Nonlinear Mechanics of Fibrous Networks
Licup, A.J.

2016

document version
Publisher's PDF, also known as Version of record

Link to publication in VU Research Portal

citation for published version (APA)

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal?

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:
vuresearchportal.ub@vu.nl
## Contents

### 1 Introduction
1.1 Biopolymer Networks: Life’s Woven Fibers .................................. 5
1.2 Elasticity of Biopolymers .................................................. 6
1.3 Network Mechanics and Nonlinear Effects .................................. 8
  1.3.1 Nonlinear stiffening .................................................. 9
  1.3.2 Negative normal stress ............................................... 9
1.4 Experimental and Theoretical Approaches .................................. 9
  1.4.1 Rheology experiments .............................................. 10
  1.4.2 Theoretical models ................................................ 11
1.5 Outline of the Thesis ....................................................... 12

### 2 Modeling athermal sub-isostatic fiber networks
2.1 Introduction ................................................................. 17
2.2 Network Architecture ..................................................... 18
  2.2.1 Lattice-based phantom network .................................... 18
  2.2.2 Mikado network ..................................................... 19
2.3 Network Elasticity .......................................................... 20
  2.3.1 Fiber elasticity: the extensible worm-like chain model .......... 20
  2.3.2 Affine/non-affine network response ................................ 21
  2.3.3 Network elastic energy density .................................... 22
  2.3.4 Rheology simulation ............................................... 23
  2.3.5 Quantitative comparison of measurements ......................... 24
2.4 Results ................................................................. 25
  2.4.1 Distinct elasticity regimes ......................................... 25
  2.4.2 Fiber buckling ...................................................... 27
  2.4.3 Linear regime ..................................................... 28
  2.4.4 Bending correlation length ....................................... 30
  2.4.5 Prefactors to the modulus ...................................... 30
  2.4.6 Nonlinear stiffening .............................................. 31
  2.4.7 Stretch-dominated elasticity ..................................... 34
2.5 Discussion and Implications ............................................. 35
2.6 Appendix: Line Density Calculation of Lattice-Based Networks ...... 37

### 3 The role of normal stress in collagen network mechanics
3.1 Introduction ................................................................. 41
3.2 Results and Discussion ................................................... 43
3.2.1 Physical picture ................................................. 45
3.2.2 Modeling collagen networks ................................. 45
3.2.3 Features of linear and nonlinear elasticity .............. 48
3.2.4 Concentration independence nonlinear stiffening ...... 49
3.2.5 Role of network geometry ..................................... 50
3.2.6 Stiffening mechanism ......................................... 52
3.2.7 Onset of strain stiffening .................................... 54
3.2.8 Generalized onset of stiffening ............................. 56
3.2.9 Normal stress controls nonlinear stiffening ............. 59
3.3 Concluding Remarks .............................................. 63
3.4 Appendix A: Materials and Methods ......................... 66
3.5 Appendix B: Shear and Normal Stresses in Networks ....... 67

4 Mechanically-controlled criticality in fiber networks ........ 71
4.1 Introduction ....................................................... 73
4.2 Models of Sub-isostatic Athermal Fiber Networks .......... 75
4.2.1 Network construction ......................................... 75
4.2.2 Network elasticity ............................................ 75
4.2.3 Fiber rigidity and volume fraction ......................... 77
4.3 Strain-driven Criticality under Simple Shear ............... 77
4.3.1 Continuous rigidity transition ............................... 77
4.3.2 Analogy with the ferromagnetic transition ............... 79
4.3.3 Divergent non-affine fluctuations ........................ 80
4.3.4 Finite-size scaling ........................................... 82
4.3.5 Crossover between elastic regimes ......................... 83
4.3.6 Evidence of crossover from collagen networks .......... 84
4.3.7 Crossover function and model fitting ...................... 87
4.3.8 Evolution of critical exponents ............................. 90
4.4 Nonlinear Mechanics under Isotropic Expansion .......... 92
4.5 Critical Slowing Down .......................................... 96
4.6 Discussion and Conclusions .................................... 96
4.7 Appendix A: Materials and Methods ......................... 99
4.8 Appendix B: Distorted Honeycomb Lattice Model ........... 100

5 The mechanics of floppy rope networks under stress ........ 103
5.1 Introduction ....................................................... 105
5.2 The Sub-isostatic Rope Network ............................... 107
5.3 Network Elasticity ............................................... 108
5.4 Results and Discussion ......................................... 110
5.4.1 Rope network response ....................................... 110
5.4.2 Rope networks stabilized by prestress .................... 110
5.4.3 Stiffening mechanism ........................................ 112
5.5 Concluding Remarks ............................................. 114

6 Decoupling of shear and Young’s moduli in extracellular networks 117
6.1 Introduction ....................................................... 119
6.2 Materials and Methods .......................................... 122
6.3 Modeling and Rheology Simulation ........................................ 122
6.4 Results ......................................................................... 124
  6.4.1 Shear response under external axial stress ....................... 124
  6.4.2 Shear response under internal contractile stresses ............ 127
  6.4.3 Stiffening mechanism .................................................. 128
6.5 Discussion and Implications .............................................. 130
6.6 Appendix: Materials and Methods ..................................... 134

Bibliography .................................................................. 137
List of Publications ......................................................... 155
Summary ........................................................................ 156
Samenvatting .................................................................. 159
Acknowledgements ........................................................ 163