Rare Event Simulation Techniques for Stochastic Design Problems in Markovian Setting

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Summary

In a stochastic model, a rare event is an event with a very small probability of occurrence, say 10^{-9} or less. Although, the frequency of these events are low, it is certainly not very wise to ignore them. For instance such small probabilities may lead to catastrophic failures – or losses – in various areas such as transportation systems, telecommunication systems, risk management and reliability theory. However, one should be very careful while simulating different types of rare events. The cause of the rarity creates a huge difference in the analysis. Hence, this monograph is dedicated to efficient simulation of rare events.

To mathematically examine rare events, we first have to model the underlying system in which the events occur. In this thesis we use Markovian setting to model different rare event problems. Although Markov chain modeling has its advantages in its computable quantities by solving systems of linear equations, for realistic systems with large state spaces direct computational methods become inefficient. In such cases, simulation turns out to be a very useful tool to analyze the rare events.

The behavior of the system can be recorded as a function of time in sample paths. By repeatedly generating sample paths one can obtain identical and independent copies of the underlying system. The most intuitive and direct estimator would be to take the average of all the outcomes of the performance function. This estimator is called the Crude Monte Carlo (CMC) estimator in the literature. However, in the case of very low probabilities CMC has important drawbacks. For example, to see one time occurrence of an event with probability 10^{-9}, we need to run the simulation on average 10^9 times. For that reason, we need special techniques to speed up the estimation process. These techniques for the simulation of rare events can be collected under two main categories: *importance sampling* and *splitting*. Both categories modify the simulation so that the rare event of interest occurs more frequently than in Monte Carlo simulation.

In Chapter 2 we give a mathematical introduction to the area of rare event simulation. We review the basic background material necessary to understand the rare event simulation techniques, such as importance sampling, cross-entropy and multilevel splitting. All the modeling issues and the rare-event simulation techniques are illustrated with the same toy example for comparison purposes. We provide sufficient detail for the reader to understand the rare event simulation methodologies together with their positive and negative aspects. Also we give insight on how to choose the most suitable simulation technique for a specific problem in hand. This chapter also provides all of the basic tools necessary to understand

the subsequent chapters.

Chapter 3 introduces a generic importance sampling technique based on the cross-entropy method. This technique is applicable to all types of rare event problems in Markovian setting. The idea of this algorithm is adapted to create both a state-dependent and a semi state-independent importance sampling measures. The algorithm is applied to two types of problems, namely reliability models and Jackson networks. The numerical example section compares the performance of the patching algorithm with many other techniques used for the same problems.

Chapter 4 studies the counting problems for combinatorial problems. We apply the splitting method introduced by Rubinstein to three well-known counting problems, namely 3-SAT, random graphs with prescribed degrees, and binary contingency tables. We present an enhanced version of the splitting method based on the capture-recapture technique, and show by experiments the superiority of this technique in terms of variance of the associated estimators, and speed of the algorithms.

In Chapter 5 we consider a Markov-fluid-driven queue, focusing on the correlation function of the workload process. We propose a simulation-based computation technique which relies on a coupling idea introduced by Glynn and Mandjes. Then an upper bound on the variance of the resulting estimator is given, which reveals the impact of the coupling time and the busy period of the Markov-fluid queue. A numerical assessment, in which we compare the proposed technique with naive simulation, gives an indication of the achievable efficiency gain.

Chapter 6 studies a completely different subject than all the other chapters. It analyzes Benford's Law and its relation with Markov chain theory. We first give a definition of Benford Markov chains. Then we derive a simple sufficient condition guaranteeing that a Markov chain is Benford. The derivation uses recent tools that established Benford behavior both for Newton's method and for finite-dimensional linear maps, via the classical theories of uniform distribution modulo 1 and Perron-Frobenius. Later on we use this result to show that almost all finite-state Markov chains are Benford, in the sense that if the transition probabilities are chosen independently and continuously, then the resulting Markov chain is Benford with probability one. The chapter is complemented with several simulations and potential applications.