Agent-Based Simulation of Episodic Criminal Behaviour

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Abstract. Criminal behaviour often involves a combination of physical, mental, social and environmental (multi-)agent aspects, such as neurological deviations, hormones, arousal, (non-)empathy, targets and social control. To study the dynamics of these aspects, this paper contributes a dynamical agent-based approach for analysis and simulation of criminal behaviour. It involves dynamically generated desires and beliefs in opportunities within the social environment, both based on literature on criminal behaviour. The approach is illustrated for the case of an Intermittent Explosive Disorder.

1. Introduction

Within Criminology the analysis of criminal behaviour addresses physical, mental, environmental and social aspects; e.g., [5, 15, 24, 27, 32]. Only few contributions to the literature address formalisation and computational modelling of criminal behaviour, usually focussing only on some of the factors involved; e.g., [3, 22, 23]. This paper is part of a large interdisciplinary research project (involving parties from computer science, criminology, psychology and social science) that has as main goal to develop a modelling approach for criminal behaviour, which integrates physical, mental, environmental and social aspects. To this end, in this research project the standard BDI-model for action preparation based on motivations [14, 28] is taken as a basis and is extended by specific models for generation of desires and for generations of beliefs in opportunities. These extensions are based on available literature on criminal behaviour and the underlying aspects. For the generation of desires, dynamical models were incorporated involving internal states, for example, for neurological, hormonal, and emotional aspects and their interaction; e.g., [24, 27]. For the generation of beliefs in opportunities, a model was incorporated formalising the well-known Routine Activity Theory within Criminology; e.g., [15]. This (informal) theory assumes motivation of the criminal and covers environmental and social aspects such as the presence of targets and social control.

The overall modelling approach for criminal behaviour involves models at two different levels. At the level of single agents, the decisions of individuals and the underlying biological and psychological aspects are addressed. At the level of the multi-agent system, the impact of such decisions on the society as a whole are addressed. The current article focuses on the latter aspect, i.e., it presents an approach to study the dynamics of a group of agents given certain assumptions about the behaviour of individuals and characteristics of the environment. For example, it aims to answer questions such as “how are crime rates influenced by the size of a city?”, or “how are crime rates influenced by the amount of police?”. Since they involve the interaction of different types of agents over time and space, such questions are usually not easy to answer analytically. Therefore, the current paper presents an approach to explore the dynamics of crime via simulation and formal techniques. As such, the main users of the approach are considered to be social scientists and researchers in applications of
modelling in criminology. However, in case it leads to interesting results, these results may also be presented to policy makers.

Since this article focuses on the social aspects of crime, the description at the level of the individual agents is kept abstract; these behaviours are modelled in terms of simple input-output relationships. However, more information about the underlying biological/psychological models in the context of the research project as a whole can be found in [6], and the way that these models are connected to the BDI-model is explained in [7].

To address the type of questions given above, an artificial society has been modelled, where on a map (represented by a labeled graph) agents move around and meet each other. Agents may be of four types: potential criminal, agent with negative appearance, potential victim, and guardian. The models for the agents and their environment have been formally specified in dynamical systems style [2, 26] by executable temporal/causal logical relationships, extended by probabilities. To obtain these, knowledge from the literature in Criminology, and the different disciplines underlying it, was exploited; e.g., [17, 24, 27, 32].

Although the model is generic, the current paper focuses on a specific type of crimes, namely those that are performed by persons with Intermittent Explosive Disorder (IED), a disorder of impulse control, characterised by short episodes of aggression. This is an interesting case study, because these types of crimes on the one hand have a biological background (the presence of the disorder highly increases the probability of these persons to perform certain assaults), but on the other hand involve a social aspect (the episodes of aggression are usually triggered by encounters with other people).

The challenge is to model the variety of physical, mental and social aspects as mentioned above in an integrated manner. On the one hand, qualitative aspects have to be addressed, such as epistemic and motivational states, certain brain deviations, and some aspects of the environment such as the presence of certain agents. On the other hand, quantitative aspects have to be addressed, such as testosterone and serotonin levels, and (in the environment) distances and time durations. Furthermore, it should be possible to model on a higher level of aggregation or abstraction, as it would not be feasible, for example, to model the brain anatomy at the level of neurons. The modelling language LEADSTO [9] fulfils these desiderata. It allows to model at higher levels of aggregation, and it integrates qualitative, logical aspects and quantitative, numerical aspects; cf. [11]. In LEADSTO direct temporal dependencies between two state properties in successive states are modelled by executable dynamic properties. The format is briefly defined as follows. Let α and β be state properties of the form ‘conjunction of ground atoms or negations of ground atoms’. In the LEADSTO language the notation \( \alpha \rightarrow e, t, g, h \beta \), means:

\[
\text{If state property } \alpha \text{ holds for a certain time interval with duration } g, \\
\text{then after some delay (between } e \text{ and } f) \text{ state property } \beta \text{ will hold for a certain time interval of length } h. 
\]

Here atomic state properties can have a qualitative, logical format, such as an expression desire(d), expressing that desire d occurs, or a quantitative, numerical format

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* For simplicity, it is assumed that every agent belongs to one of the four types. Although clearly an over-simplification of reality (e.g., a potential criminal may also be a victim, and a potential victim may also act as a guardian), this assumption considerably reduces the complexity of the model and its analysis.
such as an expression has_value(x, v) which expresses that variable x has value v. For more details of the language LEADSTO, see [9].

Section 2 discusses a summary from the literature on criminals with Intermittent Explosive Disorder. In Section 3 the simulation model is presented, and Section 4 discusses the results of the simulations by referring to an example simulation trace. Section 5 presents a number of global dynamic properties of the society and their logical formalisation and discusses automated verification of the simulation results against them. Section 6 discusses a probability-based analysis of similar properties, also automatically verified on a set of generated traces. Finally, Section 7 discusses related work, and Section 8 is a concluding discussion about the approach.

2. Case Study: a Criminal with IED

An Intermittent Explosive Disorder (IED) is a disorder of impulse control, characterised by several episodes (usually of 10 to 20 minutes each) in which aggressive impulses are released and expressed in serious assault or destruction of property, although no such impulsiveness or aggressiveness is shown in the periods (usually weeks or months) between episodes. To evoke such episodes, often only a minor stimulus is sufficient, such as an encounter with someone that has a negative, provoking, appearance or behaviour. It is estimated that about 7% of the adult population in the US can be diagnosed as having IED. Offences by persons with IED concern a disproportionate reaction, usually to an acquaintance or family member. After the episode the offender has no recollection of his actions and, when informed, has feelings of remorse [24]. The following sketch illustrates the interplay of the physical, mental and social aspects involved. Suppose the criminal meets somebody with negative, provoking behaviour (social aspect). This is interpreted by the criminal (mental aspect), provokes stress, and leads to an episode with an epileptic state of the brain (neurological, physical aspect). This state leads to changes in hormonal (physical) and emotional (mental) states, which lead to a certain type of desire, providing the motivation for some criminal action (mental aspect). As soon as an opportunity of a suitable potential victim with not much social control (social aspect) is perceived (mental aspect), the desire leads to the criminal action.

The scenario described above on the one hand involves epistemic and motivational concepts (e.g., the desire to act aggressively, and the belief that certain actions can fulfil this desire), but on the other hand biological concepts (e.g., disorders in the limbic system and high levels of testosterone [24]). In order to integrate these notions within one agent-based model, the standard BDI-model for rational reasoning (e.g., [14, 28]) has been extended by a model that generates desires based on underlying biological and psychological factors. Some of these factors, which are particularly important to model the behaviour of a person with IED, are impulsiveness, aggressiveness, emotional attitudes towards others, tendencies to become anxious or excited, and capabilities to understand the mental states of others.

In order to convert these elements into complex desires, the physiological makeup of each agent has been modelled via a number of numerical parameters (e.g., level of testosterone, serotonin, and so on). These parameters, and the relations between them, were identified in collaboration with domain experts. The model has been designed in such a way that combinations of these parameters result in an assignment of values to the characteristics mentioned above (e.g., aggressiveness, impulsiveness), on a qualitative scale. Eventually, these characteristics are combined into composed desires,
which play the role of regular desires in a BDI-model. This model (as well as a
discussion about its validation) is described in detail in [6], and its integration within the
BDI-model is described in [7]. The remainder of this paper assumes these models as
given, and focuses on the social/environmental aspect of criminal behaviour by IED
patients. To this end, in this paper the model to generate complex desires is simply
represented in terms of a (probabilistic) input-output relationship, i.e., a relationship
between incoming stimuli and the desires of the person.

3. The Simulation Model

In this section the overall simulation model as developed is described in more detail.
The combination of physical, mental and social aspects involved requires integration of
models for internal physical and mental functioning of an agent with a model at the
social level. To this end, as mentioned earlier, the simulation model has been composed
from submodels, integrating different aspects, including (but not limited to) a decision
making model based on beliefs, desires and intentions (BDI) and a model for the social
environment. The BDI-submodel describes how actions relate to desires and intentions,
when appropriate opportunities are there. It needs as input desires and beliefs in
opportunities. For these elements additional models have been developed. Thus the
simulation model is composed of four submodels:

1. a submodel for reasoning about beliefs, desires and intentions (BDI-model)
2. a submodel to determine desires needed as input for the BDI-model
3. a (small) submodel to determine how observations lead to beliefs in an opportunity as
   needed as input for the BDI-model; this model is based on the Routine Activity Theory
4. a submodel for the society; this model has two aspects namely, a geographical aspect;
   this is represented by a labeled graph of locations and connections and a multi-agent
   societal aspect; this lets agents move in the world and determines the effects of actions
   performed.

Note that submodels 1. and 2. address physical and mental aspects, submodel 4.
dresses social and environmental aspects, and submodel 3. relates society aspects to
mental aspects.

Overview of the Simulation Model: Graphical Form

A visualisation of the simulation model is provided in Figure 1. In this picture, the
circles denote state properties, and the arrows indicate causal relationships, which can
be represented by local dynamic (LEADSTO) properties (LP’s). An arc connecting
multiple lines indicates that the conjunction of multiple state properties influences
another state property. The dotted box indicates the borders of the agent: all circles that
are depicted inside the box are internal state properties, all circles depicted outside the
box are external world state properties, and all circles depicted at the left (right) border
of the box are input (output) state properties. The solid boxes indicate submodels
(which are not further worked out in the current paper).

The BDI-submodel

The BDI-submodel bases preparation and performance of actions on beliefs, desires and
intentions, e.g. [14, 28]: an action is performed when the subject has the intention to do
this action and it has the belief that the opportunity to do the action is there. Beliefs are
created on the basis of stimuli that are observed. The intention to do a specific type of
action is created if there is a certain desire, and there is the belief that in the given world state, performing this action will fulfil this desire. The generic rule to generate the action performance from the intention and the belief in the opportunity is specified within the BDI-submodel as:

LP33 The belief that there is an opportunity to perform a certain action combined with the intention to perform that action will lead to the performance of that action.

\[ \forall a : \text{ACTION}\ belief(\text{opportunity}(a)) \land \text{intention}(a) \rightarrow \text{performed}(a) \]

The effects of actions (e.g., the decrease of stimuli) are modelled in the submodel for the society. For simplicity, we assume that actions always succeed. The intention is generated by a desire and a belief in a good reason, according to the following rule:

LP32 Desire \( d \) combined with the belief that a certain action will lead to the fulfilment of that desire will lead to the intention to perform that action. Here, \( d \) is a specific combined desire that consists of multiple characteristics as described in Section 2 (see also the next submodel).

\[ \forall d : \text{DESIRE}\ \forall a : \text{ACTION}\ desire(d) \land belief(satisfies(a, d)) \rightarrow \text{intention}(a) \]

Figure 1. Graphical Overview of the Simulation Model.

Within the BDI-submodel, for reasons of simplicity, per desire only one action that can satisfy the desire is included. What remains to be generated are the desires and the beliefs in opportunities. For desires, the standard BDI-model [28] does not prescribe a generic way in which they are to be generated. Recent extensions of BDI models do comprise models for generation of desires (e.g., [25]) ; this often depends on domain-specific knowledge, which also seems to be the case for criminal behaviour. Therefore, a similar approach is adopted here. In particular, for EED patients, a number of physical aspects play a role, such as certain brain deviations and serotonin levels, as discussed below in some further detail. For beliefs in opportunities, they are strongly dependent on the (social) environment, which is another theme discussed below.

The Submodel to Determine Desires

To determine desires a rather complex submodel was built based on literature such as [5, 17, 24, 27], incorporating, for example, testosterone, serotonin, adrenaline, blood sugar levels and brain configuration aspects. These physical aspects relate to mental
aspects such as arousal, aggressiveness, impulsiveness, risk-taking, thrill-seeking, understanding others, and feeling for others. The aspects involved contain both qualitative aspects (e.g., the existence of certain brain deviations) and quantitative aspects (e.g., levels of testosterone or serotonin). To model these, both causal and logical relations (as in qualitative modelling) and numerical relations (as in differential equations) have to be integrated in one modelling framework, using the LEADSTO language.

As mentioned earlier, this submodel is explained in detail in [6]. However, since the current paper focuses on social/environmental aspects of crime, in the presented simulations an abstraction of the submodel to determine desires has been made. To be specific, it has been replaced by the following two rules:

LP30 When agent \( a_1 \), who is an IED agent is at location \( l \) and observes a ‘negative’ agent at location \( l \), then agent \( a_1 \) will have an aggressive episode.
\[
\forall a_1,a_2:\text{AGENT} \quad \forall l:\text{LOCATION} \\
\text{observes}(a_1,\text{agent\_of\_type\_at\_location}(a_1,\text{IED},l)) \land \text{observes}(a_1,\text{agent\_of\_type\_at\_location}(a_2,\text{neg\_agent},l)) \\
\rightarrow \text{has\_episode}
\]

LP42 An agent that has an aggressive episode has the desire to performs an aggressive action.
\[
\text{has\_episode} \rightarrow \text{desire\_aggressive\_action}
\]

The Submodel to Determine Opportunities

As another input for the BDI-model, the notion of opportunity is used. For the current domain, this is modelled via a single rule, based on criteria indicated in the Routine Activity Theory [15]: a suitable target and absence of a guardian. This was specified by:

LP41 When agent \( a_1 \), who is an IED agent, is at location \( l \) and observes a passer-by at location \( l \) and does not observe a guardian at location \( l \), then agent \( a_1 \) believes that there is an opportunity to assault someone.
\[
\forall a_1,a_2:\text{AGENT} \quad \forall l:\text{LOCATION} \\
\text{observes}(a_1,\text{agent\_of\_type\_at\_location}(a_1,\text{IED},l)) \land \text{observes}(a_1,\text{agent\_of\_type\_at\_location}(a_2,\text{passer\_by},l)) \land \\
\neg \text{[\text{observes}(a_1,\text{agent\_of\_type\_at\_location}(a_3,\text{guardian},l))] } \\
\rightarrow \text{belief\_opportunity\_assault}
\]

In dynamic property LP32 shown earlier, the third criterion of the Routine Activity Theory, the motivated offender, is represented by the intention to perform some action. Note that LP41 is a domain-dependent rule. For other domains, the submodel can be filled with rules that generate beliefs in opportunities for other actions than assaults. Also, the perception process that generates beliefs based on observation can be modelled in more detail. However, since the main goal of the current paper is to study the patterns that result from the Routine Activity Theory, there is no need to further refine this submodel.

The Submodel for the Society

The social, multi-agent aspect is modelled by a simple environment, in which a number of agents move around and sometimes meet at a location. One of the agents is the criminal agent with IED, the others are guardian agents, potential victims (passers-by) and agents with provoking behaviour (so that they may trigger an episode in the criminal when (s)he encounters them), from now on referred to as negative agents\(^7\).

\(^7\) According to Moir and Jessel ([24], pp. 184-194) an episode may be provoked by various types of unpleasant encounters with other people. Examples are a certain negative look or an unfriendly remark or question by someone. For simplicity, all these events are summarised here as encounters with a negative agent.
passers-by are assumed to be suitable targets, for example, because they appear rich and/or weak. However, as also the guardians are moving around, such targets may be protected, whenever at the same location a guardian is observed by the criminal: formal control\(^2\).

\[\text{Figure 2. Example World Geography (with an initial distribution of agents over locations; agent 1 is the agent with IED, agent 2 and 3 are guardians, agent 4, 5 and 6 are passers-by (potential victims), and agent 7 and 8 are 'negative' agents.}\]

The interaction between a specific agent and the environment is modelled by (1) observation, which takes information on the environment as input for the agent (e.g., at which location it is, where suitable targets are, and whether social control is present), and (2) performing actions, which is an output of the agent affecting the state of the world (e.g., going to a different location, or committing a crime). The geographical information of the world is described by a labeled graph as depicted in Figure 2. Relevant locations are indicated by nodes A, B,.., and routes connecting locations by edges E1, E2,... The agents move from location to location via these edges. Edges have lengths; travelling takes time, depending on these lengths.

To model the dynamics of an agent moving in the environment, the following cycle is used: observe, determine next action, determine effects of this action. In some more detail, the model is based on (1) properties expressing what is observed, for example, stimuli or other agents: if another agent is present at the agent’s location, then the agent will observe this, (2) properties expressing which next action is to be undertaken; for example, if the agent has stayed at its location for duration \(s\), and the next location to reach is I, then it will move to this next location (probabilities are used to make random choices between options), and (3) properties expressing the results of actions undertaken; for example, if the agent starts to move to a next location over edge \(e\) and edge \(e\) has length \(d\), then it will arrive at the next location after duration \(d\).

**Settings for the Model**

The model is initialised by setting the initial locations in the world of all agents: IED agents, guardians, passers-by, and negative agents. These inputs are included in scenarios for simulation. For the simulation trace explained in the following section, the

\(^2\) For future work, it is planned to incorporate informal social control as well, e.g., by allowing a group of passers-by to act as one guardian - and prevent crimes - as well.
settings shown in Figure 2 were chosen, i.e., consisting of 8 locations that are populated by 1 IED agent, 2 guardians, 3 passers-by, and 2 ‘negative’ agents.

4. Simulation Traces

A large number of simulation traces (200 in total) have been generated for the behaviour of the IED criminal under different circumstances using the simulation model. Below, an example simulation trace is shown in Figure 3, which was generated using the simulation model. In this picture, time is on the horizontal axis; state properties are on the vertical axis. A dark box on top of the line indicates that the property is true during that time period, and a lighter box below the line indicates that the property is false. The first eight lines display the characteristics of the agents involved. The next 5 lines show the BDI-based decision making process of agent 1, which is the agent with IED, and the rest of the trace shows the movement of the two agents (agent 1 and 8) over the environment. For simplicity, all other events, such as the generation of actions to move to another location, the locations of agents 2-7, and the physiological processes underlying agent 1’s behaviour, are not shown.

As shown by Figure 3, the example environment contains 8 agents and 8 locations. Agent 1, the agent with IED, initially does not have a desire to perform aggressive actions. However, at time point 46 this agent is at location C, where he meets a negative agent (agent 8). This causes an episode, which leads to the desire to perform an aggressive action. This desire, combined with the belief that performing an assault leads to the satisfaction of this desire, leads to the intention to assault someone. At time point 51, the IED agent is at location B, together with a passer by (agent 4, not shown) without a guardian present (agents 2 and 3 are both on location F, not shown). This leads to the belief that there is an opportunity to assault agent 4. This belief combined with the intention leads to the performance of the assault. Because of the assault, the stimuli of the world increase, which satisfies the desires of agent 1. Later, at time point 91, agent 1 again generates an aggressive episode, but because it does not encounter any opportunities, it does not perform any assaults.

As mentioned above, various similar simulation experiments have been performed. Among the different experiments, several parameter settings were varied, in particular the number of agents, the ratio between different types of agents, and the number of locations. In general, the generated traces indeed show the behaviour of crimes performed by IED patients, as described in literature such as [17, 24, 27]. Moreover, these simulation experiments may give insight into the impact of different types of populations or geographical environments on crime rates. For example, is it better to invest in more police at a particular location or to prevent passer-by from going to that location?

Obviously, when the number of simulations becomes large, it becomes impossible to study all simulation traces by hand. Therefore, in the next sections it is explained how this investigation process automated analysis techniques.

Furthermore, although the simulation examples as presented here involve only 8 agents, it has been found that the model easily scales to a society of several hundreds of agents (processing time staying within one hour). Nevertheless, complexity problems

§ Although the model allows multiple actions to fulfill a particular desire, the current paper only addresses those desires for aggression that are so strong that assault is considered the best (and most immediate) solution.
may arise when populations of (more than) thousands of (heterogeneous) agents are considered. These problems could be solved by translating the current simulation model to a stochastic model, as is done, for example, in the analysis of epidemics [1]. To make such a translation, the description of the dynamics of a population will shift from a “micro” perspective (at the level of individual agents) to a “macro” perspective (at the level of groups of agents). For example, the number of criminals, guardians, negative agents, and passers-by at certain locations may be described by global variables, which are influenced by probabilistic rules. The main advantage of these types of macro-level approaches is that they can deal with larger populations. An inevitable drawback is however that they imply a loss of detail at the individual agent level. In future work, the benefits of such approaches will be explored.

Figure 3. Example Simulation Trace.

5. Logical Analysis

When the number of simulation traces becomes large, automated support for analysis of the traces becomes very useful. To this end, the TTL Checker tool [8] may be used. This piece of software takes as input a number of simulation traces and a logical formula (represented in the predicate logic-based language TTL), and verifies whether the
property holds for the traces. Moreover, in case a property does not hold, the software automatically provides a counter example, i.e., a combination of traces, time points, and variable instantiations for which the property fails. This allows the analyst to formulate properties that (s)he expects to hold for a certain process, but also to study those situations for which the property does not hold in more detail (e.g., by investigating those simulation traces by hand), and explain what causes the unexpected behaviour.

Following this approach, a number of properties of criminal behaviour have been identified and formalised. Some of these properties have a logical character and some have a probabilistic character. Both types of properties have been automatically verified for the simulation traces. In this section the logical properties are discussed, in the next section the probabilistic ones. For a multi-agent system, dynamic properties can be identified at different aggregation levels, roughly spoken (1) the level of the behaviour of a single agent (external perspective), (2) the level of the internal functioning of an agent (internal perspective), and (3) the level of the multi-agent system as a whole (society behaviour). For each of these levels, relevant properties are identified and formalised.

**Behavioural Properties of Agents**

The properties that have been identified and formalised in the logical language TTL [8] to characterise the behaviour of the criminal agent (from an external perspective) are as follows (where t is the duration of the reaction time from observations to internal states or from internal states to actions):

**BP1** From Circumstances to Criminal Action If an IED agent meets a negative agent, and within duration t an opportunity occurs, then an assault will be performed.

\[ \forall t \forall a_1, a_2 : a g e n t \forall l : l o c a t i o n \]
\[ (s t a t e ( t, t) \models o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_2, I E D, l)) \wedge o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_2, p a s s e r_ b y, l))) \wedge \]
\[ \forall a_3 : a g e n t \]
\[ s t a t e ( t, t) \not\models o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_3, g u a r d i a n, l)) & \]
\[ \exists t \leq t : a g e n t \exists l : l o c a t i o n \ t - e \]
\[ s t a t e ( t, t) \models o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_4, n e g _ a g e n t, l))) \]
\[ \Rightarrow \exists t \leq t : a g e n t \exists l : l o c a t i o n \ t - e \]
\[ s t a t e ( t, t) \models p e r f o r m e d ( a s s a u lt) \]

Here state(t, t) |= X denotes that within the state state(t, t) at time point t in trace γ state property X holds (and with ¯ that it does not hold), with the infix predicate |= within the language denoting the formalised satisfaction relation. See [8] for more details of TTL.

**BP2** From Criminal Action to Circumstances If an assault is performed, then the opportunity was there and earlier (at most t back in time) the IED agent encountered a negative agent.

\[ \forall t \exists t_1 \exists t_2 \exists t_3 : a g e n t \exists l : l o c a t i o n \ t - e \]
\[ \exists a_{1,2,3} \exists a_{4,5} \exists l : l o c a t i o n \ t - e \]
\[ s t a t e ( t, t) \not\models o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_1, I E D, l))) \wedge o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_2, p a s s e r_ b y, l))) \wedge \]
\[ \forall a_3 : a g e n t \]
\[ s t a t e ( t, t) \models o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_3, g u a r d i a n, l))) \wedge \]
\[ \exists t \leq t : a g e n t \exists l : l o c a t i o n \ t - e \]
\[ s t a t e ( t, t) \models p e r f o r m e d ( a s s a u lt) \]

Notice that these properties summarise how the agent functions in the context of society, abstracting from the internal mechanisms underlying this behaviour. Logical consequences of these external agent behaviour properties include the following external behavioural property:

**BP3** No Opportunity No Crime If no opportunities are offered, then no criminal action occurs.

\[ \forall t \exists a_1, a_2 : a g e n t \exists l : l o c a t i o n \]
\[ \neg ( s t a t e ( t, t) \models o b s e r v e s ( a_1, a g e n t_ o f_ t y p e_ a t_ l o c a t i o n ( a_1, I E D, l))) \wedge \]
observes(a1, agent_of_type_at_location(a2, passer_by, l)) & 
\forall a\exists \text{agent} 
state(y, t) \#\text{observes}(a1, \text{agent}_\text{of_type}_\text{at_location}(a3, \text{guardian}, l)) ] 
\Rightarrow [ \forall t \text{state}(y, t) \neq \text{performed(assault)} ]

**Internal Properties of Agents**

Although for an analysis at the level of the society as a whole, details of internal mechanisms and processes are not needed, from the perspective of justifying, understanding and explaining whether, how and when such behaviour can occur, still the internal agent dynamics are interesting to formalise. Knowledge of these mechanisms may also be useful as a basis for therapy and/or medication. The following internal behavioural properties of the criminal agent were identified and formally specified***:

**IP1a (Episode Provoking)** If the IED agent observes an agent which has a negative appearance, then from t to t+ε the IED agent will have an episode.
\[ \forall t [ \text{state}(y, t) \models \text{observes(negative_event, pos)} ] \Rightarrow \exists t_1 [ t_1 \leq t + \varepsilon \Rightarrow \text{state}(y, t_1) \models \text{has_episode} ] \]

**IP1b (Episode Grounding)** If the IED agent has an episode, then at some time point between t+ε and t the IED agent observed an agent which has a negative appearance.
\[ \forall t [ \text{state}(y, t) \models \text{has_episode} ] \Rightarrow \exists t_1 [ t_1 \leq t + \varepsilon \& \text{state}(y, t_1) \models \text{observes(negative_event, pos)} ] \]

**IP2a (Crime Committing)** If the IED agent has an episode, and it believes there is an opportunity to commit a crime, then it will perform criminal action a.
\[ \forall t [ \text{state}(y, t) \models \text{belief(opportunity, pos)} \wedge \text{has_episode} ] \Rightarrow \exists t_2 t_1 \leq t_2 \& \text{state}(y, t_1) \models \text{performs_action}(a) \]

**IP2b (Crime Grounding)** If the IED agent performs criminal action a, then it believes there is an opportunity to commit a crime, and it has an episode.
\[ \forall t [ \text{state}(y, t) \models \text{performs_action}(a) ] \Rightarrow \exists t_2 t_3 t_1 \leq t_2 \& \text{state}(y, t_1) \models \text{belief(opportunity, pos)} \wedge \text{has_episode} \]

**IP3a (Belief Generation)** If the IED agent observes X, then it will believe X.
\[ \forall t [ \text{state}(y, t) \models \text{observes(X, S)} ] \Rightarrow \exists t_2 t_1 \leq t_2 \& \text{state}(y, t_1) \models \text{belief(X, S)} \]

**IP3b (Belief Grounding)** If the IED agent believes X, then before it has observed X.
\[ \forall t [ \text{state}(y, t) \models \text{belief(X, S)} ] \Rightarrow \exists t_2 t_3 t_1 \leq t_2 \& \text{state}(y, t_1) \models \text{observes(X, S)} \]

In fact, the properties IP1a and IP1b express that having an episode has a backward representation relation (within duration e) to meeting a negative agent, and IP2a and IP2b that it has a forward representation relation to conditionally performing a criminal action as soon as (within duration e) an opportunity occurs; cf. [10]. The properties IP1a, IP1b, IP2a and IP2b can be refined further into more local properties describing the criminal agent’s internal mechanisms.

*** Note that, in case the whole underlying biological and cognitive model of the IED agent is incorporated, these properties would rather be used to analyse given traces of behaviour, instead of generating them. Moreover, also probabilistic variants of these properties may be defined (see Section 6).
Society Behaviour Properties

At the level of the society as a whole, one (especially as a policy maker) may typically be interested in under which circumstances how many crimes are committed, according to a certain measure. In the case considered here, two main factors to be taken into account are the negative encounters and the opportunities. In this section, some properties have been identified and logically specified. In Section 6, from a probability analysis perspective, other types of society properties have been formalised. The logical properties identified require counting of the number of certain types of observations or actions over time. For this, a very useful summation feature is available in TTL, denoted by \( \sum_{k=0}^n \text{case}(\varphi(k),v_1,v_2) \). Here for any formula \( \varphi \), the expression \( \text{case}(\varphi,v_1,v_2) \) indicates the value \( v_1 \) if \( \varphi \) is true, and \( v_2 \) otherwise. So, for the \( k^{th} \) term, this summation adds \( v_1 \) if \( \varphi(k) \) is true and \( v_2 \) if \( \varphi(k) \) is not true. In particular, if \( v_2=0 \) is taken and \( v_1=1 \), then it counts the number of time points \( t \) at which \( \varphi(t) \) is true.

**SP1 More Negative Agents More Crime** The more often negative agents are encountered, the more often criminal actions will occur.

\[
\forall \gamma_1, \gamma_2, v_1, v_2, w_1, w_2
\]
\[
\sum_{\gamma_1} \text{case}(\text{negative_encounter}(\gamma_1, k), 1, 0) = v_1 \&
\sum_{\gamma_2} \text{case}(\text{negative_encounter}(\gamma_2, k), 1, 0) = v_2 \& v_1 \leq v_2
\]

**SP2 More Opportunities More Crime** The more often opportunities are present, the more often criminal actions will occur.

\[
\forall \gamma_1, \gamma_2, v_1, v_2, w_1, w_2
\]
\[
\sum_{\gamma_1} \text{case}(\text{opportunity}(\gamma_1, k), 1, 0) = v_1 \&
\sum_{\gamma_2} \text{case}(\text{opportunity}(\gamma_2, k), 1, 0) = v_2 \& v_1 \leq v_2
\]

Here, the following abbreviations are used:

- \( \text{negative_encounter} \): a negative encounter
- \( \text{state} \): the current state
- \( \text{opportunity} \): an opportunity

Notice that the above properties compare two traces with each other. In the language TTL, it is possible to express such properties, in contrast to, for example, modal temporal logics.

**Verification of the Logical Properties**

Verification of properties at the three aggregation levels can be done in different ways. One way is to check whether the properties hold in the different simulation traces that have been generated, using the TTL Checker tool [8]. When compared to other verification approaches such as model checking, this approach has an advantage that it is relatively cheap (since basically one checks a formula against a limited set of traces instead of ‘exhaustively’ against all possible traces of a model). As a result, the verification process is quicker, and more expressive properties can be checked. In practice, the duration of such checks usually varies from one second to a couple of
minutes, depending on the complexity of the formula and the traces under consideration. With the increase of the number of traces, the checking time grows linearly. However, it is polynomial in the number of isolated time range variables occurring in the formula under analysis. Nevertheless, for the purpose presented in this paper, all properties could be checked in a couple of seconds. For an extensive comparison between the different verification approaches, see [8] and [12].

All of the properties as discussed have been checked automatically for all 200 simulation traces using the TTL Checker. Using these checks, the behavioural and internal agent properties were all found satisfied. However, the society properties turned out not to hold for all combinations of traces. The reason for this is that, by chance, there are some traces in which there is not much crime although many negative agents are encountered (for example, because there are no opportunities). Likewise, there are some traces where there is not much crime although many opportunities arise (e.g., because the criminals have no episodes). These individual traces cannot be distinguished by checking properties such as SP1 and SP2. For this reason, a probabilistic approach is sometimes more useful. Such a probabilistic approach is worked out in the next section.

Another way of verification is by establishing interlevel relations between dynamic properties. For example, the properties IP1a, IP2a and IP3a together (logically) imply behaviour property BP1, and IP1b, IP2b and IP3b together imply BP2, by the following interlevel relations:

\[
\begin{align*}
\text{IP1a & IP2a & IP3a} & \Rightarrow \text{BP1} \\
\text{IP1b & IP2b & IP3b} & \Rightarrow \text{BP2}
\end{align*}
\]

These interlevel relations have been verified as well.

6. Probabilistic Analysis

In this section properties are analysed from a probability perspective. At the society level, a main property is the parameterised global property below, addressing the expected number of crimes occurring within a certain time interval.

GP1(t, d, EC) Crime Occurrence Expectation

The expected number of crimes that take place from \(t\) within duration \(d\) is \(EC\).

Later on an expression will be shown for the expected number of crimes \(EC\) in this property with the following parameters:

- \(M\) total number of locations that can be visited
- \(N\) total number of agents with negative appearance
- \(V\) total number of agents offering an opportunity (potential victims)
- \(G\) total number of guardian agents

To analyse property GP1 in more detail, it is related to two more refined properties:

- the probability that within a certain duration \(dl\) (for the first time) a negative agent is met
- the probability that (after meeting a negative agent) within a duration \(el\) (for the first time) an opportunity for crime is met

Here \(e\) is the assumed duration of the episode.
GP2(t1, d1, p1) Provocation Occurrence Probability
The probability that from t1 after duration d1 a negative agent is met is p1.

GP3(t2, e1, p2) Opportunity Occurrence Probability
The probability that from t2 after duration e1 a first opportunity is met and in the meantime no negative agent is met is p2.

A first step is to assume invariance over time, so that these probabilities do not depend on the time parameters. Then these parameters will be left out. As a next step it is assumed that meeting a negative agent before t1 and an opportunity after t1 are independent events. Moreover, the behavioural properties IP1 and IP2 of the criminal agent are used.

Relating the probabilities and expected crimes
As a first step the probability p in GP1 will be related to the probabilities p1 and p2 in GP2 and GP3. This is done by the following logical relation.

\[
IP1 \land IP2 \land EC = \sum_{d1} \sum_{p1} \text{with } GP2(d1, p1) \sum_{e1} \sum_{p2} \text{with } GP3(e1, p2) \ p1 \land p2 \Rightarrow GP1(d1+e, EC)
\]

This relation collects all paths that can lead to a crime, indicated by the time that a negative agent was met and the (first) time that an opportunity was met. A next step is to find out what reasonable estimations are for the probabilities in GP2 and GP3. After this step the relation above will be used to find an estimation for EC. First GP2 is addressed. For convenience the following short notations are used: \(a = (1 - 1/M)\), \(b = 1 - (1 - a^e)\).

Estimating the probability to meet a negative agent
A next assumption is that, by their moving, the agents will be present at locations according to a uniform probability distribution, so for any agent A and location L, at any point in time, the probability that agent A is at location L is \(1/M\). The probability that it is not at L is \(1 - 1/M = a\). A further assumption is that agents move independently, and hence their locations are independent. Therefore for a given location L at time point t, the probability that there is no agent with negative appearance at L is given by \(p(\text{no\,negative\,agent\,at\,L}) = a^t\), and the probability that there is at least one agent with negative appearance at L is: \(p(\text{at\,least\,one\,negative\,agent\,at\,L}) = 1 - a^t\). This gives an estimation of how the probability p1 in property GP2(d1, p1) depends on d1, or, expressed differently, it has been found that by estimation it holds: GP2(d1, (1 - a^t)).

Estimating the probability to meet an opportunity
The next step addresses the probability to meet an opportunity within duration e, as indicated by property GP3. Here, the additional condition is that at e1, it is the first time that in the interval e an opportunity is met, and that no further negative agents were met in the meantime. Then the probabilities that at that location no victims and no guardians are met are as follows (with \(a = 1 - 1/M\)):

\[
\begin{align*}
p(\text{no\,victim\,at\,L}) &= a^e \\
p(\text{no\,guardian\,at\,L}) &= a^e
\end{align*}
\]

Therefore the probability that an opportunity is met (i.e., a victim and no guardian present) is (with \(b = 1 - (1 - a^e)\)):

\[
\sum_{d_1} \sum_{e_1} \sum_{p_1} \text{with } GP2(d_1, p_1) \sum_{e_2} \sum_{p_2} \text{with } GP3(e_2, p_2) \ p_1 \land p_2 \Rightarrow GP1(d_1+e_1+e_2)
\]
\[ p(\text{opportunity\_at\_L}) = (1 - a^n) = 1\cdot b \]

The probability that no opportunities and no negative agents are met is:

\[ p(\text{no\_opportunity\_no\_neg\_at\_L}) = a^n(1 - (1 - b)) = a^n b \]

The probability that at \( e1 \) locations \( f0, \ldots, e1-1 \) of a sequence no opportunities and no negative agents are met is:

\[ p(\text{no\_opportunity\_met\_up\_to(e1-1)}) = (a^n b)^{e1} \]

Based on this, the probability that in a sequence of \( e1 \) locations at the \( e1 \)-th element a first opportunity is met, whereas at all locations before \( e1 \) no opportunity and no negative agent was met is given by:

\[ p(\text{first\_opportunity\_met\_after(e1)}) = (a^n b)^{e1}(1 - b) \]

This gives an estimation of how the probability \( p2 \) in property \( GP3(e1, p2) \) depends on \( e1 \), or, expressed differently, it has been found that by estimation the following holds:

\[ GP3(e1, (a^n b)^{e1} (1 - b)) \]

**Estimating the Expected Number of Crimes**

Now that estimations for the probabilities in \( GP2 \) and \( GP3 \) have been found, it is possible to estimate the expected number of crimes in \( GP1 \), on the basis of the following calculation for \( EC \):

\[ \sum_{0 \leq d1 \leq d} \sum_{0 \leq e1 \leq e} (1 - a^n) \cdot (a^n b)^{e1}(1 - b) \]

Substituting here the probabilities as specified by \( GP2 \) and \( GP3 \) in the form \( GP2(d1, (1 - a^n)) \) and \( GP3(e1, (a^n b)^{e1}(1 - b)) \) obtains the following for the probability in \( GP1 \) that a crime is committed within duration \( d + e \):

\[
EC = \sum_{0 \leq d1 \leq d} \sum_{0 \leq e1 \leq e} (1 - a^n) \cdot (a^n b)^{e1}(1 - b) \\
= \sum_{0 \leq d1 \leq d} (1 - a^n) \cdot \sum_{0 \leq e1 \leq e} (a^n b)^{e1}(1 - b) \\
= \sum_{0 \leq d1 \leq d} (1 - a^n) \cdot ((1 - (a^n b)^{e1}))/((1 - (a^n b))) \cdot (1 - b) \\
= d \cdot((1 - a^n) \cdot ((1 - (a^n b)^{e1}))/((1 - (a^n b))) \cdot (1 - b)) \\
\]

This implies that

\[ GP1(d+e, d \cdot((1 - a^n) \cdot ((1 - (a^n b)^{e1}))/((1 - (a^n b))) \cdot (1 - b)) \]

holds. Substituting \( b = 1 - (1 - a^n) a^n \) and \( a = (1 - 1/M) \) provides for \( EC \) an expression in the basic parameters. To evaluate the behaviour of this expression for the expected number of crimes, depending on different parameter settings for the 6 basic parameters \( M, V, G, N, d, e \), the expression has been implemented in a spreadsheet† † (in Microsoft Excel).

† † See URL: http://www.cs.vu.nl/~tbosse/crim/AAMAS07.xls
Using this spreadsheet, the impact of different parameters on the total amount of crime has been tested in a systematic manner. In the following graphs (Figure 4a - 4c) the relation between different variables and crime is shown. In each of these tests, two of the variables $M, N, V, G, d, e$ have been manipulated whilst the other four variables have been kept constant.

Figure 4a. Relation between potential victims ($V$), guardians ($G$) and crime ($EC$). Here, the following parameter values were kept constant: $M=8, N=2, d=30$ and $e=10$.

Figure 4b. Relation between negative agents ($N$), duration of an episode ($E$), and crime ($EC$). Here, the following parameter values were kept constant: $M=8, V=3, G=2$ and $d=30$. 
Figure 4c. Relation between number of locations ($M$), time window ($d$) and crime ($EC$). Here, the following parameter values were kept constant: $N=2$, $V=3$, $G=2$ and $e=10$.

Some observations that are plausible from the context are indeed shown by these tests, as well as by the implementation. For example:

- $EC$ is monotonically increasing in its dependence on each of $N$, $V$, $d$, $e$
- $EC$ is monotonically decreasing in $G$
- for $N=0$ or $V=0$ or $M$ very large, $EC$ becomes 0

Furthermore, the expected number of crimes has been automatically verified against a set of simulation traces. To this end, another set of $n=200$ simulation traces has been generated. These traces were similar to the ones mentioned in Section 4, but used a fully connected graph for the geographical model (because of the assumption that the location of agents is independent of their previous location). For these traces, the following TTL formula:

$$\exists w \left[ w = \sum_{k=1}^{n} \sum_{t=0}^{d} \text{case}(\gamma(k), t) \mid \text{performed(assault)}, 1, 0) / n \& EC - \delta < w \& w < EC + \delta \right]$$

was checked with suitable values for parameters $EC$ (given above) and $\delta$. For the expected number of crimes $EC$, the value of 4.14 was chosen, as predicted by the above probabilistic analysis (with $M=8$, $N=2$, $V=3$, $G=2$, $d=40$, $e=10$). For $\delta$, the value of 0.1 was chosen (i.e., about 2.5% of $EC$). Based on these parameter values, the TTL formula mentioned above indeed succeeded (in a few minutes), since in the 200 traces under investigation 809 crimes were performed. This is an average of 4.04 crimes per trace, which lies just within $\delta$ from the number of 4.14 expected crimes. This is an indication that the probabilistic analysis is an adequate alternative for the simulation-based approach, as long as the analyst is interested in overall numbers, rather than in the local mechanisms that cause certain types of criminal behaviour.
7. Related Work

Although it is recognised that computer support in the area of crime investigation is an interesting challenge, only few papers on simulation and formal analysis of criminal behaviour can be found in the literature (see [21] for an extensive overview); they usually address a more limited number of aspects than the approach presented in this paper. For example, Brantingham and Brantingham [13] discuss the possible use of agent modelling approaches to criminal behaviour in general, but do not report a specific model or case study. Moreover, in [3] a model is presented with emphasis on the social network and the perceived sanctions. However, this model leaves the mental and physical aspects largely unaddressed. The same applies to the work reported in [23], where an emphasis is on the environment, and police organisation. Finally, [19] explores the usefulness of collaborative network theory to study organised crime, but does not provide a computational model. The contribution put forward in the current paper and its counterparts [6, 7] shows that an agent-based modelling approach is possible where both a complex internal agent model is involved (addressing physical and mental aspects) and a model for the multi-agent society.

Other agent models for human-like behaviour incorporating more cognitive and social aspects (such as trust and theory of mind) are described in [29, 30, 31]. These references focus on the internal architecture of an agent, and the applications aimed at are mainly in the area of games and virtual reality. An interesting extension of the work reported in the current paper would be to design more complex internal models for criminals (incorporating, for example, aspects such as trust and theory of mind in a more detailed manner) and perform social simulations with them. In such extensions the challenge how criminal agents come to their decisions in the context of a large variety of internal aspects can be addressed in more detail.

8. Conclusion

This paper presents results from an interdisciplinary research project that is aimed at the development of an agent-based modelling approach to analyse criminal behaviour in its social context. Agent-based modelling approaches often either address the internal functioning of an agent in an extensive manner but leave the social context limited, or address the social interactions at the level of the multi-agent system as a whole, thereby taking the internal models of the agents of limited complexity. As in many cases the interaction of physical, mental and social aspects is crucial, a model covering both levels is required. The proposed model adopts a general BDI-agent-model [14, 28] extended by specific models to generate desires and beliefs in opportunities, exploiting literature on criminal behaviour, in particular [17, 24, 27]. It involves both qualitative aspects (such as the anatomy of brain deviations, and presence or absence of agents at a specific location in the world), and quantitative aspects (such as distances and time durations in the world and hormone and neurotransmitter levels).

One of the challenges met when designing an agent model for criminal behaviour, is the large variety of different types of criminals and the amount of literature of different scopes about them. Often knowledge is formulated in a manner that does not make it clear how much certainty can be attached to it and/or in which context it would be valid. By focussing on the Intermittent Explosive Disorder (IED) type of criminal and using knowledge about this type of criminal that is confirmed in different sources in the literature, this challenge was addressed. It has been found that the model indeed shows
The behaviour as known from the literature of this type of offender within the given social context, as described in criminological literature.

The presented approach in general involves models at two different levels: submodels at the level of the biological/physiological aspects of single agents and submodels about the multi-agent society as a whole. The current paper specifically focuses on analysis of the dynamics of the latter. At this level, typical questions asked by criminologists are “how are crime rates influenced by the size of a city?”, or “how are crime rates influenced by the amount of police?” Due to the high number of parameters and interactions involved, these questions are difficult to be answered analytically. Therefore, this paper presents an approach (based on simulation and formal analysis) that can be used as an experimental tool to address such questions, by offering the analyst the possibility to predict crime rates given various characteristics of the population and the environment (often called “what if”-scenarios). As such, the tool can be used by researchers in modelling applied to criminology, and social scientists, but (in the long term) also by policy makers. In future work, the possibilities will be explored to apply these methods to real data, to be able to make predictions about crime in existing cities.

In addition, to analyse the model in more detail, a number of dynamic properties have been formalised in the TTL language, and (using an automated checker tool) have been (successfully) verified against a large set of simulated traces. These dynamic properties, both of logical and probabilistic type, comprise not only behavioural and internal properties of the agents involved, but also properties that address the society as a whole. Especially the latter type of properties may have a complex structure, e.g., because they compare multiple traces with each other, or because of the probabilistic aspects involved. The language TTL and its software environment turned out useful for these purposes.

In literature such as [14, 28], within standard BDI-models no general model for generation of desires is included. In many cases desires are just assumed to be there, or even communicated to the agent as goals it should adopt. Recently, extensions of BDI models are being developed in which this is the case, e.g., in Jadex [25]. One aspect that is addressed particularly here is the revision of desires as a result of undertaken actions that fulfill them. Another aspect relevant for desire generation is the biological substrate of the agent. Sometimes desires are just inherent to a certain biological makeup or state. The project of which the current paper reports results, takes a similar approach, namely to incorporate both biological and psychological factors into a submodel for generation of desires, see [6, 7]. Within the project, a number of biological aspects as found in the literature have been taken into account in the dynamic generation of desires, varying from specific types of brain deviations, and serotonin and testosterone levels, to the extent to which a substrate for theory of mind was developed. For the current paper, however, this model has been abstracted to a more high-level behavioural model. Moreover, the generation of beliefs in opportunities has been based on environmental and social aspects involving two specific criteria (suitable target, presence of guardian) as indicated by the Routine Activity Theory in [15]. Within the BDI-submodel, for reasons of simplicity, per desire only one action that can satisfy the desire is included (and one intention for that action). When a number of intentions are possible for one desire, then the model can be extended by a more specific decision making approach, such as utility-based multi-objective decision making; cf. [7, 16].

Further future work will address a number of extensions to the model. Among the factors that will be added are attractiveness and reputations of locations, informal social
control by passers-by, adaptivity of individual agents, and different surveillance strategies (e.g., random, planning-based, or area-based) of the guardians.

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