Self-destructive percolation, invasion percolation and related models

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This thesis concerns itself with three different stochastic models: the self-destructive percoaltion model, the invasion percolation model and a stochastic dynamic for the incipient infinite cluster model. These models are all defined on a graph in the plane denoted by $\mathbb{Z}^2$, which is obtained by taking the points of $\mathbb{R}^2$ with integer coordinates as vertices and two vertices are connected with an edge if their Euclidean distance is 1. All the three models hebben iets te maken met constructing random subgraphs of $\mathbb{Z}^2$.

The first model, the self-destructive percolation, can be considered to be a simplified model for forest fires. Imagine that each vertex of $\mathbb{Z}^2$ represents a location where a tree can grow. Consider a tree that has grown at the vertex v. The cluster of this tree is the maximal set of trees that can be reached from v via a sequence of trees that are of distance 1 from each other. We say that a tree is in an infinite cluster if its cluster contains infinitely many trees. Now, suppose that initially there are no trees at any location. Then, at each location a tree grows with probability $p \in [0, 1]$ independently of the other locations. In the next step a destruction of trees, supposedly by fire, takes place: if there are infinite clusters then they get destroyed and the locations of the trees in the infinite clusters become empty again. The finite clusters remain intact. Finally, trees grow again at each site where there is no tree present at the moment. The probability of a tree growing at this stage is $\delta \in [0, 1]$ andgrowth again happens indipendently for different locations.

This model can be considered as a simple model for forest fires, where first a number of trees grow, then the big clusters of trees are destroyed and finally trees can grow again. One can naturally ask what is the probability of having an infinite cluster in the final configuration for a given pair of (p, δ). We denote this probability by $\Theta(p, \delta)$. In Chapter 2 we study the function $\Theta(., .)$. Computer simulations suggest that there is a special line-segment $\{\hat{p}\} \times [0, \hat{\delta}]$ such that $\Theta(., .)$ is not continuous at $(\hat{p}, \hat{\delta})$. We unfortunately do not have a mathematically rigorous proof for this. The main question studied in Chapter 2 is the continuity of $\Theta(., .)$ outside this line-segment, where we show $\Theta(., .)$ to be indeed continuous.

In our second model a random subgraph of $\mathbb{Z}^2$ is created by a stochastic growth mechanism. Every edge of $\mathbb{Z}^2$ has a number attached to it according to a specified probability density. Initially only a special vertex of $\mathbb{Z}^2$, called the origin, belongs to this subgraph. At each step a new edge, and subsequently a new vertex, is added to graph. The next edge to be added is determined by the numbers attached to the edges, namely the edge on the boundary of the graph with the smallest number on it is added. This construction is repeated infinitely many times, resulting in a random subgraph of $\mathbb{Z}^2$ which is called the invaded region.

In Chapter 3 we study properties region with focus on certain parts of it called the invasion ponds. We derive power law results for the distributions several characteristic quantities of these ponds. Furthermore, we compare the invaded region with another random graphs called the incipient infinite cluster and show that although they are locally similar, globally they differ significantly.

The last model considered in this thesis is a stochastic dynamics for the incipient infinite cluster. This cluster is yet another infinite random subgraph of $\mathbb{Z}^2$ obtained by a complicated procedure. In order to study its distribution, in Chapter 4 we propose three dynamic models that we hope to approximate the distribution of the IIC. Since these models are defined on an infinite graph, the existence of these models, in the sense that they form well-defined stochastic processes, is not immediately clear. In Chapter 4, we give a formal construction to all the three dynamic models.