresponding C_t^S. The resulting aggregate resource consumption $\hat{S}_0$, the sum of C_t^S and C_t^P, is compared to the

Table C.1: Parameter values for the numerical exercises

Model's parameters		
Initial resource stock	S_0	100
Social weight	γ	0.7
Discount rates	ρ	0.05
	δ	0.08
Political time horizon	T	∞
Elasticity of intertemporal	η	0.5
Substitution	ψ	0.5

initial stock S_0. If $\hat{S}_0 \neq S_0 + |\epsilon|$, where ϵ is an error margin, we start with a new initial guess C_0^P. The parameters used are exactly the same as displayed in Table C.1, but for the values of γ and δ, which are found endogenously in the numerical model.