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Summary

Starting in 1944 with the publication of 'Theory of Games and Economic Behavior' by John von Neumann and Oscar Morgenstern, cooperative games have been studied now for 65 years. As counterpart of the subject of conflicting interests studied by non-cooperative games, von Neumann and Morgenstern introduced the notion of cooperative game as a particular type of non-cooperative games in which commitments are fully binding and enforceable (Harsanyi, 1966). Although cooperative games can be considered as a special case of non-cooperative games, historically cooperative games are formulated in a form that abstract away from describing the negotiation process and enforcement procedures explicitly. Instead it concentrates on the possibilities for agreement and studies questions like 'what coalitions will form?' and 'how will the payoff to a coalition that forms be divided between its members?'.

In this thesis we follow this approach and consider the standard situation that each coalition of players can reach a certain payoff by cooperating together. It is profitable for a group of players to join together if the payoff that can be obtained to the coalition of all players of the group is at least as high as when they all stay single or form several subgroups. After a coalition is formed, the next question is to agree on a distribution of the total payoff of the coalition amongst its members. Within cooperative game theory it is widely accepted that the payoffs that can be obtained by every subcoalition when operating on its own, are taken into account in obtaining a distribution of the total payoff of the coalition. A procedure that gives a distribution of the total payoff of a coalition taking into account the payoffs of all subcoalitions is called a solution. One of the main goals of cooperative game theory is the construction of a 'fair' solution. What does the word fair mean here? There are various definitions of this notion and the number of different approaches to this notion grows in the number of various solutions.

A possible notion of the 'fairness' of a distribution is to evaluate fairness by some distance measure between the payoffs to any subcoalition of players assigned by the solution and the payoffs that these subcoalitions can attain on their own. The traditional assumption in studying this type of problems is that every coalition is feasible and can form to attain their payoff. When defining fairness by some distance measure, this means that information about the payoffs of every subcoalition can be used. However, in many real life situations not every group of players has the opportunity to cooperate and to collect

their own payoff. We say that we deal with cooperative games with restricted cooperation when not all coalitions can form. The reason of restrictions on the collection of feasible coalitions can be various, for instance restrictions induced by law, restrictions on the maximum number of players that are allowed to cooperate, restrictions because their is no full communication between players or restrictions because players need consent of their superiors to form coalitions with others. These type of situations can easily occur, but such situations are ignored by traditional cooperative game theory in which it is assumed that every group of players can form. Modelling situations of real life, a typical question is which properties have to be taken into account and which properties of the particular situation can be ignored. The question whether it is sufficient to model only the most important properties or to complicate the model including more specific properties, is not easy to answer. Within the framework of restricted cooperation the question is whether the standard model will be used, assuming that all coalitions may form, or a more specific model taking into account that not all coalitions are feasible. To answer this question we have to consider whether or not a more extended model provides us with a better prediction of the outcome. When a more complicated model leads to new and important information about the outcome, then it might be useful to use this model.

Myerson (1977) is one of the first studies in which restricted cooperation is taken into account. He considered a situation in which communication between agents is modelled by a non-directed graph on the set of agents. A group of players is able to communicate and thus feasible if and only if the corresponding subgraph is connected. The paper constructs and axiomatizes a solution for this situation of restricted cooperation. Even in case the graph on the group of all players is connected and thus the coalition of all players, the grand coalition, is feasible, Myerson's result shows that it is important to take into account the restricted cooperation. In fact, the model shows that the distribution of the total payoff of the grand coalition not only depends on how much payoff the subsets of players can collect, but also on the structure of the collection of feasible coalitions. His contribution has inspired many authors to study models with restricted cooperation and to develop solutions for these models.

A collection of coalitions is union stable when for every two coalitions in the collection with non-empty intersection, also the union of the two coalitions is in the collection. The collection of coalitions that are connected within a graph is union stable. Another interesting and simple property of a collection of coalitions is union closedness. We say that a collection of coalitions is union closed if the union of every pair of coalitions from this collection is also member of this collection. An example of a situation that results in a union closed collection of feasible coalitions is when agents are organized is some hierarchical structure represented by a directed graph, referred to as games with a permission structure. In such a situation, a player is a predecessor (or direct superior) of another player, when there is a (directed) edge from the former to the latter player. A coalition of players is feasible, when every player in the coalition has (some of) its predecessors (if any) in the coalition. Every player with at least one predecessor needs permission of some of (a subset of) its predecessors to cooperate with other players.

Notice that the class of games with union closed collection of feasible coalitions is a subclass of games with union stable collection of feasible coalitions. Of course, the latter class is a subclass of the class of games without restrictions on the collection of feasible coalitions. Properties of solutions that are satisfied for every game in a class of games, remain true for every game in a subset of this class. On the other hand, every uniqueness theorem that characterizes a solution on a class of games as the unique solution satisfying some properties, is weaker than the equivalent theorem that characterizes a solution by the same properties on a subset of this class. So, uniqueness theorems provide more information about the nature of a solution when the class of games becomes smaller.

Most research in the field of games with restricted cooperation are either concerned with the class of union stable collections of feasible coalitions or they consider just a particular case within the smaller class of games with a union closed collection of feasible coalitions, for instance games with a union closed system induced by a permission structure, or games with an antimatroid as the collection of feasible coalitions. Uniqueness theorems are either stated on the class of union stable collections of feasible coalitions or on a subclass of games with a union closed collection of feasible coalitions. This thesis is mostly concerned with games on union closed systems. On the one hand, solutions are considered and characterized on the general class of games with union closed systems, on the other hand specific properties of solutions and algorithms to compute solutions, in particular the nucleolus, are considered on specific subclasses of the class of games on union closed systems.

Outline of the thesis

Chapter 2 contains known facts about cooperative games. In this chapter properties of solutions and the solution concepts core, Shapley value, (pre)nucleolus and (pre)kernel are given. The notion of Davis-Maschler consistency, which plays an important role within this thesis, is given. Also basic notions and definitions of graph theory are introduced. Finally, the concept of restricted game is discussed. In most papers about games with restricted cooperation a solution is obtained from a solution concept for a standard cooperative game without restrictions. The standard approach is to define a modified game without restrictions, called the restricted game, in which in some way payoffs for the nonfeasible coalitions are induced by the payoffs of their feasible subsets. Then the solution for the game without restrictions is used to obtain a solution for the original game with restrictions.

The other seven chapters of the thesis are divided into two parts. Part I consists of the chapters 3-6 and deals mainly with properties and the characterizations of solutions for games with restricted cooperation. Part II contains the chapters 7-9 and is mostly devoted to algorithms for the computation of the nucleolus for games with a permission structure (chapters 7, 8) or some other particular feature (chapter 9).

In chapter 3 games on union closed systems are described. The class of such games is a subclass of the class of games with restricted cooperation, namely that the collection of feasible sets is closed under union, but generalizes the classes of games with a permission structure and games on antimatroids. In this chapter an important notion for games on union closed systems is introduced: the superior graph. This graph can be constructed for every set system which is closed under union. Then, for every game with union closed system, a modified game with a permission structure can be defined by using the superior graph of the union closed system. Properties of the core, the Shapley value, the prenucleolus and the prekernel are considered on the class of games with union closed systems.

Also chapter 4 deals with games on union closed systems. In this chapter two new solutions are defined and axiomatized. The first of them (the superior rule) is based on the superior graph and is defined as the conjunctive permission value on this graph. The second (the union rule) is based on the approach which is similar to the approach described by Myerson (1977) applying the Shapley value directly to a modified game.

Chapter 5 differs from the other chapters in the sense that this chapter does not deal with restricted cooperation. In this chapter a class of new solutions for standard cooperative games is introduced. This class is based on properties which hold for both the prekernel and the prenucleolus. The prekernel is the maximal (with respect to inclusion) solution on this class and the prenucleolus the minimal. All other solutions in the class are intermediate between the prenucleolus and the prekernel. Each of these solutions can be described by a positive integer k. To characterize these solutions the known properties of reconfirmation and converse consistency are generalized to k-reconfirmation and k-converse consistency. Also a generalization of Kohlberg's theorem is formulated and proved.

Chapter 6 deals with a special subclass of games with a permission structure, the class of peer-group games. A peer-group game is a game with a permission structure induced by a tree as the underlying graph, and with an additive game as the underlying game. Without restricted cooperation, the class of additive games is trivial and the only efficient and individually rational solution is to give every player its own contribution. Therefore the interesting question is to analyse the impact of the permission structure on the distribution of the total payoff among the players. Moreover, the class of peer-group games is worthwile to consider because it has various interesting economic applications. In this chapter a characterization of the Shapley value on the class of peer-group games is given and also some interesting properties of the nucleolus are discussed.

The second part of the thesis is devoted to computational aspects in cooperative game theory, in particular algorithms to compute the nucleolus are given. The nucleolus is a very specific solution because its definition is not constructive. It is possible to prove that for every cooperative game the nucleolus exists and consists of only one point, but for the general class of games, algorithms to find the nucleolus do not exist. There is no simple way to construct the nucleolus for an arbitrary cooperative game, but it is possible to design algorithms for particular classes of games.

Chapter 7 presents a polynomial time algorithm for finding the nucleolus of games with a disjunctive permission structure, under some conditions on the game and the directed graph. In particular the graph has only one top-player (that is a node without superior) and is acyclic.

In chapter 8 this algorithm is modified to a polynomial time algorithm for situations that allow for the more general case that the directed graph has more top-players, but the underlying cooperative game is restricted to be additive. In fact it is shown that under these conditions the game decomposes into several subgames satisfying the conditions of chapter 7.

In chapter 9 a special class of games is considered: the class of co-insurance games. In principle every non-negative monotone game can be considered as a co-insurance game and this consideration gives new tools to describe arbitrary non-negative monotone cooperative games. Also a new class of games is considered in this chapter: the class of veto-removed games. Each game in this class can be presented as the Davis-Maschler reduced game of a veto-rich game. By investigation of this class it is possible to find new properties of some monotone games. The chapter concludes with a simple algorithm to compute the prenucleolus of a veto-removed game.