Phase transitions in Bak-Sneppen avalanches and in a continuum percolation model

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Summary

This thesis concerns itself with the study of two stochastic models: the Bak-Sneppen model and a continuum percolation model. Both of these models consist of a large number of microscopic particles arranged in some spatial structure. The behaviour of these particles is governed by probabilistic rules. The challenge with such models is to deduce the macroscopic behaviour of these systems given the behaviour of the microscopic particles. The final chapter is devoted to the continuum percolation model with the rest of the thesis treating the Bak-Sneppen model.

The Bak-Sneppen model is a relatively well-known model and is a 'toy' model of evolution. In its original setting, N species are arranged on a circle. Each species is given a rating between 0 and 1 called its *fitness*. The fitnesses are to represent how well-adapted the species are. The model changes in (discrete) time by the following procedure. The species with the smallest fitness is located and the fitness of this species and those of its two neighbouring species are replaced by independent uniform $(0, 1)$ random variables.

The Bak-Sneppen model enjoys non-trivial behaviour with the fitnesses converging to a critical state over time. For this reason, the model has attracted attention as an example of self-organised criticality. In order to analyse the model it is often convenient to divide it up into *avalanches*. In short, a p-avalanche is a period of time starting when all fitnesses are greater than p and ending the first time this property is repeated. A full introduction to the model can be found along with some fundamental results in Chapter 2 of this thesis.

Although much of the research about the model focuses on the limit behaviour, there is also interest in how the model converges to its critical state. Chapter 3 takes this approach by considering the behaviour of the initial avalanche. The somewhat surprising result that the expected duration of the initial avalanche is infinite. Perhaps the main result of the thesis can be found in Chapter 4, where rigorous comparisons between the Bak-

Sneppen model and site percolation are made via a coupling argument. This coupling approach allows bounds for the critical values of the Bak-Sneppen model to be calculated.

In Chapter 5, a new continuum percolation model is introduced. This model is a generalisation of the nearest-neighbour model, where vertices inhabiting a continuous space connect themselves to their nearest-neighbouring vertices. This creates a random graph and the interest is in deciding for which parameter values there are infinitely many vertices connected to each other. Although some of the results for the original model still hold in the generalised model, there are a few important differences. In particular, it appears the the critical value of the model is no longer monotone in the dimension.