Computation of symbolic dynamics of low-dimensional maps

Sella, L.

2009

document version
Publisher's PDF, also known as Version of record

Summary

Dynamical systems are mathematical objects which model quite effectively many phenomena in nature. Frequently they are very complex and it is indispensable to develop theoretical and practical tools to understand them.

This thesis gives a contribute to this task by providing techniques to analyse discrete time dynamical systems over real numbers. The general framework where this thesis sets is the translation of dynamical systems over the reals to symbolic dynamical systems. Symbolic systems consist of infinite sequences of symbols over a finite alphabet, the dynamics is given by the shift operator which maps a sequence to a sequence shifted of one position. These kind of systems are easier to analyse and understand than systems over real numbers, in particular it is simpler to compute properties such as topological entropy which give a measure of the level of complexity of the system.

In Chapter 1 we explain the basic theory of symbolic systems and the main results in the literature on translating real value systems to symbolic systems. In one dimension one of the main results is Kneading theory developed by Milnor and Thurston, while in two dimensions one of the main results is the theory of fixed point tangles.

In Chapter 2 we present some algorithms for the computation of symbolic dynamics of piecewise continuous systems in one dimension, these techniques are based on covering relations. The state space where the system is defined is partitioned in pairwise disjoint open sets which represent the alphabet of the symbolic system. The covering relations describe how each element of the partition map an other, covering it totally, partially or not at all. Refinement strategies of the partitions allow improvement of the approximation of the symbolic dynamics and ensure convergence of the algorithm to the exact dynamics. A combination of covering relations with Kneading theory provides an remarkable improvement in the running time of the algorithm.

In Chapter 3 we present algorithms to compute the symbolic dynamics of piecewise affine dynamical systems in two dimensions. For these purpose we need more sophisticated technique than in one dimension, covering relations do not provide in general a good lower approximation of the symbolic

dynamics. The explained method uses algebraic homology, more precisely the decomposition of the Conley index to compute a lower approximation of the symbolic dynamics of the map. To implement this method we develop a calculus of generally non convex polyhedra, by which we model the index pair. The methods reveal to be useful also for discontinuous maps, and so also to study discrete dynamics of hybrid systems via symbolic dynamic of a suitable return map.

Finally in Chapter 4 we prove that the discrete abstraction of hybrid systems is in general uncomputable. We prove this results by showing two examples of converging sequences of hybrid systems whose entropy of the discrete dynamics does not converge to the entropy of the discrete dynamics of the limit. This work exploit the result of a previous work by Misiurewicz.