Rabinowitz Floer Homology for Tentacular Hamiltonians

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SUMMARY

Imagine a satellite in a gravitational field of a system of celestial bodies or an electric particle in an electro-magnetic field. The evolution of these physical systems can be described by Hamilton's equations. However, even though the equations are known and the energy of the system is preserved, computing the exact trajectories might be often too complex. Even a simple question like that of existence of closed trajectories on a fixed energy level might be difficult to answer. One effective approach to investigate the evolution of Hamiltonian systems in time is to apply techniques from symplectic geometry.

Rabinowitz Floer homology captures the relation between closed solutions of Hamilton's equations with a fixed energy and the geometry of the corresponding energy hypersurface. Even in cases where explicit solutions of Hamilton's equations cannot be computed, Rabinowitz Floer homology might still be computable and provide information about the existence of closed orbits. Moreover, unlike the Hamilton dynamics on the hypersurface, the Rabinowitz Floer homology is invariant under small perturbations. The most prominent observation is that whenever the Rabinowitz Floer homology of an energy hypersurface is different from its singular homology, it implies the existence of closed solutions of Hamilton's equations on this hypersurface.

In this thesis we present the first attempt to extend the definition of Rabinowitz Floer homology to include examples of non-compact energy hypersurfaces. First, we present a general framework for the construction of Rabinowitz Floer homology under suitable conditions on the hypersurface and the associated Hamiltonian system. Later, we introduce a class of Hamiltonians, called *tentacular Hamiltonians*, which satisfy the conditions and for which the Rabinowitz Floer homology is therefore well defined. The main result of this part of the thesis is the existence of uniform bounds on the Floer trajectories corresponding to a tentacular Hamiltonian. It is established by observing that in the neighborhood of the non-compact hypersurface the tangential component of the Floer trajectories is bounded due to the Morse-Bott property and therefore the Floer trajectories cannot escape along the non-compact hypersurface. Finally, we show some explicit examples of tentacular Hamiltonians.