In the first part of this thesis, uniform probability measures on countable sets are discussed. These probability measures are finitely additive, but not countably additive. In Chapter 2, a notion of uniformity is introduced that is called ‘weak thinnability’, which is a generalization of the property of uniform probability measures on [0, 1] or on a finite spaces that if we condition on any suitable subset, the resulting conditional probability measure is again uniform on that subset. It is showed that in the class of probability measures satisfying this notion, the probability of some subsets of natural numbers that do not have a natural density, is uniquely determined.

In Chapter 3, we mathematically model the supertask, introduced by Hansen [26, 27], in which an infinity of gods together select a random natural number by each randomly removing a finite number of balls from an urn, leaving one final ball. We showed that this supertask is highly underdetermined, i.e. there are many scenarios consistent with the supertask. In particular we show that the notion of uniformity for finitely additive probability measures on the natural numbers emerging from this supertask is unreasonably weak.

The second part of this thesis is dedicated to epistemic probability and more specifically to the theory of belief functions. In Chapter 5, the calculus of belief functions is discussed, based on a frequentist interpretation of belief functions. With respect to conditioning, instead following the distinction of Dubois and Prade [14] between ‘focusing’ and ‘updating’, a distinction between learning that something is contingent true (or has occurred contingently) and learning that something is necessarily true (or has necessarily occurred). Contingent conditioning coincides with what Dubois and Prade call ‘focusing’ and corresponds to a rule of conditioning that was already described by Demspter [9]. The rule that corresponds to necessary conditioning can be seen as the special case of Dempster-Shafer conditioning in which we have Bel(H\(^c\)) = 0, if H is the set on which we condition. Having introduced these two notions of conditioning, it is explained how Dempster-Shafer conditioning, the rule of conditioning that corresponds to what Dubois and Prade call ‘updating’, as a ‘mix’ between contingent and necessary conditioning. After interpreting both Dempster-Shafer conditioning and strong independence, it is argued that there is no justification for Dempster’s rule of combination within the interpretation of Chapter 5.

In Chapter 6, the proposed calculus of Chapter 5 is justified from a betting
SUMMARY

interpretation of belief functions. The first step is the justification of belief functions themselves from a betting interpretation, in an argument analogous to the Dutch Book argument. The theory of Peter Walley [61], which already provides betting schemes with a distinction between buying and selling prices, serves as a natural starting point here. Then it is argued that, on top of the constraint of coherence that Walley gives, reasonable agents should adhere to an additional constraint.

In Chapter 7, the theory developed in Chapter 5 is applied to forensic problems. This is done for two variants of the classical island problem and a case of parental identification. In these cases, a crucial problem of the classical approach is that one needs to assign prior probability to the criminal (in case of the island problem) and father (in case of parental identification). A uniform distribution is what is typically used, this does not really represent ignorance. With the theory of belief functions, we are able to take an actual uninformative prior. It is showed that doing so, leads to different answers than the classical answers and we compare these answers.

In Chapter 8, the theory is used to study an infinite regress problem. In this problem, we have an infinite sequence $E_0, E_1, E_2, \ldots$ of propositions. The central question is: if we know the conditional beliefs of $E_n$ given $E_{n+1}$ of some agent for every $n$, what can we infer about his or her (unconditional) beliefs about the individual propositions, and proposition $E_0$ in particular. For the analysis this problem, some basic theory of belief functions on infinite spaces is developed in this chapter. It is showed that, contrary to certain claims in the literature, this regress problem does in general not have a unique solution.