Topics in Markov Chain Theory and Simulation Optimisation

Berkhout, J.

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1. OVERVIEW OF THIS THESIS

This thesis covers different topics from Markov chain theory together with a side-step to the topic of simulation optimisation. On the basis of my ‘PhD research journey’ I would like to introduce the different research lines and further motivate its diversity.

The divergent topics of this thesis are the results of many questions which naturally emerged during the PhD trajectory and my determination to address them. It all started with the study of a real-life pharmacy inventory problem which we modelled as a *Markov chain*.

Markov chains are named for A. A. Markov who introduced its basic concepts in 1907. Since then much research has been conducted in the field of Markov chain theory with a wide range of applications, e.g., in queuing theory, statistics, social sciences and in internet applications such as the acclaimed Google PageRank for ranking websites on the internet. Background on discrete-time finite state space Markov chains can be found in the book of Kemeny and Snell in [115]. Kemeny and Snell further extended it to the continuous-time setting in [114] and to denumerable state spaces in [116].

In the pharmacy inventory study the goal was to establish a global optimisation algorithm for finding the best inventory policy (in particular, via series expansion techniques). The modelling of the inventory problem led to a Markov *multi*-chain, i.e., a Markov chain consisting of multiple closed communicating classes, see also [31]. As the long-term behaviour of multi-chains fails to be unique this leads to difficulties when directly calculating Markov chain concepts such as the ergodic projector, [161]. The ergodic projector gives insight into the long-term behaviour of a Markov chain and is one of the key concepts. As Markov chain literature [115, 104] is mainly concerned with uni-chains we couldn’t find satisfactory direct computation approaches for multi-chains.

Determined to address this gap, we tried to develop a series expansion that was applicable to multi-chains. Series expansions of Markov chains can be used as an efficient numerical approximation technique for ergodic projectors, [47, 91, 92]. Eventually the series expansion research for multi-chains led to a publication in the *Procedia Computer Science* [32] and was even nominated for the best paper award by prof.dr. V. Alexandrov. Supported by the numerical power of the series expansion approach to multi-chains we developed the technique further to an approximation framework for the ergodic projector called *jump start power method* (JSPM). JSPM enhances the classical power method for Markov chains, [115], by ensuring a ‘jump start’ towards its ergodic projector leading to faster convergence. Moreover, numerical experiments showed that JSPM allows efficient evaluation of the notorious *nearly decomposable Markov chains* via a power method approach, for a treatment of this particular subject see [173]. Basically a Markov chain may be considered to be nearly decomposable if the chain can be divided into two or more subchains such that interactions between subchains are infrequent relative to the interactions within each subchain, [148]. The results of this research can be found in Chapter 5. The paper corresponding with Chapter 5 is submitted to a refined international journal and, backed by all the positive feedback and the generality of the results, I have no doubt that it will find its way into a good scientific journal.

Afterwards, we tried to obtain insight in the approximation error of JSPM. We found that the JSPM error reduction is related to the deviation matrix of the Markov chain, which is a fundamental matrix in Markov chain theory that can be used to calculate virtually everything that one would like to know from Markov chain modelled systems, [144, 115]. Eventually, we managed to make this insight fruitful by obtaining an approximate expression for the deviation matrix which can be efficiently calculated once a JSPM approximation has been found. These results are captured in Chapter 6 where the techniques are applied to Markov chain modelled networks, such as social networks [157]. In particular, a nearly decomposability detection algorithm for Markov chains is introduced which is applicable to homofily analysis, [141], and data clustering.

The approximation techniques from Chapters 5 and 6 lead to the development of an alternative ranking for Google PageRank in Chapter 7. Google PageRank was introduced by Page and Brin in 1998, [44], and has become a standard ranking methodology, [131]. The newly proposed ranking allows for a more structural ranking for nodes in general networks. This research was awarded the *Best Student Paper Award* at the international *Workshop on Discrete Event Systems 2016* and will be published in the corresponding proceedings [30].

When modelling real-life processes as Markov chains, such as the Markov chain modelled pharmacy inventory problem, the underlying parameters are usually unknown and have to be estimated by statistical methods, [94]. This motivates our study of the impact of parameter uncertainty in terms of performance measures such as long-term average inventory costs. This research line led to a publication in the proceedings of *Workshop on Discrete Event Systems 2014* [31]. The risk analysis techniques were further elaborated and applied to generalized Jackson networks which led to [171]. The results of this work can be found in Chapter 3.

Due to the knowledge accumulation of series expansions for Markov (multi-)chains during the above research lines, I got involved into research of B. F. Heidergott and K. Abbas on *perturbation bounds* for Markov chains. The goal of perturbation bounds theory is to efficiently obtain insight into the effect that a change of probability transition matrix has on the ergodic projector (or stationary distribution), see also [145]. This research led to the introduction of a series expansion based perturbation bound. Moreover, in this study we showed that existing perturbation bounds are most of the time not useful in practice and the new proposed perturbation bound showed promising results. All these finding presented in Chapter 4 were combined and accepted for publication in the journal *Markov Processes and Related Fields*.

A last line of research parallel to the above Markov chain research concerns the topic of simulation optimisation. Simulation optimisation considers problems that are difficult to formulate (in contrast to Markov chains for example) and for which simulation is the main tool for analysis. This kind of problems is present in every sector of society such as, for example, facility planning, job scheduling, revenue management, complex inventory policies, and portfolio management. Solving such problems, both conceptually and numerically, is a hard task and is topic of research in simulation optimisation [74, 73]. In Chapter 2 a state of the art solving methodology called nested partition hybrid algorithm, [169], is improved in such way that a computational speed-up of 18% on average is achieved.

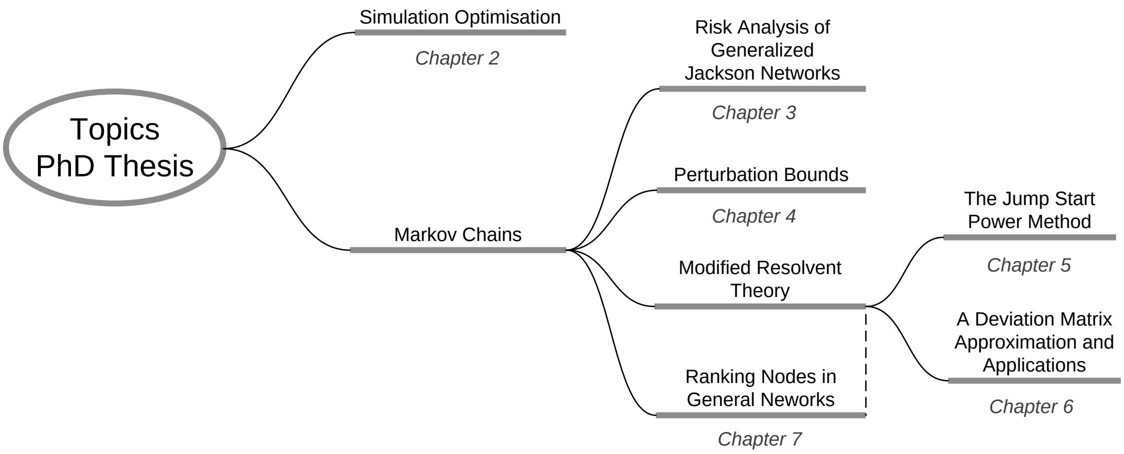

Fig. 1.1: Overview of PhD thesis.

The work led to a publication in *Discrete Event Dynamic Systems* [28].

Each chapter starts with a short overview of the main results and is followed by the outline of the chapter. Furthermore, each chapter contains a detailed introduction into the specific research topic which includes an exposition of the current literature. All chapters are self-contained in the sense that each chapter (re)introduces the required notation. Figure 1.1 gives an overview of all research topics corresponding to the chapters of my PhD journey.