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## Computer Assisted Proofs in Dynamics and Delay Equations

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# Summary

In this thesis, we develop techniques that allow for the construction of computer assisted existence proofs of solutions of infinite-dimensional dynamical systems. The methods developed rely on using a numerically obtained approximate solution as a foundation for the existence proof. In particular, our aim is to show the existence (and uniqueness) of a solution close to the numerically obtained approximate solution.

Our approach is essentially based on the Banach contraction theorem. We rewrite the problem in question as a fixed-point problem of a Newton-like operator on a suitable Banach space. Moreover we construct this Newton-like operator such that any fixed point corresponds to a solution of the original problem. It can subsequently be shown that this Newton-like operator is contractive whenever a *finite* set of inequalities is satisfied. If this is the case, then the existence and (local) uniqueness of a fixed point is guaranteed by the Banach contraction theorem, which in turn proves the existence of a solution.

The inequalities constructed in this method are the fruit of careful pen-and-paper analysis and numerical methods. However, since the inequalities contain (possibly many) terms that depend explicitly on the numerical solution used, they themselves cannot be verified by hand but must be evaluated using specialised computer software. Hence the end result of the procedures described above is a computer program that checks whether the inequalities hold, and as a by-product provides the radius  $r$  of a ball around the numerical solution in which the true solution can be found.

In this thesis, we apply these techniques to two different classes of infinite-dimensional dynamical systems. The first, described in Chapter 2, concerns finding radially symmetric stationary solutions of PDEs. The remainder of the thesis focuses on the study of delay equations. In Chapter 3 we apply these techniques to the problem of finding periodic solutions of delay equations. Finally Chapter 4 is dedicated to developing techniques that allow for the construction of the unstable manifold of stationary and periodic solutions of delay equations. As we show in Chapter 4, these techniques allow for an efficient numerical approximation of compact subsets of the unstable manifolds, as well as a natural starting-point for a computer assisted existence proof. The latter falls outside the scope of Chapter 4, but nonetheless a relatively simple example of such a proof is included as an example in the Introduction of this thesis.