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1989

SERIE RESEARCH MEMORANDA

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Research Memorandum 1989-28
July 1989

VRIJE UNIVERSITEIT
FACULTEIT DER ECONOMISCHE WETENSCHAPPEN
EN ECONOMETRIE
AMSTERDAM

Exact Solutions For Central Service Systems With Breakdowns

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Abstract A central service system with a fixed number of users is studied for three possible types of breakdowns : independent, active and delayed. For each of these an explicit recursive expression for the stationary queue length distribution is derived. As a particular application, a star-type CSMA communication network is hereby solved.

Keywords Breakdown, central service system, communication network, global balance equations.

1. Introduction

Breakdowns or off-periods are a most common feature in stochastic service systems of which most notably computer and communication networks, such as resulting from a processor failure, an error detection, a maintenance operation, a service interuption, a blocked or broken communication link, a priority job or some external disturbance factor.

As breakdowns ussualy have a considerable negative impact on sytem performance, they have become a general concern in performance evaluation and currently receive special attention under the name of performability analysis (cf [15], [19], [20]).

Unfortunately, simple explicit expresions such as Jackson's celebrated product form do no longer apply for systems with breakdowns. Closed form expresions have been limited to special situations (see remark 2.2). Approximations and efficient computational pro cedures have therefore been widely investigated (cf. [3], [7], [8], [11], [15], [17], 19], [20]).

This paper will provide an explicit recursive expression for the stationary queue length distribution of a central processor system under each of the following types of breakdowns :

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independent : breakdowns occur randomly
active      : breakdowns occur only when the processor operates
delayed     : breakdowns (off-periods) are delayed until idling
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The recursive expressions can be easily computed from which other relevant performance measures such as a mean queue length or system efficiency are readily obtained.

The organization of this paper is as follows. In section 2 the model and the three different breakdowns are described and motivated by examples. The stationary expressions are presented in section 3 and proven in section 4. A brief discussion such as on possible extensions as well on the limitations concludes the paper.

2. Model

Consider a central processor as sketched in fig. 1 with a fixed number of N sources (e.g. users or terminals)

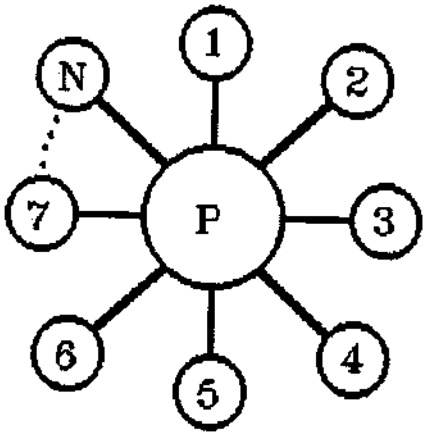


fig 1 processor connected with N sources

Each source alternates between idle and busy periods as follows. After an exponential period with parameter λ, during which it is called idle, a source requests to receive an exponential amount of service from the procesor with parameter μ and is called busy until completion of this service. The processor, however, alternates between "on" (or up) and "off" (or down) periods as due to breakdowns which renders it inoperative for exponential periods with parameter γ_0. When the processor is "on" each busy source is served at a unit speed i.e. it receives one unit of service per unit of time. When it is "off" no service is provided and the busy sources have to wait for the next "on" period to be served.

As for the times at wich breakdowns can occur, we distinquish between three types of breakdowns. The first two are standard (cf [10]). The third one arises from a particular communication application as will be clarified further on.

1. Independent breakdown.

   A breakdown occurs randomly at an exponential rate γ_1 independently of whether the processor is servicing or not.

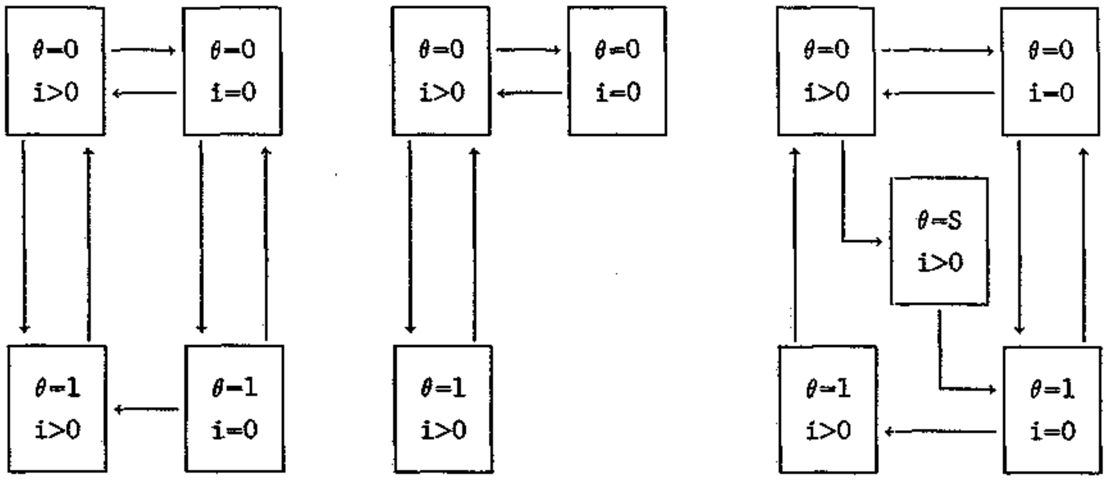
2. Active breakdown.

   A breakdown occurs at an exponential rate γ_1 only when the processor is servicing i.e. when at least one source is busy.

3. Delayed breakdown.

   A breakdown (or rather off-period) is scheduled at an expontial rate with parameter γ_1 independently of whether the processor is servicing or not. After a breakdown has been scheduled an off-period will not start before all sources are idle. As long as this off-period has not yet started sources can become busy.

The different cases are graphically presented in fig. 2 below.



independent case   active case   delayed case

$$\theta = \begin{cases} 0 : \text{processor on} \\ 1 : \text{processor off} \\ S : \text{processor starved} \end{cases}$$

fig 2   i = number of active sources

Remark 2.1 :

One way to see that these different types are not effectively the same is the following. When all sources have been idle simultaneously, the first source that becomes busy may have to wait to be served in the independent case while it is always immediately served in the active case. In both the active and independent case, after an off-period services may still have to be completed. In the delayed case, in contrast, an off-period always leaves all sources idle.

Examples 2.2 :

1. Independent case.

Consider a computer processor unit with N users and a memory module. Upon service completion data are to be retrieved and stored at the module. This module however is subject to off-periods regardless of the system state. For instance, an off-period may represent a regular maintenance period or a period in wich the module is required for another CPU (see fig 3).

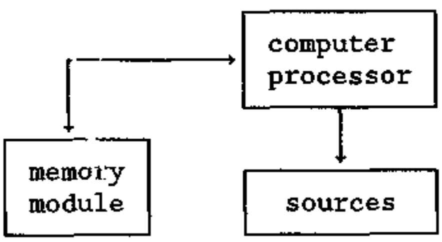


fig 3

2. Active case.
Here one can think of a processor unit which uses some resource such as a storeage device or a generator in order to provide service. This resource itself may have breakdowns or may need off-periods when used. For example a storage device may give errors from time to time or a printer requires installment of writing material at regular periods.

3. Delayed case.

a) Lower priority job :
Consider a processor with N users in which it is possible that some external job requires (or stops) the full processor capacity. For example, it may represent a regular maintenance operation. The external job has a lower priority than the regular jobs and thus has to wait for all users to become idle. Its service however cannot be interrupted so that regular jobs (busy users) may have to wait for this external job to be completed.

b) CSMA communication network :
In figure 1 now let all nodes 1..N as well as the central node represent transmitters of a star communication network. The subnodes independently schedule messages at an exponential rate λ and can transmit simultaneously. Their message lenghts are exponential with parameter μ. The central node schedules messages at an exponential rate γ_1 with an exponential message length with parameter γ_0. A subnode and the central node, however, cannot transmit at the same time, as graphically reflected by the links. In practice, this is effectuated by a so-called carrier sense multiple acces protocol (CSMA) (cf[1], [2], [13], [21], [22]). As messages cannot be interrupted, nodes can become starved when blocking arrises. Upon a blocked message request of a subnode it becomes starved until message completion of the central node. Upon a blocked message request of the central node it becomes starved until all subnodes are idle again (Note that various subnodes may meanwhile have become active and completed a message transmission)

Remark 2.3 (literature) :
To the best of our knowledge no explicit expressions for the stationary queue length distribution of the above or closely related systems have been reported in the literature. From [9] one can roughly conclude that none of these systems can have a geometric or product form type distribution. Sources can become busy even though the processor is down, so that the rate into that particular state due to a source is positive while the rate out of that state due to that source is zero. Necessary partial balance notions such as balance per source, for concluding a product form thus fail. Below we briefly discuss results that have been obtained for these systems.

(i) The independent and active breakdown model have both been analysed in [10] with general service times, a single server and the finite sources replaced by Poisson input. In that case the generating function of the queue length distribution has been obtained. However, even not under exponential services an explicit expression for the stationary queue length distribution can be extracted for the present system. In [4] simple performance bounds for the independent case have therefore been established while similar bounds can be provided for the active case.

(ii) For the particular communication network application of the delayed breakdown model as described in example 3b, it is argued in [20] that a product form result cannot hold as starvations destroy reversibility ([12]). An approximation procedure was therefore developed. In contrast, if upon blocking messages are lost product form results for communication schemes such as ALOHA, CSMA and BTMA can be concluded under wide conditions (cf [5]) The prime motivation for this paper, however, was just to investigate for the communication network of example 3b up to what extent explicit expressions are still obtainable when blocking tends to starvation rather than to lost messages.

(iii) In [7] most accurate approximations are provided for various performance measures as well as the queue length distribution without exponentiallity assumptions. In the special exponential case this even leads to a recursive expression for the queue length distribution. Their paper, however assumes Poisson arrivals, a single server, FCFS-dicipline and independent breakdowns only. In [3] a discrete time model with messages of fixed length is studied.

(iv) In [16] and [18] multi-server systems are studied in which each server can randomly breakdown and require a repair. The moment generating fuction for the queue length distribution is obtained for less or equal than two servers while tractible numerical methods are proposed for the general case.

(v) Though somewhat related results for vacation models such as in [14] and reviewed in [6] are not directly transformable as essentially different technicalities are involved. For instance, in an vacation model a server may instantaneously take a "break" upon work completion, while in our formulation a "break" will be scheduled randomly.

(vi) The closed form expressions in [15] are given for an approximate model based upon decomposition and are thus ro be seen as approximations.

3 Results

Let the state of the system be described by the pair (θ, i) denoting that the system is "on" when $\theta=0$, "off" when $\theta=1$ and, only in the delayed breakdown model, "starved" (breakdown scheduled) when $\theta=S$, while i is the number of busy sources. Let $\pi(\theta, i)$ be the corresponding stationary probability. For presentational convenience we introduce the following notation :

$$A_{ik} = \begin{cases} \dfrac{1}{(N-i)\lambda + \gamma_0} & i=k \\ \dfrac{(N-i)A_{i+1k}}{(N-i)\lambda + \gamma_0} & 0<i<k \\ NA_{i+1k} & i=0 \end{cases}$$

$$B_{ik} = \begin{cases} i\mu & i=k \\ i\mu + (N-i)\lambda + \gamma_1 & i=k-1 \\ \dfrac{B_{i+1k}}{(i+1)\mu}\left\{i\mu + (N-i)\lambda + \gamma_1\right\} - \dfrac{B_{i+2k}}{(i+2)\mu}(N-i)\lambda & i\le k-2 \end{cases}$$

for the independent and active breakdown model, and additionally

$$C_i = \begin{cases} \dfrac{1}{i\mu + (N-i)\lambda + \gamma_1 - (i+1)(N-i)\lambda\mu C_{i+1}} & 0\le i\le N \\ 0 & i=N+1 \end{cases}$$

$$D_i = \begin{cases} 1 & i=0 \\ \dfrac{(N+1-i)\lambda}{\gamma_0 + (n-i)\lambda} D_{i-1} & 1\le i\le N \end{cases}$$

$$E_{ik} = \begin{cases} i\mu & i=k \\ i\mu + (N-i)\lambda & i=k-1 \\ \dfrac{E_{i+1k}}{(i+1)\mu}\left\{i\mu + (N-i)\lambda\right\} - \dfrac{E_{i+2k}}{(i+2)\mu}(N-i)\lambda & 1\le i\le k-1 \\ \dfrac{E_{i+1k}}{\mu}\left\{\gamma_0 + N\lambda\right\} & i=0 \end{cases}$$

for the delayed breakdown model. The following results will then be proved in section 6.

Independent breakdown (reference state $\pi(0,0)$)

$$\text{(1a)}\quad \pi(0,k) = \frac{1}{k\mu}\left\{B_{0k}\pi(0,0) - \gamma_0 \sum_{j=0}^{k-1} \pi(1,j)\frac{B_{j+1k}}{(j+1)\mu}\right\}$$

$$\text{(1b)}\quad \pi(1,0) = \pi(0,0)\gamma_1/(N\lambda+\gamma_0)$$

$$\text{(1c)}\quad \pi(1,k) = \gamma_1 \sum_{j=1}^{k} \lambda^{k-j} A_{jk}\pi(0,j) + A_{0k}\lambda^{k}\pi(1,0) \qquad 1\le k\le N$$

Active breakdown (reference state $\pi(0,0)$)

$$\text{(2a)}\quad \pi(0,1) = \pi(0,0)N\lambda/\mu$$

$$\text{(2b)}\quad \pi(0,k) = \frac{1}{k\mu}\left\{(B_{0k} - B_{1k}\gamma_1/\mu)\pi(0,0) - \gamma_0 \sum_{j=1}^{k-1} \pi(1,j)\frac{B_{j+1k}}{(j+1)\mu}\right\}, \qquad 2\le k\le N$$

$$\text{(2c)}\quad \pi(1,k) = \gamma_1 \sum_{j=1}^{k} \lambda^{k-j} A_{jk}\pi(0,j) \qquad 1\le k\le N$$

Delayed breakdown (reference state $\pi(1,0)$)

$$\text{(3a)}\quad \pi(1,k) = D_k\pi(1,0)$$

$$\text{(3b)}\quad \pi(0,0) = \gamma_0 \sum_{j=0}^{N} j!\mu^{j}\pi(1,j) \prod_{i=0}^{j} C_i$$

$$\text{(3c)}\quad \pi(0,k) = \frac{1}{k\mu}\left\{B_{0k}\pi(0,0) - \gamma_0 \sum_{j=0}^{k-1} \pi(1,j)\frac{B_{j+1k}}{(j+1)\mu}\right\}$$

$$\text{(3d)}\quad \pi(s,k) = \frac{1}{k\mu}\left\{E_{0k}\pi(1,0) - \gamma_1 \sum_{j=0}^{k-1} \pi(0,j)\frac{E_{j+1k}}{(j+1)\mu}\right\} \qquad 1\le k\le N$$

4 Extensions and Limitations

The models presented are relatively simple but seem extendable in various ways as will be discussed below. As the precise details are rather lengtly while the proofs are essentially the same, the details are omitted and left to the reader. Roughly, as long as the state can be presented in a two-dimensional manner recursive structures as used above will remain. Some limitations are therefore also apparent as will be briefly adressed.

Extensions

1. State dependent service speeds

Clearly the transition and thus recursive structure is not affected by replacing the "unit sevice speed" per busy source by a speed which depends on the total number of busy sources. Service diciplines such as processor sharing or first come first served can hereby be modelled. Particularly one may note that the "symmetry" condition (cf. [10]) is not involved.

2. Busy source limit

The total number of busy sources can be prohibited to exceed a certain limit L, such as reflecting a restricted number of sources or storage capacity. Assuming that service requests upon saturation of this limit are lost, the transition structure applies with truncation at n=L .

3. Poisson input

As under 1 for service speeds, also the arrival intensities can be made arbitrarily dependent on the number n. Particularly, the finite source input can be replaced by a Poisson input.

4. Delayed breakdown model

After a breakdown has been scheduled in the delayed breakdown model, idle sources can still become busy and be served before the "off" period starts. The corresponding transitions can simply be deleted in the transition diagram. When service requests are lost when a breakdown has already been scheduled a similar expression can thus be obtained.

5. Nonexponential "on"- and "off"-periods

By letting θ include the residual time up to the next "on"- and "off"-period, non exponential "on"- and "off"-periods can be modelled. Particularly using mixtures of Erlang distributions and measuring the number of residul exponential phases, a recursive discrete structure, though more complex than above, will be retained leading to recursive explicit expressions. However, as per numerical counterexamples (eg.[]), one cannot expect an insensitivity result.

Limitations

1. Source independent characteristics

As the recursive structure is based on the two-dimensional representation, an extension to source dependent characteristics, which would require detailed information on any source, is not obvious.

2. Exponential services

For the same reason as under 1, one cannot include specifications of residual service times. Exponential service assumptions thus seem to be necessary.

3. Single breakdowns

Our description assumes a breakdown or "off"-period for the total system and not for individual servers. Extensions to multiple breakdowns are not apparent.

5. proofs

In this section we prove the results stated in section 3. First, in 6.1, we present the global balance equations. The common two-dimensional structure but also the differences under the different types of breakdowns are hereby illustrated. Next in section 6.2 we show how these equations can be solved recursively. For convenience we introduce

(4) $\pi(0,k-1)=0$ for $k\leq 0$
$\pi(0,k+1)=0$ for $k\geq N$.

5.1 Global balance equations

In the global balance equations stated below the normalization condition is omitted. Also, given the model descriptions one easily argues that the underlying processes are irreducible and finite so that these equations have a unique solution upto normalization.

The global balance equations for the independent breakdown model are :

(5a) $\pi(0,i)(\gamma_1 + i\mu + (N-i)\lambda) = \pi(0,i-1)(N-i+1)\lambda + \pi(0,i+1)(i+1)\mu + \pi(1,i)\gamma_0$
(5b) $\pi(1,i)(\gamma_0 + (N-i)\lambda) = \pi(0,i)\gamma_1 + \pi(1,i-1)(N-i+1)\lambda$

with $0\leq i\leq N$ and (4). The equations are graphically shown in fig. 4.

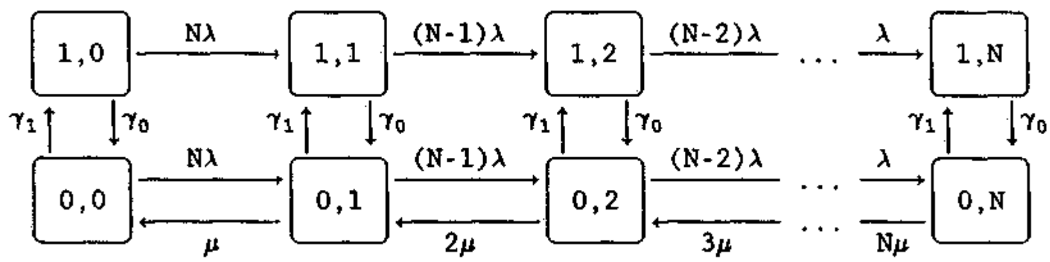


fig 4 independent breakdown model

Active breakdown model

Almost equal to (5) the global balance equations now become

(6a) $\pi(0,0)N\lambda = \pi(0,1)\mu$
(6b) $\pi(0,i)(\gamma_1 + i\mu + (N-i)\lambda) = \pi(0,i-1)(N-i+1)\lambda + \pi(0,i+1)(i+1)\mu + \pi(1,i)\gamma_0$
(6c) $\pi(1,i)(\gamma_0 + (N-i)\lambda) = \pi(0,i)\gamma_1 + \pi(1,i-1)(N-i+1)\lambda$

with $1\leq i\leq N$, (4) and $\pi(1,0)=0$. These equations are graphically shown in fig. 5.

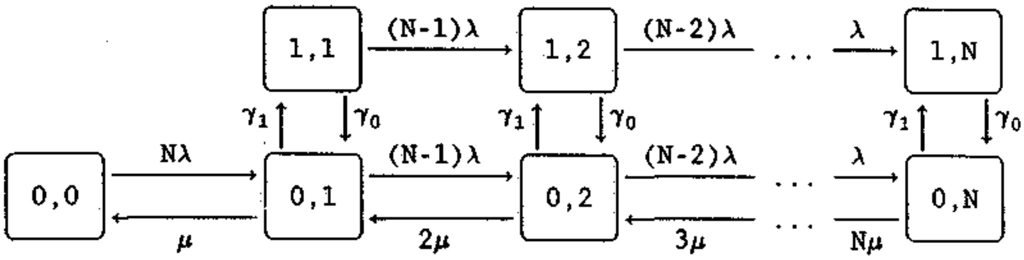


fig. 5 active breakdown model

Delayed breakdown model

The global balance equations are somewhat different as given by

(7a) $\pi(0,i)(\gamma_1 + i\mu + (N-i)\lambda) = \pi(0,i-1)(N-i+1)\lambda + \pi(0,i+1)(i+1)\mu + \pi(1,i)\gamma_0$
(7b) $\pi(0,0)(\gamma_1 + N\lambda) = \pi(0,1)\mu + \pi(1,0)\gamma_0$
(7c) $\pi(s,i)(i\mu + (N-i)\lambda) = \pi(s,i-1)(N-i+1)\lambda + \pi(s,i+1)(i+1)\mu + \pi(0,i)\gamma_1$
(7d) $\pi(1,i)(\gamma_0 + (n-i)\lambda) = \pi(1,i-1)(n-i+1)\lambda$
(7e) $\pi(1,0)(\gamma_0 + N\lambda) = \pi(0,0)\gamma_1 + \pi(s,1)\mu$

with $1 \leq i \leq N$, (4) and $\pi(s,0)=0$, and illustrated by fig. 6.

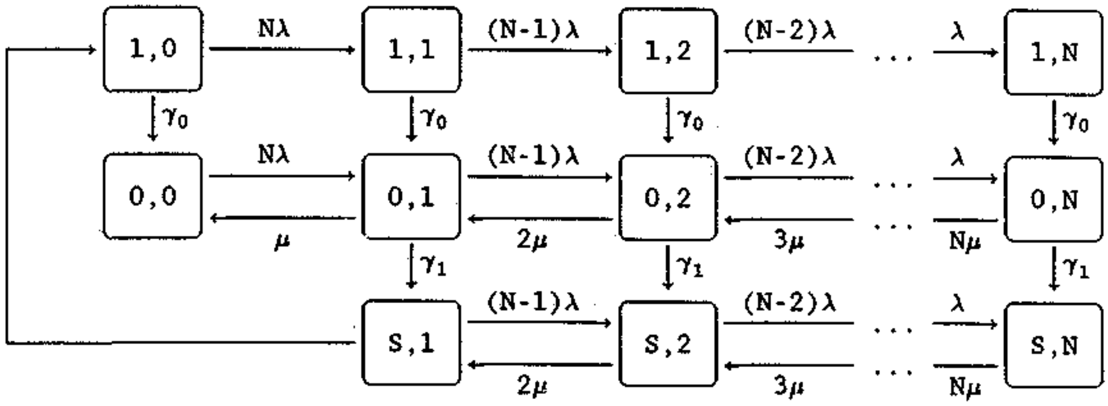


fig. 6 delayed breakdown model

5.2 Recursions

Below we will show how the above global balance equations can be solved recursively. The recursions for (5) and (6) are almost equal while that for (7) is somewhat different.

These recursions can be executed directly in actual copmutations. However, for both computational and theoretical interest, such as to obtain quantitative and relational inseights, we will show that they also lead to the explicit expressions (1), (2) and (3).

For each case we first present the recursive calculation scheme. For the independent and delayed breakdown case these will be worked out in detail while that of the active case follows similarly to that of the independent case.

Independent breakdown model

From figure 4 we derive the following calculation scheme for solving the balance equations upto normalization :

1) Set $\pi(0,0) = 1$ $\pi(1,0) = \pi(0,0)\gamma_1/(\gamma_0+N\lambda)$ by (5a)   2) Suppose $\pi(0,j)$ and $\pi(1,j)$ are known for $j\le k$ and $k\le N-1$ then   first calculate $\pi(0,k+1)$ by (5a)   next calculate $\pi(1,k+1)$ by (5b)

Suppose that $\pi(0,j)$ and $\pi(1,j)$ are known for $j\le k$. Then by rearranging (5a) for $i=k$, and keeping (4) in mind, we obtain for $k\ge 2$:

$$\pi(0,k) = \frac{1}{k\mu}\left\{\pi(0,k-1)((N+1-k)\lambda+\gamma_1+(k-1)\mu) - \pi(0,k-2)(N-k+2)\lambda - \pi(1,k-1)\gamma_0\right\}$$

$$= \frac{1}{k\mu}\left\{B_{k-1k}\pi(0,k-1) - \frac{B_{kk}}{k\mu}\pi(0,k-2)(N-k+2)\lambda - \frac{B_{kk}}{k\mu}\pi(1,k-1)\gamma_0\right\}$$

$$= \frac{1}{k\mu}\left\{\frac{B_{k-1k}}{(k-1)\mu}\Big[\pi(0,k-2)((N+2-k)\lambda+\gamma_1+(k-2)\mu) - \pi(0,k-3)(N-k+3)\lambda + \right.$$

$$\left. - \pi(1,k-2)\gamma_0\Big] - \frac{B_{kk}}{k\mu}\pi(0,k-2)(N-k+2)\lambda - \frac{B_{kk}}{k\mu}\pi(1,k-1)\gamma_0\right\}$$

$$= \frac{1}{k\mu}\left\{B_{k-2k}\pi(0,k-2) - \frac{B_{k-1k}}{(k-1)\mu}\pi(0,k-3)(N-k+3)\lambda + \right.$$

$$\left. + \frac{B_{k-1k}}{(k-1)\mu}\pi(1,k-2)\gamma_0 + \frac{B_{kk}}{k\mu}\pi(1,k-1)\gamma_0\right\} .$$

By iterating this relation and rearranging terms we obtain

$$\pi(0,k) = \frac{1}{k\mu}\left\{\frac{B_{1k}}{\mu}\left[\pi(0,0)(N\lambda+\gamma_1) - \pi(1,0)\gamma_0\right] - \frac{B_{2k}}{2\mu}\pi(0,0)N\lambda - \gamma_0\sum_{j=1}^{k-1}\pi(1,j)\frac{B_{j+1k}}{(j+1)\mu}\right\}$$

$$= \frac{1}{k\mu}\left\{B_{0k}\pi(0,0) - \gamma_0\sum_{j=0}^{k-1}\pi(1,j)\frac{B_{j+1k}}{(j+1)\mu}\right\}$$

The expression (1a) of $\pi(0,i)$ in (1) is hereby proven. We now proof the relation for $\pi(1,i)$ by induction to i. First, from (5b) and the definition of A_{jk} we obtain

$$\pi(1,1) = \frac{N\lambda}{(N-1)\lambda+\gamma_0}\pi(1,0) + \frac{\gamma_1}{(N-1)\lambda+\gamma_0}\pi(0,1)$$

$$= A_{01}\gamma_0\pi(1,0) + A_{10}\lambda\pi(0,1)$$

which proofs expression (1b) for $\pi(1,i)$ for $i=1$. Now suppose that this expression is valid for $i\leq m<N$. Then we need to verify the relation for $i=m+1$. To this end first note that

$$A_{jm+1} = (N-m)A_{m+1m+1}A_{jm}$$

It then follows from equation (5b) and the induction hypothesis that

$$\pi(1,m+1) = \frac{1}{(N-(m+1))\lambda+\gamma_0}\left\{\pi(1,m)(N-m)\lambda + \pi(0,m+1)\gamma_1\right\}$$

$$= \frac{1}{(N-(m+1))\lambda+\gamma_0}\left\{\left[\gamma_1\sum_{j=1}^{m}\lambda^{m-j}A_{jm}\pi(0,j) + A_{0m}\lambda^{m}\pi(1,0)\right](N-m)\lambda + \right.$$

$$\left. + \pi(0,m+1)\gamma_1\right\}$$

$$= \gamma_1\sum_{j=1}^{m}A_{jm+1}\lambda^{m+1-j}\pi(0,j) + A_{0m+1}\lambda^{m+1}\pi(1,0) + \gamma_1A_{m+1m+1}\pi(0,m+1)$$

$$= \gamma_1\sum_{j=1}^{m}A_{jm+1}\lambda^{m+1-j}\pi(0,j) + A_{0m+1}\lambda^{m+1}\pi(1,0)$$

This is the expression in (1a) for $i=m+1$ which completes the proof of (1) □.

Active breakdown model

From fig. 5 we directly conclude that the calculation scheme is exactly the same to that of the independent breakdown model except for $\pi(0,1)$ and $\pi(1,1)$. Precisely the calculation scheme here becomes

1) Set $\pi(0,0) = 1$
   $\pi(0,1) = \pi(0,0)N\lambda/\mu$ as by (6a)
   $\pi(1,1) = \pi(0,1)\gamma_1/(\gamma_0+(N-1))$ as by (6c)
2) Suppose $\pi(o,j)$ is known for $j\le k$ and $k\le N-1$ then
   first calculate $\pi(0,k+1)$ by (6b)
   next calculate $\pi(1,k+1)$ by (6c)

The explicit expression (2) can now be derived identical to (1) □.

Delayed breakdown model

From fig. 6 we now extract the following calculation scheme :

1) Set $\pi(0,0)=1$ and then calculate $\pi(1,k)$ for $k=1..N$ by (7d)
2) By downwards recursion express $\pi(0,k)$ in $\pi(1,j)$ $j=k..N$ for $k= N..1$
3) Compute $\pi(0,0)$ by using relation (7b)
4) Recursively compute $\pi(0,k)$ for $k=1..N$ by (7a)
5) Recursively compute $\pi(S,k)$ for $k=1..N$ by (7c)

Note that (7e) is hereby implicitly satisfied. Let us employ these steps to prove (3). First, step 1 and the definition of D_k immediately gives (3a). Next, step 2, from (7a) and the definition of C_n we obtain

$$\pi(0,N) = C_N\left\{\pi(0,N-1)\lambda + \pi(1,N)\gamma_0\right\}$$

$$\pi(0,N-1) = \frac{1}{(N-1)\mu + \gamma_1 + \lambda}\left\{\pi(0,N)N\mu + \pi(0,N-2)2\lambda + \pi(1,N-1)\gamma_0\right\}$$

$$= C_{N-1}\left\{\pi(0,N-2)2\lambda + \pi(1,N-1)\gamma_0 + \pi(1,N)NC_N\mu\gamma_0\right\} .$$

where the latter relation is obtained by substituting the first. Iterating in this manner, that is with repeated substitution of the former one we find

$$\pi(0,1) = C_1\pi(0,0)N\lambda + \gamma_0 \sum_{i=1}^{N} i!\mu^{i-1}\pi(1,i)\prod_{j=1}^{i} C_j$$

From(7b) and rearranging terms we obtain for $\pi(0,0)$

$$\pi(0,0) = C_0\gamma_0\pi(1,0) + \gamma_0 \sum_{i=1}^{N} i!\mu^{i}\pi(1,i)\prod_{j=0}^{i} C_j = \gamma_0 \sum_{i=0}^{N} i!\mu^{i}\pi(1,i)\prod_{j=0}^{i} C_j$$

Which proofs (3b). The proof of (3c) is similar to that of (1b) as based upon (7a) again and will therefore be omitted. We conclude with the proof of (3d)

From (7c) and $k\geq 2$ it follows that

$$\pi(s,k) = \frac{1}{k\mu}\left\{\pi(s,k-1)((N+1-k)\lambda+(k-1)\mu) - \pi(s,k-2)(N-k+2)\lambda - \pi(0,k-1)\gamma_1\right\}$$

$$= \frac{1}{k\mu}\left\{E_{k-1k}\pi(s,k-1) - \frac{E_{kk}}{k\mu}\pi(s,k-2)(N-k+2)\lambda - \frac{E_{kk}}{k\mu}\pi(0,k-1)\gamma_1\right\}$$

$$= \frac{1}{k\mu}\left\{\frac{E_{k-1k}}{(k-1)\mu}\left[\pi(s,k-2)((N+2-k)\lambda+(k-2)\mu) - \pi(s,k-3)(N-k+3)\lambda + \right.\right.$$
$$\left.\left. - \pi(0,k-2)\gamma_1\right] - \frac{E_{kk}}{k\mu}\pi(s,k-2)(N-k+2)\lambda - \frac{E_{kk}}{k\mu}\pi(0,k-1)\gamma_1\right\}$$

$$= \frac{1}{k\mu}\left\{E_{k-2k}\pi(s,k-2) - \frac{E_{k-1k}}{(k-1)\mu}\pi(s,k-3)(N-k+3)\lambda + \frac{E_{k-1k}}{(k-1)\mu}\pi(0,k-2)\gamma_1 + \right.$$
$$\left. + \frac{E_{kk}}{k\mu}\pi(0,k-1)\gamma_1\right\} .$$

By iterating the latter expression we obtain

$$\pi(s,k) = \frac{1}{k\mu}\left\{E_{1k}\pi(s,1) - \gamma_1\sum_{j=1}^{k-1}\pi(0,j)\frac{E_{j+1k}}{(j+1)\mu}\right\} \quad (\pi(s,0)=0)$$

$$= \frac{1}{k\mu}\left\{\frac{E_{1k}}{\mu}\left[\pi(1,0)(N\lambda+\gamma_0) - \pi(0,0)\gamma_1\right] - \gamma_1\sum_{j=1}^{k-1}\pi(0,j)\frac{E_{j+1k}}{(j+1)\mu}\right\}$$

$$= \frac{1}{k\mu}\left\{E_{0k}\pi(1,0) - \gamma_1 \sum_{j=0}^{k-1} \pi(0,j)\frac{E_{j+1k}}{(j+1)\mu}\right\}$$

by which (3d) is derived and thus the proof of (3) completed □.

6. Evaluation

Service or communication systems can be inoperative from time to time due to some type of "breakdown" or "off" period. Explicit recursive expressions for the steady state queue length distribution appear to be obtainable for different types of breakdowns under a specific two dimensional transition structure. These structures allow extensions to state dependent servicing or cappacity restrictions as well as apply to starvations in centralized CSMA communication structures.

7. References

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