Product forms for random access schemes

van Dijk, N.M.

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Nico M. van Dijk

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Product Forms for
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Nico M.van Dijk
Free University, Amsterdam, The Netherlands

Abstract A framework is presented to investigate product form expressions for circuit or packet switching random access protocols such as multihop-CSMA. Acceptation, retransmission and delay or acceleration functions are included. The transmission times and packet lenghts are generally distributed. A concrete invariance condition on the system functions is given which guarantees an insensitive product form. This condition unifies and extends known results. Several new examples are obtained. In particular, recently derived product form expressions for multihop-CSMA protocols are generalized.

Keywords Product form, insensitivity, circuit/packet switching random access schemes, invariance condition, multihop-CSMA.

1. Introduction

Various packet or circuit switching random access schemes for computer, broadcasting or telecommunication networks have been introduced and investigated over the last decades (cf. [2], [10], [13], [14], [15], [16], [20], [21], [22], [23], [27]). Most notably among these are the ALOHA (e.g. [13], [20]) and CSMA (e.g. [13], [14], [20]) packet switching protocols and their various extensions (e.g. [2], [16], [23]). Particularizing to CSMA-protocols, explicit product form expressions for the steady state distribution have been established under exponentiality assumptions and simple interactions such as arising in single-hop radio packet networks. Recently, in [16] for the so-called "rude CSMA"-protocol and in [2], these results were extended to multihop random access schemes which take into account the well-known "hidden terminal problem". Relaxations to non-exponential packet lenghts (cf. [2]) and transmitter dependent parameters (cf. [5]) were also established. However, transmissions are still assumed to be exponential and several random access schemes of practical interest are not yet covered.

This paper aims to show that a conceptually simple framework unifies and extends the above CSMA-product form results, while it also provides new product form results for several other random access schemes. Particularized to the recent multihop CSMA-results from [2], [5] and [16] the extensions are the following:

(i) Non-exponential transmissions and packets.
(ii) More general random access mechanisms.
(iii) State dependent transmission speeds.
(iv) Link selective characteristics.

Generally, the main results are:

1) An insensitive product form expression.
2) A concrete condition in terms of system protocols.
3) A generalization of product form random access protocols.

Here, insensitivity means that the underlying random distributions (transmission times, packet lengths) play a role through only their means. A product form stands for factorization to individual components or stations. This product form result is related to product form results in the extensive literature on queueing networks (cf. [3], [4], [7], [8], [12], [26]), but has as such not been reported or recognized for the system under study. It can be shown that it conceptually fits in the framework of reversibility (cf. [12]) or of job-local-balance (cf. [8], [9]), provided appropiate conditions are met. However, sufficient conditions in terms of concrete system protocols are hereby left open and not obvious. To this end, a general invariance condition will be provided.It so turns out that various known product form telecommunication examples can be unified (e.g. examples 2.1-2.4). But also new product form transmission examples (e.g. examples 3.4-3.6) and a generalization of the multihop-CSMA protocols from [2] and [16] (see section 5) are easily concluded.

The organization is as follows. First, in section 2 the model is outlined. Next, in section 3 the condition upon the system protocols is presented and illustrated by some examples. The product form is derived in section 4. Finally, the particular models of [16] (Rude-CSMA) and [2] are extended as special examples. An evaluation concludes the paper.

2. Model

Consider a system of N nodes, numbered 1,...,N. Each of these nodes alternates between idle and busy periods as follows. After a think time, during which a node is called idle, a node h requests to become busy. If upon this request also other nodes $h_1,\ldots,h_n$ are already busy, this request is accepted with probability

$$A(h|h_1,\ldots,h_n)$$

and node h starts a holding time, during which it is called busy. When this request is not accepted, node h has to restart a new think time and thus remains idle. Conversely, upon completion of a holding time node h requests to become idle. When other nodes $h_1,\dots,h_n$ are currently busy, this request is accepted with probability

$$D(h|h_1,\dots,h_n)$$

and node h starts a think time. When this request is not accepted, node h has to restart a new holding time and thus remains busy. A think time of node h corresponds to a random service with distribution function T_h. A holding time of node h corresponds to a random service, with distribution function H_h. When nodes $h_1,\dots,h_n$ are busy, then

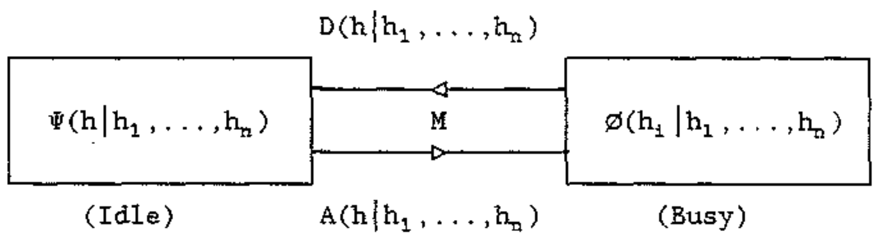
$$\Psi(h|h_1,\dots,h_n), \qquad h\neq h_1,\dots,h_n,$$

is the service speed of idle node h, while

$$\emptyset(h_i|h_1,\dots,h_n), \qquad i=1,\dots,n,$$

is the service speed of busy node h_i, $i=1,\dots,n$.

Queueing model correspondence. The description above can be visualized by



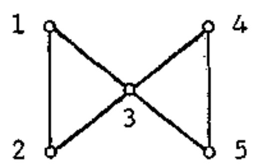
with the interpretation of a queueing example in which M jobs are sent back and forth between two stations with accessibility constraints (reflected by $A(.|.)$ and $D(.|.)$) and processor sharing servicing (reflected by $\Psi(.|.)$ and $\emptyset(.|.)$). The same description applies also to seemingly more complex communication or broadcasting systems as will be illustrated below. Herein we choose $D(.|.) = \Psi(.|.) = \emptyset(.|.) = 1$.

Throughout let $H = \{h_1, \ldots, h_n\}$ and denote by $H \pm h$ the state in which node h is added (+) or deleted (-) as a busy node.

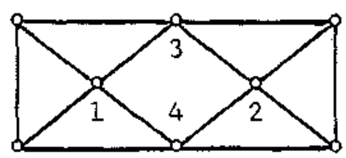
Example 2.1. (Interference graph; standard CSMA) (cf. [14], [16], [18], [20]). Let the nodes in a graph represent transmitters with the restriction that adjacent nodes cannot transmit at the same time. Let N(h) be the set of all neighbors of node h. Then the above description applies with

$$A(h|h_1, \ldots, h_n) = \begin{cases} 1 & \text{if } h_1, \ldots, h_n \notin N(h) \\ 0 & \text{otherwise.} \end{cases}$$

For example, in the two-hop CSMA-figure below (a hop means that all nodes within this hop can hear each other) node 3 prohibits all other nodes to transmit at the same time

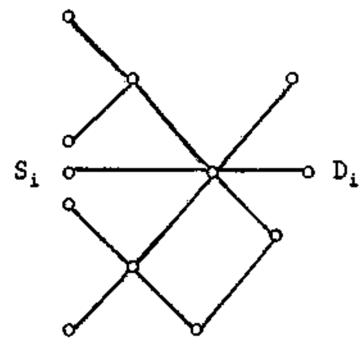


Example 2.2. (Multihop CSMA; hidden terminal problem) (cf. [2], [16]). As in example 2.1, again consider a graph of nodes with its neighbors all nodes that can hear this node. However, a node is not allowed to hear two nodes at the same time.



For instance in the above structure nodes 1 and 2 cannot transmit simultaneously as they are both heard by node 3 (and 4). (This is referred to as the hidden terminal problem). Though this structure cannot be modeled as a graph in which merely neigbors exclude each other, the parametrization of example 2.1 still applies if we replace N(h) be the set of neighbors that is either transmitting or hearing. Clearly, these two-hop interactions can be extended to multi-hop interactions.

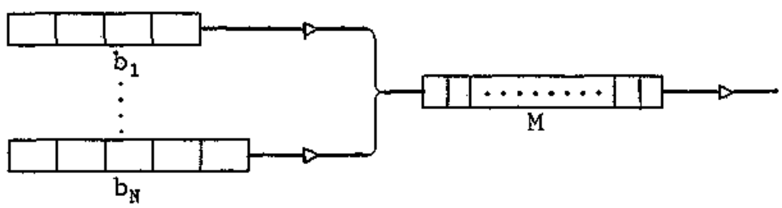
Example 2.3. (Circuit switching) (cf. [3], [20]). A circuit switching transmission may typically have a structure of the form



where messages from a particular source S_i are to be transmitted along a particular path P_i to a destination D_i. A transmission requires one trunk from each trunkgroup along this path. Interference thus arises because of limited trunkgroups and messages using the same trunkgroups. With $H = \{h_1, \ldots, h_n\}$ representing the different messages, $N_i(H)$ the number of these using trunkgroup i, and M_i the number of trunks in trunkgroup i, we can use

$$A(h|H) = \begin{cases} 1 & \text{if } N_i(H \cup h) \le M_i \text{ for all } i \\ 0 & \text{otherwise} . \end{cases}$$

Example 2.4. (Synchronous servicing) (cf. [6], [10], [11]). As a typical feature of digital transmissions, a transmission may use several time slots from a limited number of M time slots. The following figure visualizes that a type-i message simultaneously requires b_i time slots.



With $n_i(H)$ the number of type-i messages and t(h) the message type of node h, this is parametrized by

$$A(h|H) = \begin{cases} 1 & \text{if } \Sigma_i \; b_i \; n_i(H) + b_{t(h)} \le M \\ 0 & \text{otherwise} \end{cases} \quad .$$

Remark 2.5. As in these examples, many applications will involve only the function $A(.|.)$ while the other functions can be set equal to 1. The inclusion of the function $\emptyset(.|)$ may naturally arise to model a state dependent speed for transmitting, translating or processing a message at a node. The functions $D(.|.)$ and $\Psi(.|.)$ do not complicate the analysis at all. They make the model totally symmetric in idle and busy nodes which can be handy for both analysis and modeling purposes. For instance, delay factors such as due to error detection (see examples 3.4 ii and 3.5 ii), service accelerations(see example 2.5 i), or message interruptions (see example 3.6) can so be modeled.

Remark 2.6. The assumption of a restarting think or holding time upon blocking is common for communication systems (cf. [6], [10], [11], [15], [16], [17], [20], [27]). For an exponential think or holding time, it coincides with interrupting this time to evolve if the idle or busy status respectively is currently not allowed to change (cf. [17]).

Remark 2.7. Clearly we could have combined the functions $A(.|.)$, $\Psi(.|.)$, $D(.|.)$ and $\emptyset(.|.)$. However, as they naturally correspond to separate system features, we prefer not to.

3. Interference invariance condition

In this section we will impose a concrete condition upon the system functions that will guarantee an explicit product form expression later on. To this end, let a state $(h_1,\ldots h_n)$ denote that nodes $h_1,\ldots,h_n$ are busy, where $h_1,\ldots,h_n$ are given in increasing order, while the other nodes are idle. The monotone ordering is introduced merely for notational convenience in the condition below but does not play any role itself. Let state $\emptyset$ denote that all nodes are idle and without loss of generality assume that there exists an irreducible set H of states containing $\emptyset$, i.e. a set of states such that out of any state from this set any other state within this set and no state outside this set can be reached.

Condition 3.1. For any $H = (h_1, \ldots, h_n) \in \mathcal{H}$ and some value

$P(H)$

we have for some $\ell \le n$:

$$D(h_\ell | H-h_\ell)\ \emptyset(h_\ell | H) > 0, \tag{3.1}$$

while for all $i = 1, \ldots, n$:

$$D(h_i | H-h_i)\ \emptyset(h_i | H) = 0 \quad \Longleftrightarrow \quad A(h_i | H-h_i)\ \Psi(h_i | H-h_i) = 0, \tag{3.2}$$

and for all permutations $(i_1, \ldots, i_n) \in (1, \ldots, n)$:

$$\prod_{k=1}^{n} \frac{A(h_{i_k} | h_{i_1}, \ldots, h_{i_{k-1}})\ \ \Psi(h_{i_k} | h_{i_1}, \ldots, h_{i_{k-1}})}{D(h_{i_k} | h_{i_1}, \ldots, h_{i_{k-1}})\ \ \emptyset(h_{i_k} | h_{i_1}, \ldots, h_{i_k})} = P(H) \tag{3.3}$$

Condition (3.1) guarantees that the product in (3.3) has a positive denominator for at least one permutation, while (3.2) guarantees that if the denominator of this product is zero than also the numerator is equal to zero, so that the product can be chosen equal to P(H). Thus effectively only permutations with non-zero denominators need to be considered.

Condition (3.1) could be avoided but is included as it simplifies the presentation while it excludes only the extreme case that none of the current busy nodes is allowed to become idle again. Condition (3.2) is essential and corresponds to the property of "instantaneous attention" in the queueing literature (cf. [3], [4], [8], [9], [12]). Condition (3.3) is related to the well-known Kolmogorov criterion (cf. [12]) for a Markov chain to be reversible. Indeed, for the exponential case it will lead to reversibility. In the non-exponential case, however, reversibility is not satisfied.

Remark 3.1. (Decomposed $A(.|.)$ and $\emptyset(.|.)$ conditions). As mentioned in remark 2.4, in various applications the functions $D(.|.)$ and $\Psi(.|.)$ are equal to 1. Clearly, condition (3.3) is then guaranteed if for certain functions $P_1(H)$ and $P_2(H)$:

$$\prod_{k=1}^{n} A(h_{i_k} | h_{i_1}, \ldots, h_{i_{k-1}}) = P_1(H) \tag{3.4}$$

$$\prod_{k=1}^{n} \emptyset(h_{i_k} | h_{i_1}, \ldots, h_{i_{k-1}}) = P_2(H) \tag{3.5}$$

for all permutations $(i_1, \ldots, i_n)$ for which these products are positive. These conditions are satisfied for example, if for certain functions $g(n)$ and $h(n)$:

$$\begin{aligned} A(h|h_1, \ldots, h_n) &= g(n) \\ \emptyset(h_i|h_1, \ldots, h_n) &= h(n) \; . \end{aligned} \tag{3.6}$$

Remark 3.2. (Coordinate convex interferences). An important subclass of interferences with only 0 and 1 values (i.e., no randomized blocking) satisfying (3.4) is obtained by

$$A(h|h_1, \ldots, h_n) = \begin{cases} 1 & \text{if } (h, h_1, \ldots, h_n) \in C \\ 0 & \text{otherwise,} \end{cases} \tag{3.7}$$

where C is some set of states such that for all j

$$(h_1, \ldots, h_n) \in C \quad \Rightarrow \quad (h_1, \ldots, h_{j-1}, h_{j+1}, \ldots, h_n) \in C \tag{3.8}$$

In words that is, departures from C are prohibited where C satisfies (3.8). In correspondence with [6] and [11], such interferences are called "coordinate convex". Note that the corresponding function $P_1(H)$ is equal to 1 for all $H \in C$.

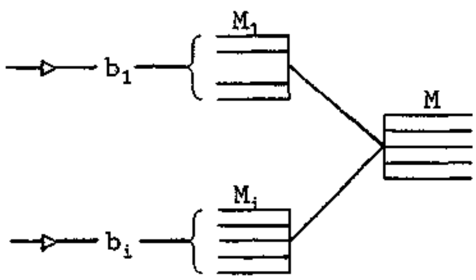
Below we will present several examples satisfying (3.4) and (3.5). The coordinate convex examples 3.3 have been individually studied in the literature (cf. [6], [10], [11]). The examples 3.4-3.6 have not been reported. Herein, all functions not specified are identical to 1.

Examples 3.3 (Coordinate convex interferences). One easily verifies that the examples 2.1-2.4 are "coordinate convex" with

(i) $C = \{H \mid H$ has no neighbors$\}$ in example 2.1
(ii) $C = \{H \mid H$ has no one or two-step neighbors$\}$ in example 2.2
(iii) $C = \{H \mid N_i(H) \le M_i$ for all trunkgroups $i\}$ in example 2.3
(iv) $C = \{H \mid \Sigma_i b_i n_i(H) \le M\}$ in example 2.4.

Examples 3.4 (Randomization). In some examples the functions $A(.|.)$ or $D(.|.)$ include randomization and thus have values other than 0 or 1.

(i) (Random grading). The following extension of the classical "Engset ideal grading" satisfies (3.4). There are different types of nodes. A type-i node transmits type-i messages. All messages share the same group of output channels. Type-i messages, however, can only be transmitted through M_i inputchannels.



When a node of type i wishes to transmit a message, it randomly hunts over b_i from the M_i input channels to find a free channel. Further, a transmission simultaneously requires an input and output channel. With $n_i[H]$ the number of type-i messages, n the total number of messages and t_h the type of node h, this is modeled by

$$A(h|H) = 1(n < M) \left\{1 - \binom{n_i[H]}{b_i} \Big/ \binom{M_i}{b_i}\right\}, \qquad (i = t_h).$$

The invariance condition (3.4) holds as a special example, since it holds with arbitrary functions $g(.)$ and $g_i(.)$ for

$$A(h|H) = g(n)\; g_i(n_i[H]), \qquad (i = t_h), \text{ with}$$

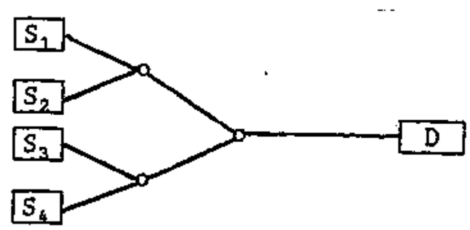
$$P_1(H) = \prod_{k=1}^{n} g(k-1) \prod_{i} \{ \prod_{k=1}^{n_i[H]} g_i(k-1) \}.$$

(ii) (Error detection). Consider a number of sources that share a common multi-channel transmission cable. During a transmission an error in the message may arise depending upon the current load of the cable. An error is not detected (e.g. by acknowledgements) before completion of the transmission and requires the complete message to be retransmitted. Let

$$D(h|H)$$

be the probability of an error in a message from source h if the sources $H = (h_1,\ldots,h_n)$ are currently transmitting. Then (3.3) is satisfied if condition (3.4) holds with $A(.|.)$ replaced by $D(.|.)$.

For example, consider the following circuit switching structure with four source types and let n_i be the number of busy type-i sources (i.e. currently sending a message from S_i to D).



As a message uses a trunk from each trunkgroup (edge) along its trajectory, the following error probabilities can be involved:

$$D(h|H) = \begin{cases} P_i(n_i)\ Q_1(n_1+n_2)\ Q_3(n_1+n_2+n_3+n_4), & (t_h=i\in(1,2)) \\ P_i(n_i)\ Q_2(n_3+n_4)\ Q_3(n_1+n_2+n_3+n_4), & (t_h=i\in(3,4)) \end{cases}$$

where P_i and Q_j are arbitrary functions with values between 0 and 1. The invariance condition (3.4) or rather (3.3) is easily verified with

$$P(H)^{-1} = \prod_i \{ \prod_{k=1}^{n_1} P_i(k)\} \prod_{k=1}^{n_1+n_2} Q_1(k) \prod_{k=1}^{n_3+n_4} Q_2(k) \prod_{k=1}^{n_1+n_2+n_3+n_4} Q_4(k).$$

Example 3.5. (Delay/acceleration factors)

(i) (Acceleration factors). As a simple acceleration example, assume that the transmission speeds are doubled upon threspassing a threshold M on the total number of transmissions. Then (3.5) is guaranteed by

$$\emptyset(h|H) = \begin{cases} 1, & n < M, \\ 2, & n \geq M, \end{cases}$$

$$P_2(H) = 2^{[n-M]^+}.$$

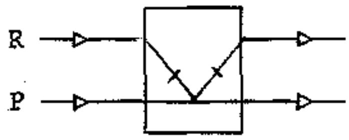
In analogy with example 3.4 (i), the above example is extendable to type-dependent thresholds M_i. More precisely, (3.5) is satisfied by substituting $\emptyset(.|.)$ for $A(.|.)$ and P_2 (.) for $P_1(.)$ in example 3.4(i).

(ii) (Delay factors). A standard delay example is a processor sharing service mechanism in which each job to be served (e.g. program to be run by a central processor unit) gets an equal share of the total capacity as prescribed by

$$\emptyset(h|H) = 1/n.$$

This can be extended to more detailed delay interferences. For example, the circuit switching example of 3.4 (ii) can be reread verbatim with $D(.|.)$ replaced by $\emptyset(.|.)$ representing a delay factor.

Example 3.6 (Priority messages). Various transmission systems are subject to "priority" (e.g. emergency) messages that have priority over regular messages in a preemptive manner. For example, consider a transmission device which can handle only one message at a time. Upon arrival of a "priority" message a regular transmission is interrupted and temporarily held up. Upon completion of the "priority" transmission, the regular transmission is continued.



Under exponential transmission times one easily argues that the stationary behaviour of the above system is the same under the following protocol. Once started, a regular transmission is continued until completion without interruptions by priority messages. The device can transmit one regular and one priority message simultaneously but, as before, a regular transmission can be started only when the device is idle while otherwise it is lost. Moreover, a regular message is to be retransmitted if upon completion of its transmission a priority message is currently transmitted.

Let R and P denote the sets of nodes that generate "regular" and "priority" messages respectively. Then the latter system, and thus also the original priority system under exponential transmission times, satisfies (3.3) with

$H = \{h_i \mid i=1,\ldots,M\} \cup \{(h_i, h_j) \mid h_i \in R \text{ and } h_j \in P\},$

$A(h_i \mid H) = 0 \quad \text{for } h_i \in R \text{ and } H \neq \emptyset,$

$A(h_i \mid h_j) = 0 \quad \text{for } h_i \in P \text{ and } h_j \in P,$

$D(h_i \mid h_j) = 0 \quad \text{for } h_i \in R \text{ and } h_j \in P,$

$A(.\mid.) = D(.\mid.) = 1 \quad \text{otherwise},$

$P(.) = 1.$

4. Product form

This section contains the main result of the paper. Without loss of generality assume that the think and holding time service functions T_h and H_h have continuous density functions $f_h(.)$ and $q_h(.)$ with means σ_h and τ_h respectively. Let the state

$$(S,T) = ((s_1,t_1),\ldots,(s_N,t_N))$$

denote that node i is idle when $s_i = 1$ and busy when $s_i = 2$ with a residual time t_i up to completion of the current think time ($s_i = 1$) or holding time ($s_i = 2$) respectively, $i = 1,\ldots,N$. For a given node specification $S = (s_1,\ldots,s_n)$ let H be the corresponding set of busy nodes. Let $\pi((S,T))$ and $\pi(H)$ be the steady state distributions. The next two theorems will then be proven. The first, of which the proof is given at the end of this section, is the key theorem. The second is the more practical consequence showing that the distributional forms of the think and holding times do not play a role.

Theorem 4.1. Under condition 2.1 with P(H) given by (3.3) and c a normalizing constant, we have for all (S,T) with $H \in \mathcal{H}$:

$$\pi((S,T)) = c\, P(H) \prod_{h:s_h=1} [1-T_h(t_h)] \prod_{h:s_h=2} [1-H_h(t_h)]. \qquad (4.1)$$

As an immediate consequence, by noting that

$$\int_0^\infty [1-T_h(t)]dt = \sigma_h \qquad \text{and}$$

$$\int_0^\infty [1-H_h(t)]dt = \tau_h \ ,$$

we obtain by integration over all possible residual times t_h and substituting $\bar{c} = c(\sigma_1)(\sigma_2)\ldots(\sigma_N)$:

Theorem 4.2. Under condition 3.1 with P(H) given by (3.3) and $\bar{c}$ a normalizing constant, we have for all $H \in \mathcal{H}$:

$$\pi(H) = \bar{c}\ P(H) \prod_{h \in H} [\tau_h/\sigma_h] \qquad (4.2)$$

Remarks 4.3.

1. Note that expression (4.2) is determined by only mean think and holding times as well as P(H) calculated by (3.2) in terms of concrete systems functions.

2. In principle the verification of condition 3.1 and the calculation of P(H) can be computationally complex. However, in most practical situations one either easily finds a counterexample with 0 and 1 values or one can recursively calculate P(H) as based upon "basic" paths or cycles. (Related results along this line can be found in [7], [8] and [12]).

3. Similarly to [24], the above results can be extended to allow different levels of think and holding times for a node. These levels can be "averaged out" leading to expression (4.2) with σ_h and τ_h representing "averaged" means. Multi levels may reflect for instance different interrupted phases of a transmission.

4. Similarly to [25], also the "arrival theorem" can be shown to hold which here would read as: "The steady state distribution as seen by a node upon think time completion is given by (4.2) for the system without that node". The well-known mean value algorithm (cf. [19]) to efficiently compute performance measures can thus be applied. A computational approach to compute the normalizing constant based upon a statistical mechanics technique can be found in [18].

5. In various standard ways (e.g. by letting $N\to\infty$ as in [1] or by including a "dummmy node" as in [8]), similar results can be provided to model "infinite or open" transmission systems with Poissonian inputs.

Proof of theorem 4.1.

We need to verify the global balance or forward Kolmogorov equations assuming without loss of generality that these have a unique solution. To this end, for a given state (S,T) and node i, let

$$(S,T)-(s_i,t_i)+(\tilde{s}_i,\tilde{t}_i)_i$$

denote the same state with the node i specification changed from (s_i,t_i) in $(\tilde{s}_i,\tilde{t}_i)$. Further, we use the symbol 0^+ to indicate the right hand limit at 0. Then, for a fixed state (S,T) with H representing its busy sources, the global balance equations become:

$$\sum_{h:s_h=1} \Big\{ \frac{\partial}{\partial t_h} \pi((S,T))\ \Psi(h|H) +$$

$$\pi((S,T) - (1,t_h)_h + (2,0^+)_h)\ \emptyset(h|H+h)\ D(h|H)\ q_h(t_h)$$

$$\pi((S,T) - (1,t_h)_h + (1,0^+)_h)\ \Psi(h|H)[1-A(h|H)]\ q_h(t_h) \Big\} +$$

$$\sum_{h:s_h=2} \Big\{ \frac{\partial}{\partial t_h} \pi((S,T))\ \emptyset\ (h|H) +$$

$$\pi((S,T) - (2,t_h)_h + (1,0^+)_h)\ \Psi(h|H-h)\ A(h|H-h)\ f_h(t_h)$$

$$\pi((S,T) - (2,t_h)_h + (2,0^+)_h)\ \emptyset(h|H)[1-D(h|H)]\ f_h(t_h) \Big\} = 0. \tag{4.3}$$

Assume that (4.3) has a unique probability density solution $\pi(.)$. It thus suffices to verify (4.3) with (4.1) substituted for $\pi(.)$. First conclude from (3.2) that for h with $s_h = 1$ and $\Psi(h|H) = 0$ or for h with $s_h = 2$ and $\emptyset(h|H) = 0$ all three terms within braces {...} corresponding to that node are equal to 0.

From (4.1), the permutation invariant expression (3.3) for P(.), noting that $T_h(0^+) = H_h(0^+) = 0$ and recalling that $T_h(.)$ has a derivative $q_h(.)$, we conclude for a node h with $s_h = 1$:

$$\frac{\partial}{\partial t_h} \pi((S,T))\mu = -q_h(t_h) \times$$

$$\pi((S,T) - (1,t_h)_h + (1,0^+)_h) \tag{4.4}$$

$$\pi((S,T) - (1,t_h)_h + (2,0^+)_h) = \frac{A(h|H)\ \Psi(h|H)}{D(h|H)\ \emptyset(h|H+h)} \times$$

$$\pi((S,T) - (1,t_h)_h + (1,0^+)_h), \tag{4.5}$$

provided $D(h|H)\ \emptyset(h|H + h) > 0$. However, $D(h|H)\ \emptyset(h|H+h) = 0$ would imply that $A(h|H)\ \Psi(h|H) = 0$ by virtue of (3.2). Hence, by also assuming $\Psi(h|H) > 0$ as argued above we then have $A(h|H) = 0$. As a consequence, in either case and by substituting (4.4) and (4.5) the term within {...} in (4.3) for h with s_h=1 is equal to

$$\pi((S,T) - (1,t_h)_h + (1,0^+)_h) \quad x$$

$$q_h(t_h)\ \Psi(h|H)\ \{-1 + A(h|H) + [1-A(h|H)]\} = 0. \tag{4.6}$$

One similarly argues that for h with $s_h = 2$ the term within {...} in (4.3) equals

$$\pi((S,T) - (2,t_h)_h + (2,0^+)_h) \quad x$$

$$f_h(t_h)\ \emptyset(h|H)\ \{-1 + D(h|H) + [1-D(h|H)]\} = 0, \tag{4.7}$$

regardless of whether $\Psi(h|H-h)\ A(h|H-h) > 0$ or not. We have thus verified (4.3), which completes the proof of the theorem.

5. Multihop-CSMA protocols

As illustrated in section 4, the framework of section 2 both unifies and extends standard product form communication examples. In this section we will show that also the multihop-CSMA protocols from [16] and [2] are included and generalized within this framework.

5.1 Extended rude CSMA (cf. [16])

As an extension of example 2.2, consider a set of nodes representing transmitters. Let N(h) be the set of all neighbors of node h, i.e. all nodes that it can hear, where it is assumed that if node i can hear node j than also node j can hear node i. As yet, in contrast with example 2.2, we do not exclude that neighbors can transmit at the same time. For a given set of busy (i.e. transmitting) nodes $H = (h_1, \ldots h_n)$, let $B_0[H]$ be the number of pairs of neighors that are both not transmitting and let $B_1[H]$ the number of pairs of neighbors that are both transmitting. Consider arbitrary funtions $g_0(n)$ and $g_1(n)$ and assume that for all reachable states $H \in \mathcal{H}$: $g_0(B_0[H])g_1(B_1[H]) > 0$ and for all $h \notin H$:

$$A(h|H) = \frac{g_0(B_0[H + h])}{g_0(B_0[H])} \quad \frac{g_1(B_1[H + h])}{g_1(B_1[H])} \tag{5.1}$$

which by scaling of the functions $g_0(.)$ and $g_1(.)$ can be assumed to be less than or equal to 1. Further, for simplicity assume that the other functions $\emptyset(.|.)$, $\Psi(.|.)$ and $D(.|.)$ are identical to 1. Setting

$$P(H) = g_0(B_0[H]) \; g_1(B_1[H]), \tag{5.2}$$

we have for all H, H + h $\in \mathcal{H}$:

$$P(H+h) = P(H) \; A(h|H), \tag{5.3}$$

which is to be seen as the detailed balance equation for reversibility (cf. [12], p.22) of a continuous time Markov chain with rates $q(H \to H + h) = A(h|H)$ and $q(H + h \to H) = 1$. The invariance condition (3.3) is then a direct consequence of the Kolmogorov criterion (cf. [12], p.23) for reversibility.

As a special case the rude-CSMA protocol from [16] is obtained by

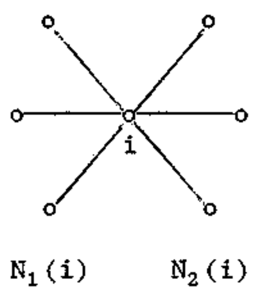
$$g_0(B_0[H]) = x^{-B_0[H]}$$
$$g_1(B_1[H]) = y^{B_1[H]} \tag{5.4}$$

$$A(h|H) = x^{N_0^h(H)} \; y^{N_1^h(H)} \tag{5.5}$$

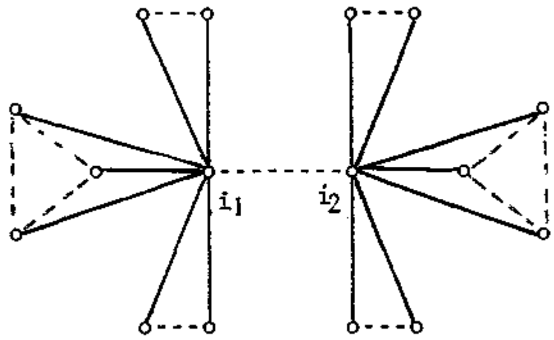
where $N_0^h(H)$ and $N_1^h(H)$ are the numbers of idle (not transmitting) and busy (transmitting) neighbors from h in state H and where x and y are given system parameters. For instance x=1, y=1 corresponds to the ALOHA-protocol (no collisions), x=1, y=0 models the standard CSMA protocol of example 2.1 and other values of x and y may reflect for instance that sensing of channels is not always reliable (cf. [16]). As the framework allows node dependent transmission times and packet lengths the extension of [5] is hereby covered.

5.2 Link selective multihop CSMA (cf. [2])

Again consider a set of nodes representing transmitters. Now, however, we allow that a node may transmit different messages to different sets of neighbors. For instance, a different transmission rate may be scheduled for each different neighbor or link. Say, node i can transmit a message type j to neighbors $N_j(i)$ for $j=1,\ldots,m(i)$, where the sets $N_j(i)$ are not required to be disjoint. Also, it is not excluded that a node transmits more than one message at the same time. The transmission scheduling times and message lengths are all assumed to be independent. For example, a node i can transmit 2 message types to disjoint sets $N_1(i)$ and $N_2(i)$.



Such a system can be transformed into the framework of section 2 as follows. Consider a new multi-node system in which each node corresponds to a different message type of a node. For example, as illustrated below, a node with two message types to two disjoint sets of neighbors will lead to separate nodes i_1 and i_2. These nodes i_1 and i_2 will be connected depending on whether or not the original node can or can not transmit both messages at the same time. Also each of the original neighbors is to be splitted in as many neighbors as it has message types, such as 2 for the lower and upper and 3 for the middle original neighbors.



The original system is thus modified in the standard multihop-CSMA model from example 2.2, which satisfies the coordinate convex condition (3.7) and (3.8) (see example 3.3(ii)) and thus the invariance condition (3.3) with $P(.) = 1$.

Now let τ_m^h, σ_m^h be the mean message length and transmission time respectively of message type m from transmitter h and denote by

$$(H,M) = \{(h, M(h)); h \in H\}$$

the state in which nodes $h \in H$ are transmitting and where node h currently transmits messages of types $M(h) = \{m_1, \ldots, m_{x(h)}\}$ for some x(h). Let $(\mathcal{H},\mathcal{M})$ be the corresponding state space of admissible states. Then by virtue of the above transformation of the original system into the standard multihop-CSMA system of example 2.2, we obtain from theorem 4.2:

$$\pi((H,M)) = \tilde{c} \prod_{h \in H} \prod_{m \in M(h)} [\tau_m^h/\sigma_m^h] , \quad (H,M) \in (\mathcal{H},\mathcal{M}) \qquad (5.6)$$

as steady state distribution with $\tilde{c}$ a normalizing constant. In particular, assuming that a node can transmit only one message at a time, so that M(h) is always a singleton, and aggregating over the message types we obtain:

$$\pi(H) = \bar{c} \prod_{h \in H} \bar{\rho}^h, \qquad H \in \mathcal{H}, \tag{5.7}$$

$$\bar{\rho}^h = \sum_{i=1}^{m(h)} \tau_m^h / \sigma_m^h$$

Thus $\bar{\rho}^h$ is the averaged transmission intensity for node h. The result from [2] is hereby included setting $1/\tau_m^h = \mu_h$ (the packet lengths or transmission times are the same for all links) and $1/\sigma_m^h = g_{hm}$ (representing a scheduled transmission rate for link m of node h), so that $\bar{\rho}^h = g_h/\mu_h$ with $g_h = \Sigma_m g_{hm}$ the total transmission rate of node h.

Evaluation. A framework is presented by which the possibility of product form results for various telecommunication packet or circuit switching random access schemes can be investigated. Exponentiality assumptions are avoided. A condition is provided, in terms of concrete system protocols, that guarantees an explicit product form expression depending upon only mean transmission times and packet lengths. This condition unifies and extends standard product form telecommunication examples, but also leads to a number of new product form examples for circuit or packet switching and resource sharing random access schemes. For instance, synchronization, random grading, error detection, delays or accelerations and priority messages can be involved. Particularly, generalizations are given of recently reported product form results for multihop-CSMA protocols. Extensions of this framework such as to include multi-stage or ordered transmissions seem possible.

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