# Aggregate dynamic economic-ecological models for sustainable development

van den Bergh, J.C.J.M.; Nijkamp, P.

1990

Faculteit der Economische Wetenschappen en Econometrie



# Serie Research Memoranda

## Aggregate Dynamic Economic-Ecological Models for Sustainable Development

J.C.J.M. van den Bergh
P. Nijkamp

Research Memorandum 1990-73
December 1990

*vrije* Universiteit *amsterdam*

# AGGREGATE DYNAMIC ECONOMIC-ECOLOGICAL MODELS FOR SUSTAINABLE DEVELOPMENT.

Jeroen C.J.M. van den Bergh
and
Peter Nijkamp

Faculty of Economics
Free University
De Boelelaan 1105
1081 HV Amsterdam

November 1990

* This study was supported by the Foundation for Advancement of Economic Research (Ecozoek), which resorts under the Dutch Organization for Scientific Research (N.W.O.), project no. 450-230-007.

**ABSTRACT.**

The aim of the present paper is to formulate a general integrated aggregate dynamic model for sustainable development, that will be both simple in structure and able to deal with the main objectives, processes and constraints applying to sustainable development in closed economic-ecological systems. General characteristics of models to be used for sustainable development are discussed. It turns out that such models are not existing.

Short critical descriptions are given of representative analytical models that have arisen in theories on economic growth with renewable and non-renewable resources, and pollution, or that provide an interesting alternative view of economic-environmental interactions. These models appear to provide incomplete descriptions of economic, ecological and interactive processes.

In order to clarify the logic and special features of our general model, we address three issues. First, a general aggregate ecological model is proposed that is consistent with a macroeconomic system in terms of geographical coverage. Such a general model would have to be able to deal with a specific set of general functions and characteristics of ecological systems. Second, the meaning of some economic, ecological and physical concepts for modelling is discussed. Finally, we look at the potential for inclusion of sustainable development conditions.

Next, the general structure of an economic-ecological model for sustainable development is presented. It includes the general ecological model, descriptions of economic activities and dynamics, material and economic balances, constraints, and behaviourial feedback mechanisms related to the sustainable development conditions. It is shown how concern for the well-being of future generations can be included. Simulation is used to obtain insight into the behaviour of the system under different policies and scenarios. Several environmental policies can be studied, including waste treatment, recycling, research and development, and environmental cleaning. Resource use strategies may include sustainable use, specific division of extraction between renewable and non-renewable resources. Resource use and waste emission strategies may be based on variable degrees of concern for future generations.

The relevance of this model is that it allows for a study of the medium and long term dynamic economic and environmental consequences of sustainable development conditions and variable degrees of concern for future generations. Its purpose is not to investigate the conditions under which the entire system will (n)ever breakdown.

## 1. INTRODUCTION.

In this paper a general model for sustainable development is constructed. Two questions immediately arise. First, why is this done? Second, what characterizes such a general model? The answer to the first question is that in order to study sustainable development one needs a general conceptual basis which includes a description of the object of study, namely an economic-ecological system and general characteristics of its dynamic behaviour. As the complexity of our object of study is high, one needs a simplified model which describes the main general characteristics of economic-ecological systems and can be used for analyzing dynamic patterns of such systems. A dynamic mathematical model seems the most logical choice in helping to perform these tasks. For reasons of too complicated dynamics and multi-dimensional or satisficing goals optimal control algorithms are then usually insufficient, so that scenario analysis in combination with simulation experiments may have to be used.

General implications for models arising from the notion of sustainable development are discussed in section 2. Concise descriptions of some representative analytical models in the area of economic growth with resources and pollution are discussed in section 3. Most of these models are incomplete or unbalanced in terms of their description of relevant economic-ecological interactions. In the fourth section three important modelling issues are dealt with. First, a general structure of an ecological model is proposed. In addition, the inclusion in dynamic models of specific economic, ecological and physical concepts is shortly discussed. Lastly, we examine the possible conditions for sustainable development. Section 5 is devoted to the formulation of a general model for sustainable development. In section 6 a set of useful scenarios that can be studied with the proposed model is presented and some simulation results for one of them is shortly discussed.

## 2. SUSTAINABLE DEVELOPMENT AND MODELS.

Which specific type of model is relevant for gaining insight into sustainable development issues, or for tracing sustainable development paths? This crucial question may be tackled by studying the following list of considerations:

1. A general - in contrast to a partial - approach; partial approaches are likely to become less relevant when considering long term horizons; the entire economic structure with both productive and non-productive uses of the environment should be included; a general description of the natural environment should therefore be included as well.
2. The impacts of economic productive and consumptive activities upon the natural environmental processes and conditions, in terms of materials extraction, waste emission and pollution, and non-material disturbances.
3. Presence of a feedback from the ecology to the economy; inclusion of feedbacks of ecological impacts of general economic activity to the economic system is essential

for an adequate description of long term processes in economic systems; feedback to decision-making with respect to productive activities also occurs via perception of resource scarcity and pollution levels (or environmental quality)[1].

4. Not only material or priced services should be included, but also, as much as possible other services, such as for instance productive conditions (e.g., soil quality) and amenity services; the latter may be included in the evaluation or welfare function, but equally by way of behaviourial feedback mechanisms; furthermore, multi-functionality of ecosystems, and resource systems (as opposed to single one-dimensional resources) can be dealt with in a systems description.
5. Inclusion of concern for future generations; this requires a judgement criterion to be chosen for the evaluation of intergenerational distributions; furthermore, various conditions may be imposed on natural capital, pollution or economic capital to assure an equitable intergenerational distribution; if the repercussions of concern for future generations are taken in a behaviourial rather than an evaluative or constraining sense it implies that behaviourial or policy feedbacks aiming at intergenerational equity are endogenous; a long time horizon implies that short term processes are as much as possible excluded; furthermore, the long term horizon implies that linear models will not be adequate for a description of relevant processes, and then scenario analyses may be used for dealing with the various types of long term uncertainty.
6. It must be possible to describe qualitative (structural) change, either implicitly or explicitly; this means that one has to allow for a description of irreversible processes, thresholds, nonlinear structures and time-delays.
7. The model assumptions should not conflict with physical constraints; sufficient limitations on substitution in production and utility functions should be built in; furthermore, dependencies between substitution of production factors, investments and technological progress have to be considered simultaneously; in addition, model assumptions should not conflict with thermodynamic laws (e.g., 100% recycling), while the material balance conditions can be included explicitly.

## 3. SIMPLE MODELS OF DEVELOPMENT.

### 3.1. ECONOMIC GROWTH THEORY.

The theory of economic growth studies the trend of the time paths for macroeconomic variables in developed economies (see Wan, 1971). Economic growth models are

[1] The second and third point together may be summarized as 'economic-ecological integration'. This terminology has been employed to denote economic process models which include environmental variables (e.g., waste emission or resource extraction) and ecological process models with economic variables (e.g., stress factors). The considerations above indicate that a stronger concept is necessary to deal with sustainable development, namely one that integrates economic and environmental-ecological processes rather than a process and a variable (see also Braat and van Lierop, 1987).

especially concerned with processes of growth in capital endowments, resulting from investment, and substitution of inputs in production. These models take a long term perspective, and can be used to study intertemporal (including intergenerational) distributions of welfare, consumption, and endowments (e.g., capital, resources, pollution). In a technical sense, growth theory is concentrating on the study of the existence, uniqueness and stability of dynamic equilibria, asymptotic and catenary properties of dynamic paths (for infinite and finite time horizons, respectively), and comparative dynamics related to changes in parameter values, initial conditions, or functional relationships.

One (relatively small) research area in economic growth theory deals with the inclusion of variables referring to environmental components, such as non-renewable and renewable resources, or pollution (see for a systematic review Kamien and Schwartz (1982)). A full elaboration and analysis of all combinations of the extensions mentioned above has not been pursued and seems to be hindered by limitations on the mathematical tractability of systems of non-linear dynamic equations[2].

## 3.2. BASIC MODELS OF ECONOMIC GROWTH: ACCUMULATION OF CAPITAL.

The basic neoclassical optimal growth model 'without ecology' can be formulated as follows (see e.g. Burmeister and Dobell 1970, Chapter 11):

Choose the optimal investment schedule (or capital accumulation path) in order to

$$\max \int_0^\infty U[C(t)]\exp(-rt)dt \qquad (1)$$

$$\text{s.t.:} \quad dK(t)/dt = I(t) - \delta_K K(t), \quad K(0) = K_0 \qquad (2)$$

$$C(t) + I(t) = F(K(t),L(t)), \qquad (3)$$

$$dL(t)/dt = gL, \ L(0) = L_0, \ g>0, \qquad (4)$$

$$C,K > 0, \qquad (5)$$

with $U[\cdot]$ a generation's welfare function, that is monotonously increasing and strictly concave; r is the discount rate, C consumption, K capital, L labour and g the growth rate of labour. The assumption of additive separability underlying the specification of the social welfare function in (1) is based on the idea that welfare levels of generations separated by time are independent. Such an independence does not exist between individuals in one generation, so that overlap of generations casts some doubts on this assumption.

Some interesting observations are:

1. A discounted bequest value $\exp(-rT)*B[K(T)]$ can be added to the welfare criterion (1) to replace the upper bound of the integral by a finite time horizon T.

---

[2] We concentrate here on models of growth and development that have a tradition in economics. Alternative approaches to integration of growth, economics and environment can be found in the energy language models of Odum (1987), who pursues a simulation approach. Still other approaches use complicated systems descriptive models such as some of the so-called world models (see for an overview Meadows et al., 1982).

2. An alternative interesting intertemporal welfare criterion - though not a social welfare function - is the maximin criterion. It can be based on the ethical ideas of Rawls (1972), although the depth of his moral theory cannot be included in a simply formulated criterion. The objective in such a case is formulated as (see Arrow, 1973; and Solow, 1974): maximize over $t \in [0,T]$, subject to (2) to (5), minimum$\{ U[C(t)] \}$. Fulfilment of this criterion will give rise to greater intergenerational equity - in terms of welfare - than maximization of (1). Continuous growth in welfare will then not be stimulated, as it will demand sacrifices of earlier generations in terms of welfare.
3. In analyses based on the above model a neoclassical production function F(K,L) is usually assumed, having the following properties: non-negative and smooth; $F(0,0)=0$; non-negative first derivatives; homogeneity of degree one; and, strict quasi-concavity. It seems realistic to impose $F(0,L)=F(K,0)=0$.
4. Capital depreciates exponentially at a rate $\delta_K$, while investment in capital is irreversible, i.e., $I \geq 0$. Labour force growth occurs only when g is strictly positive.
5. Several characteristics of technological progress as included in such aggregate models are: (a) it is either embodied (in newly produced capital equipment and/or in new generations of workers), or disembodied (and affecting all generations of machines or workers in the same way); (b) it is output-augmenting, capital widening or capital deepening (the latter two coinciding with labour saving and labour augmenting, respectively); (c) it follows a growth path which is progressive (e.g., exponential), linear, regressive, or otherwise (e.g., logistic growth; see Ayres, 1978); and (d) it results from an endogenous process of investment in R&D (see Kamien and Schwartz 1978).

Many directions for expanding and improving the realism of models of this type are available, which are limited only by mathematical requirements for analysis. In the next three sections we will discuss growth models that include a linkage with the environment. For the input side of an economic system resources should therefore be included, while at the output side waste materials and pollution are to be considered. However, if this is done, one does not yet arrive at integrated economic-ecological models.

### 3.3. PRODUCED CAPITAL AND NON-RENEWABLE RESOURCES.

Non-renewable resources do not possess the capacity to regenerate themselves. Examples are ores, fossil fuels, land, and scenic beauty. Non-renewable resources have both been dealt with in a macroeconomic growth context and in a microeconomic context. The type of analyses belonging to the latter include amongst others studies on extraction to optimize profits, monopolistic or competitive extraction, and exploration (see for overviews Peterson and Fisher, 1977; and Dasgupta and Heal, 1979).

The simple basic growth model (i.e., inclusion of man-made capital) with non-renewable resources can be formulated as follows: Choose a resource extraction path and an investment path that maximizes (1) subject to (2),(4) and

$$C(t) + I(t) = F(L(t),R(t),t), \quad (6)$$
$$dS(t)/dt = -R(t), \; S(t) \geq 0, \; t>0, \; S(0)>0. \quad (7)$$
$$C,K,I \geq 0, \quad (8)$$

where R(t) denotes the extracted amount of the non-renewable resource at time t, and $S_0$ its initial stock.

Some interesting observations are:

1. In order to analyze optimal non-renewable resource use, the utility function U[C(t),S(t)] expresses preservation motives originating from ethical considerations (intrinsic value of nature), ecological considerations (diversity, irreversible processes), or the presence of environmental amenities (e.g., for recreation).
2. In the above model, consumption and capital goods are manufactured with only one resource. Each additional unit of a consumption or capital good implies an extra demand for material from this resource, so that it may be assumed that F(K,L,0,t)=0.
3. In addition to the remarks about the aggregate production function in the last section, the objections raised by Georgescu-Roegen (1971) are relevant. The above production function mixes the flow element R with the fund elements K and L. Substitution between flows, or between fund elements is in general feasible. However, substitution between these two classes of production inputs is not so evident.
4. Three types of processes may enhance economic efficiency: substitution of capital or labour for resources, resource augmenting technological progress and recycling of waste materials.

Although the non-renewable resource models consider the input of resource in the economic system, they cannot be used to trace all effects of resource extraction, transformation and derived waste outflow from the economic system. Therefore using these models for analysis gives only partial insights. Moreover, they provide no framework for studying the relationship between for instance the regenerative natural resource base and the use of fossil fuels. It is however important to be able to investigate which limit is reached sooner: the exhaustion of energy or mineral resources, or the critical level of pollution - e.g., one that leads to drastic changes on the biosphere level - that is generated by production and consumption using these resources.

### 3.4. PRODUCED CAPITAL AND RENEWABLE RESOURCES.

Renewable resources in their function as generating materials and amenities have not often been included in models of economic growth. One reason is that they were considered - as opposed to non-renewable resources - not to be limiting factors for economic growth. Another reason may be that various difficulties are met in modelling the regenerative character of the environment at a high level of aggregation. One such difficulty is the non-homogeneous character of regenerative processes. Examples of such very different processes are self-regeneration of ecosystems, reproduction and growth of populations, biogeochemical cycles, and water, wind and tidal flow generation.

Attention for renewable resources has usually been directed to partial microeconomic analyses of specific types of sector-resource interactions (see e.g., Clark, 1976). Integrated macroeconomic analyses with renewable resources have emphasized the assimilative function of regenerative natural systems. This aspect has been included in aggregate models via decay functions of pollution and will be looked at more closely in the next section. Issues that are relevant in a macro- as opposed to a micro-context are the allocation of resources to different uses, and the allocation of production factors (capital, labour, land, resources) to processes of resource extraction, resource replenishment, production, etc.. Finally, with aggregate models intergenerational aspects can be studied.

A drawback of simple models for description of resource regeneration processes is that characteristics such as complexity, diversity, stability, succession of ecosystems, different concepts of irreversibility, and thresholds for pollution, crowding and other stress factors cannot satisfactorily be dealt with.

## 3.5. PRODUCED CAPITAL AND POLLUTION.

A simple picture of reality shows that pollution flows from the production and consumption processes in the economic system to a stock in the natural environment. Pollution as a stock has a negative influence on the regenerative and assimilative capacity of the environment, on the efficiency and productivity of economic activity, and on human welfare via the public goods character of the environment.

Here we will present a general model, similar to the one in Keeler et al. (1972), that takes both the flow and stock of pollution into account.

$$\max \quad \int_0^\infty U[C(t),P(t),dP(t)/dt]\exp(-rt)dt \tag{9}$$

$$\text{s.t.:} \quad Q(t) = F(K_1(t),P(t),M), \tag{10}$$
$$dK(t)/dt = (1-\beta)Q(t) - C(t) - \delta_k K(t), \tag{11}$$
$$dP(t)/dt = w(K^1) + M - h(K_2(t)) - d\beta Q(t) - \delta_p P(t), \tag{12}$$
$$K(t) = K_1(t) + K_2(t), \tag{13}$$
$$C(t),K_i(t)(i=1,2),P(t)\geq 0, \tag{14}$$
$$K_i(0)=K_{i0}(i=1,2),\ P(0)=P_0. \tag{15}$$

Pollution - both as a stock and a flow - is assumed to influence social welfare. A production process generates output Q, with capital input $K_1$, and material input M, and may be negatively influenced by the stock of pollution P. A second process concerns pollution reduction, which will occur with a capital input $K_2$ at a rate $h(K_2)$; output has to be allocated to capital accumulation, consumption or pollution control. The stock of pollution increases as a result of waste generation by the production process, composed of the by-product $w(K_1)$ and the material outflow M. The flow M occurs essentially only after consumption. The stock of pollution has a natural decay rate $\delta_p$, which may be positive or zero, depending on the type of pollution. The rate $\delta_p$ may be dependent on P, since pollution may affect the self-cleansing capacity of the environment. The stock of pollution is also reduced by pollution control activities, or directly reduced at a rate of d units per unit of expenditure, which is a fixed part (ß) of total expenditures (Q). The materials that are taken from the stock of pollution as a result of pollution reduction activities and capital depreciation are not accounted for. Further restrictions on w are necessary to ensure this; in any case the condition $w(K_1) \geq \delta_k * K_1$ should hold.

In contrast to the models in the foregoing sections, here depreciated capital may enlarge the amount of waste in the environment and thus indirectly has a negative impact on welfare. In the present case recycling will not only increase the effective resource base, but also diminish the stock of pollution. The effects of pollution may be varying: it may harm the efficiency of the production process, its inputs or almost finished goods (see Siebert, 1987). It may also negatively affect the regenerative capacity of a replenishable resource.

### 3.6. MODELS WITHOUT PRODUCED CAPITAL.

In this section we present a model by Siebert (1982) that does not include man-made capital as a stock variable. The economic process described is consumption of extracted amounts of a renewable resource (environmental capital), when (1) consuming and/or extraction cause accumulation of a stock of pollution and (2) the stock of pollution has a negative impact on the regenerative capacity of the renewable resource. His model is specified as follows:

$$\max \quad \int_0^T U(C)\exp(-rt)dt \tag{16}$$

$$\text{s.t.:} \quad dN/dt = g(N) - a*P - C \tag{17}$$

$$dP/dt = b*C - \delta_p*P \tag{18}$$

$$C,N,P \geq 0 \tag{19}$$

$$N(0)=N_0,\ P(0)=P_0 \tag{20}$$

We notice, that this model does not include a stock of man-made capital and productive activity. It is interesting to regard it as the opposite of the basic economic growth model, as here only naturally produced goods or services are consumed, instead of only those produced with man-made capital as in the models of sections 3.2 to 3.5.

Barbier (1989) circumvents the problem of too many stock variables by introducing an environmental degradation variable. His problem is defined as optimizing a social welfare function as in (16), except that the utility function has the flow of consumption and the level of environmental degradation as arguments, subject to a dynamic equation for degradation (D). The latter increases (decreases) as a result of waste emission W in excess (shortage) of assimilative capacity A, renewable resource extraction $R_N$ in excess (shortage) of natural regeneration G, and extraction of non-renewable resources $R_S$ (non-negative, so no increase of D only if its level is zero).

$$dD/dt = (W-A) + (R_N-G) + R_S. \qquad (21)$$

The flows of waste emission and both types of resource extraction are functions of the economic process (Barbier makes them dependent on the flow of consumption).

## 4. ELEMENTS OF A GENERAL MODEL FOR SUSTAINABLE DEVELOPMENT.

### 4.1. TOWARDS A GENERAL AGGREGATE ECOLOGICAL MODEL.

The use of models in ecology is directed at either a disaggregate numerical analysis of particular ecosystems (i.e., dynamic simulation with detailed descriptive systems models) or a more rigid analysis of single renewable resource models (for instance, populations, water resources, and multiple interacting resources). The latter type of models is simple in structure, the most familiar and basic example being the logistic growth model (see Watt, 1968; and Walters, 1986). While the use of such models follows from a choice for a highly aggregate level of study, they usually refer to a single, rather homogeneous system. From an economic point of view however, these approaches take a micro or meso level of aggregation, comparable with individual firms or consumers, and sectors in economic systems, respectively. This becomes also clear from the great many simple integrated economic-ecological models that have been studied, most of them dealing with interactions between a single firm or sector and a single resource (Clark, 1976 and 1985; Holling, 1978; and Walters 1986).

A general aggregate ecological model should be consistent with a macroeconomic system in terms of geographical coverage. Therefore, it should have a global character, i.e. describing the essential features of a collection of various (possibly interacting) homogeneous ecological systems. Such a general model would have to be able to deal with the following functions and characteristics of ecological systems: (1) regenerative capacity, (2) assimilation of pollution, (3) resource supply, (4) storage of waste materials, (5) non-material services for consumption, (6) decreasing performance of all functions for higher levels of pollution, resource extraction, and other disturbances, and (7) possibility of irreversible development as a result of too much pollution or a high extraction rate. Ecological (internal) functions cannot be represented in a global model having to do with regulation, reserves-keeping, transport of water, minerals and seeds, etc. Furthermore, many social, scientific, and cultural functions are neither represented by a global ecological model (at least, not explicitly).

We propose the following simple dynamic aggregate structure as a suitable representation of a large scale (global, national or regional) ecological system[3]:

$$E = H(N,P,Pop,O_c), \tag{22}$$
$$dN/dt = G(N,E) - R_N, \tag{23}$$
$$dP/dt = -M(P,E) + W_{em}, \tag{24}$$

Two natural processes are described in equations (23) and (24) to be able to deal with above mentioned characteristics (1) and (2). In addition, the stock levels are changed by the above mentioned functions (3) and (4). The fifth function can be linked via a welfare function (if it is included in a wider model) to the environmental quality, so that it indirectly related to ecological resource and pollution characteristics as well as human impacts such as population pressure/density and direct environmental cleaning. The last two characteristics mentioned above are included by the chain of effects via the two stock values, the environmental quality indicator E and the natural processes G( ) and M( ), and by taking certain conditions on the specific functional characteristics and parameter values into account. The general assumptions applying to these and initial stock values are:

$N(0)=N_0,\ P(0)=P_0,$
$0\leq H(N,P,Pop,O_c)\leq 1,$
$\delta H/\delta N\geq<0,\ \delta H/\delta P<0,\ \delta H/\delta Pop<0,\ \delta H/\delta O_c\geq 0,$
$G(N_{min},E)=G(N_{max},E)=0,$
for some $(N,E)$, with $0<N_{min}<N^*<N_{max},\ E>0$: $G(N^*,E)>0,$
$\delta G(N,E)/\delta E\geq 0,$
$E,N,P,Pop,O_c,R_N,W_{em},N_0,P_0,N_{min},H,G,M > 0.$

### 4.2. ECONOMIC, ECOLOGICAL AND PHYSICAL CONCEPTS.

In this subsection several concepts are checked upon their relevance for and feasibility of inclusion in a general aggregate economic-ecological model for sustainable development such as the one which will be presented in section 5. Economic concepts suitable for inclusion at an aggregate level of description are: production, consumption, substitution, material flows and technical progress. More difficult to include are externalities, prices/monetary variables, and growth-development distinctions. Ecological concepts which can be included are regenerative, assimilative, and carrying capacity. Very hard include are resilience, diversity, biotic-abiotic distinctions, and succession. An important physical concept that can be included is the materials balance principle. This

---

[3] E is an indicator for environmental quality that depends on the regenerative resource capacity N, the stock of pollution P, the congestion indicated by the population level Pop, and the outlays for environmental cleaning activities $O_c$. The regeneration of the resource is given by G( ), which depends on the present stock of resources and the environmental quality. $N_{min}$ is the minimum viable level of the regenerative resource. $N_{max}$ denotes its maximum level (also referred to as carrying capacity). The assimilation function M( ) depends on the present level of accumulated pollution, and the environmental quality. This ecological model includes 'outlays for environmental cleaning'($O_c$), an indicator for the level of (various types of) disturbances (Pop), resource extraction ($R_N$), and waste emissions ($W_{em}$) as exogenous variables.

has mainly been applied to provide for a correlation between the quantities of material inputs to and outputs of economic production systems in a static general equilibrium (input-output) setting (see Kneese et al., 1970). d'Arge and Kogiku (1973) and Mäler (1974) have applied material balance conditions in growth models to study the relationship between growth and pollution. Gross and Veendorp (1990) analyze growth with a non-renewable resource under production constrained to materials balance conditions. However, the relationship in terms of materials balance conditions in the cycle of extraction, production, waste emission, assimilation and natural regeneration has not been studied yet. How to explicitly include the entropy law is not so evident, however (see also Georgescu-Roegen, 1971).

## 4.3. SUSTAINABLE DEVELOPMENT CONDITIONS.

We distinguish here between constraints on the level of welfare (for a whole generation or per capita) over generations and restrictions on physical-ecological stocks and flows. In the first case one may choose between the following types of conditions[4]: (1) requiring welfare always to exceed some minimum level (e.g., a subsistence level), or (2) requiring a monotonous non-decreasing movement of welfare over time.

The second type of conditions may involve constraints on stocks or on flows in and between economic and environmental systems. For instance, the notion of stock constancy (or non-decreasing stocks) may be applied to the sum of man-made economic and natural stocks, to a stock concept such as environmental quality or environmental degradation, to the sum of all natural stocks, or to each stock separately. Instead of directly applying such stock conditions to a model one may use derived flow conditions.

To illustrate the second type of sustainable development conditions by considering equation (29) once more. The condition may be that dS/dt is non-positive, i.e. no (further) degradation. This may be assured by requiring that the total economic impact $W+R_N+R_S$ does not exceed the natural capacity A+G to accept it, which can be interpreted as extensive compensation for negative impacts (see Klaassen and Botterweg, 1976). As a special case the following two constraints is relevant (see Barbier, 1989): R+E=G and W=A, requiring the effect of pollution to be separated from that of resource extraction, while renewable resources may substitute (compensate) for a decline in non-renewable resource stocks. In the model described in the next section the latter type of conditions are used in combination with feedback mechanisms that become operational when they are (bound to be) violated.

---

[4] Pezzey (1989, p.13) gives a systematic account of possible simple formulations in this respect. He further links it to the distinction between combinations of on the one hand growth, development and resource use, and on the other hand survivability and sustainability.

## 5. A GENERAL AGGREGATE MODEL FOR SUSTAINABLE DEVELOPMENT.

### 5.1. INTRODUCTION.

In this section a general economic-ecological model for analyzing sustainable development is presented. This framework should be general in nature, so as to encompass most of the particular specifications of models presented in the literature. A fully dynamic model for the economic-ecological system will be presented, consisting of economic and natural production functions, accumulation, decumulation, and regeneration functions. The models describes the material flows in both the economic and ecological systems and those between them. The ecological model used is already described in section 4.1 (except for the non-renewable resources). The economic model will be described in the next sub-section. It is supply-oriented, reflecting a long-term horizon, so that markets are assumed to be in equilibrium. The complete model further includes economic balance equations, materials balance equations to link the cycle of extraction, production, waste emission, assimilation and natural regeneration (see section 4.2), and inequality constraints to prevent unrealistic patterns. In addition, some behaviourial feedback mechanisms are included for describing the reactions in production activity to sustainable development conditions with regard to resource scarcity and accumulation of waste (see section 4.3). These mechanisms are related to subjective perceptions based on ethical concerns for future generations. Four types of environmentally beneficial activities are included, namely waste treatment (abatement), recycling, research and development to develop resource efficient production techniques, and environmental cleaning. These activities are competitive for economic (financial) means, and can be stimulated by government policies. Strategies with respect to resource use allow for a choice between non-renewable and renewable resource use, and specific use patterns over time for each separately (including sustainable use). Waste emission strategies are also possible, for instance sustainable waste emission. The model dynamics arise from logical/causal structures. For an explanation of symbols the reader is referred to the appendix.

### 5.2. ECONOMIC ACTIVITIES.

The description of the economic activities is based on the formulation of an aggregate production function that is a standard appearance in growth models. However, the general character of the production function is different from the neoclassical or Harrod-Domar (fixed production coefficients) in four respects. First, it satisfies materials balance conditions so that waste and output can be regarded as related to the input by way of joint production. Second, the level of environmental quality may impact upon the production efficiency (for instance, in agriculture). Third, the production activity is influenced by feedback mechanisms that become operational when sustainable development conditions with respect to resource use and waste emission are violated. These mechanisms include subjective perception of scarcity of resources that depends on the ethical concern for future generations (see section 5.7 for a complete explanation). Fourth, an absolute limit to the level of production may become active.

Output is produced by means of capital K, labour L and resource inputs $R_Q$. A higher quality of the environment E and the cumulative total of R&D outlays $T_{rd}$ are assumed to affect the productive efficiency positively; scarcity of resource inputs is reflected via an indicator for perceived shortage in resource supply $R_{short}$ and a waste policy indicator $W_{sust}$, and the perceived total availability of resources $R_{sup}$:

$$Q = F(K,L,R_Q,E,T_{rd},R_{short},W_{sust},R_{sup}). \qquad (25)$$

The level of output is determined by either production capacity or resource availability restrictions. Furthermore, the ratio $Q/R_Q$ is changed through technological progress (indicated by $T_{rd}$) that changes the resource use per unit of output (indicated by c( )). Thus we may specify equation (25) as:

$$Q = MIN\{ F(K,E,R_{short},W_{sust}),R_{sup}/c(T_{rd}) \}, \qquad (25a)$$
$$U = L - J,\ L=a(Pop),\ J=b(K),\ \text{and} \qquad (25b)$$
$$R_Q = c(Trd)*Q, \qquad (25c)$$

where U denotes unemployment, and a( ) relates the labour supply L to the population level Pop, while b( ) relates the employment J to utilized capital K. Output Q cannot exceed input $R_Q$, which means that c( )$\geq$1 (>1, assumed that always some loss will result). Technological change is assumed to generate more efficient production processes in terms of a lower ratio of $R_Q$ to Q, so that c'( )<0. $R_{short}$ and $W_{sust}$ represent part of the behaviourial characteristics of a feedback from perceived resource scarcity and waste emission levels to economic activity. $R_{sup}$/c( ) is the upper limit to the output level for a given state of technology.

Two other activities are described, which are not productive in the sense of generating output, but relate to environmental beneficial activities (policies). One process treats production waste in such a way that part of it does not end up in the environment and consequently causes no negative environmental impacts. The amount of waste treated $R_{wa}$ is determined by the level of waste arising directly from production $W_Q$, and the outlays for abatement activity $O_a$:

$$R_{wa} = f_a(W_Q,O_a). \qquad (26)$$

A second process recycles materials, its amount $R_{rec}$ being determined by the waste flow suitable for recycling $W_{rec}$ and the outlays for recycling activity $O_{rec}$:

$$R_{rec} = f_r(W_{rec},O_{rec}). \qquad (27)$$

### 5.3. ECONOMIC DYNAMICS.

We regard as economic dynamics here the dynamic formulations of changes in production capital, population, technology or knowledge, and treated or abated waste. The change in the capital stock depends on the investment level and the rate of depreciation, which in turn is related to the present level of capital. The stock of capital depreciates at a rate given by D(K) which is strictly monotonically increasing:

$$dK/dt = I - D(K). \tag{28}$$

The population change is determined by the present number of people Pop and the level of material consumption per capita (an partial indicator for welfare):

$$dPop/dt = B(C/Pop)*Pop. \tag{29}$$

The (non-material) cumulative amount of R&D outlays $T_{rd}$, serving as an indicator for technological progress, increases by the rate of such outlays:

$$dT_{rd}/dt = O_{rd}. \tag{30}$$

Furthermore, a stock of abated waste $S_{wa}$ is filled as a result of abatement activities:

$$dS_{wa}/dt = R_{wa}. \tag{31}$$

The initial conditions are

$$K(0)=K_0,\ Pop(0)=Pop_0,\ T_{rd}(0)=0,\ S_{wa}(0)=0. \tag{32}$$

## 5.4. ECONOMIC BALANCE EQUATIONS.

The economic system is closed, so that the total production equals the sum of produced consumption goods $C_Q$, investments I, and total (material) outlays O. The total environmental outlays are subdivided into outlays for abatement, recycling, cleaning, and R&D, so that these four activities are included consistently from an economic point of view, and compete or can be stimulated by policies. The aggregate character links here to the assumption that the production output is homogeneous and can be used for either consumption, capital accumulation or environmental policies/activities:

$$C_Q + I + O = Q, \text{ and} \tag{33}$$
$$O_a + O_{rec} + O_c + O_{rd} = O. \tag{34}$$

Material consumption C consists of produced consumption $C_Q$ and resource consumption $C_R$.

$$C = C_Q + C_R. \tag{35}$$

Resource consumption is that part of consumption which is directly obtained from the natural environment without going through one or more stages of production. It is either a free variable or linked to the level of produced consumption per capital (a partial indicator for welfare), in the following way:

$$C_R = d(C_Q/Pop)*Pop, \tag{36}$$

with $d'(\ ) \leq 0$.

## 5.5. ECOLOGICAL DYNAMICS.

The regenerating and assimilating ecological system is described by equations (22) - (24), and the conditions together with the conditions on functions, parameters and stock values mentioned in section 4.1. The non-renewable resource dynamics is represented by

$$dS/dt = - R_S,\ S(0)=S_0, \tag{37}$$

where S denotes the present stock of non-renewable resources, and $R_S$ denotes the extraction rate.

## 5.6. MATERIAL BALANCE EQUATIONS.

The production activity gives rise to a material balance equation, as the amount of resource input $R_Q$ equals the material goods output Q and the waste output $W_Q$ from production, or:

$$W_Q = R_Q - Q. \tag{38}$$

The total resource input of the production process $R_Q$ equals the sum of newly extracted amounts of renewable and non-renewable resources $R_{Qn}$ and the amount of recycled materials $R_{rec}$, which is only used for production (i.e., it cannot be used for consumption directly). Therefore, the demand for newly extracted resources is equal to

$$R_{Qn} = R_Q - R_{rec}. \tag{39}$$

The total demand for extracted resource now equals

$$R_{dem} = C_R + R_{Qn}. \tag{40}$$

The waste amenable for recycling equals the sum of depreciated capital D(K), consumptive waste C, and other (material) outlays O. It is assumed here that waste arising from production cannot be recycled (or is implicitly included, i.e., recycling which increases efficiency in terms of a lower ratio of waste output to resource input of production):

$$W_{rec} = D(K) + C + O. \tag{41}$$

The total waste, before abatement and recycling, equals the sum of production waste $W_Q$ and recyclable waste $W_{rec}$:

$$W = W_Q + D(K) + C + O. \tag{42}$$

The waste emission level $W_{em}$ equals the total waste before abatement and recycling W minus the amounts of abated and recycled waste material, $R_{wa}$ and $R_{rec}$ respectively:

$$W_{em} = W - R_{wa} - R_{rec}. \tag{43}$$

## 5.7. FEEDBACK FROM ECOLOGY TO ECONOMY: SUSTAINABLE DEVELOPMENT CONDITIONS AND BEHAVIOUR.

The model allows for feedback from resource scarcity and pollution to the decision with regard to the activity level of production. This goes as follows. At the core are sustainable development (flow) conditions applying to the flows of resources and waste, of the last type mentioned in section 4.3. These are transformed, via behaviourial parameters, into variables that indicate the perceived levels. This involves an ethical choice with regard to the degree of concern for the well-being of future generations.

For the resources, it implies that the perceived amount of resources that is allowed to serve as an input to the economic processes of production and consumption is given by:

$$R_{sup} = f_{sup}(N,S,G). \qquad (44)$$

The function $f_{sup}(\ )$ may be specified as the sum of N and S (no concern for future generations), or the sum of S and G (sustainable use), or the sum of p*S ($0 \leq p < 1$) and G (much concern for future generations). The variable $R_{short}$ indicates whether the consumptive and productive demand for newly extracted resources can be met by the above mentioned perception of supply:

$$R_{short} = f_{short}(r,R_{sup},R_{dem}). \qquad (45)$$

This function may be specified for instance as min $\{1,r^*R_{sup}/R_{dem}\}$, or MAX$\{0,r^*(R_{sup}-R_{dem})\}$, where the first is preferred because it generates a dimensionless number. The parameter r is used to include some measure of cautiousness, because of uncertainty or again intergenerational considerations. A value of r equal to 1 may then be regarded as being neutral (risk-neutral and neutral towards the well-being of future generations), much smaller than 1 means that there is either much care for future generations or much uncertainty, and the opposite holds for values of r much larger than 1. $R_{short}$ returns in the production function to impact negatively on production when its value indicates a perceived shortage and neutrally otherwise.

For a feedback from pollution to economic behaviour the following approach is taken. The goal is again given in by concern for future generations and may vary from net decay of the stock of pollution to zero growth (sustainable waste emission) or even a certain positive rate of waste accumulation. In the case of sustainable waste emission the following simple equation controls the feedback of pollution stock accumulation to production:

$$W_{sust} = \text{MIN}\{1,m^*M/W_{em}\}, \qquad (46)$$

where M denotes the variable assimilative capacity, and m a safety margin which in the case of sustainable waste emission is equal to 1, and in the case of an allowed positive rate of accumulation in the stock of pollution is higher than 1. Similar remarks hold for m as for r above. The smaller m, the more careful the economic behaviour towards negative ecological consequences and thus the more concern for future generations is included.

## 5.8. INEQUALITY CONDITIONS.

In order to prevent the model from generating on forehand evidently unrealistic patterns several conditions are imposed. They include non-negative conditions on all stock and flow variables, and all functions in the model. Also the extraction rate should not be higher than the level of the stock from which extraction is taking place. The total extraction of resources should not be higher than the demand for them, while - under a given allocation schedule of extraction - total extraction should be as high as possible (i.e., equal to or minimizing the difference with the demand), so that three conditions result:

$$R_S \leq S \text{ and } R_N \leq N, \text{ and} \quad (47)$$
$$R_S + R_N \leq R_{dem}. \quad (48)$$

Furthermore, abated waste and recycling should not overrate production waste and waste amenable for recycling, respectively:

$$R_{wa} \leq W_Q \text{ and} \quad (49)$$
$$R_{rec} \leq W_{rec}. \quad (50)$$

### 5.9. A SIMPLE ANALYSIS.

The model presented in sections 5.2 to 5.8 includes 7 variables where values are not endogenously determined. These can be considered as control variables of the system: 2 out of the set $\{C_Q, I, O\}$; 3 variables out of the set $\{O_a, O_{rec}, O_c, O_{rd}\}$ (equations (33) and (34) will determine the remaining variable in each group, respectively); and $R_N$ and $R_S$ (inequalities (47) and (48) only restrict their values). Furthermore, if equation (36) is deleted, then $C_R$ is also a free variable. Three distribution mechanisms - for production output Q, government outlays O, and resource demand R - have to be decided for in order to solve the model. Such distribution schemes may be constructed in several ways, with allocations as a fixed or changing part of the related sum-variable (Q,O or R), or being determined on the basis of behaviourial relationships (which will mean a more complex model), or resulting from optimizing the intergenerational welfare function.

From the equations (28) to (43) and the ecological dynamic equations from section 4.1 we can derive that

$$dK/dt + dS_{wa}/dt = R_S + R_N - W_{em}, \quad (51)$$

which can be interpreted as $[\text{stock changes}]_{\text{economic system}} =$ $[\text{input - output flows}]_{\text{economic system}}$. A material balance equation to express the closeness of the total system and the complete description with the presently available stock variables is stated as follows:

$$dK/dt + dS_{wa}/dt + dN/dt + dP/dt + dS/dt = 0 \quad (52)$$

By substituting for $dK/dt + dS_{wa}/dt$, using equation (51) we obtain

$$dN/dt + dP/dt = W_{em} - R_N, \quad (53)$$

which can be interpreted as $[\text{stock changes}]_{\text{ecological system}} =$ $[\text{input - output flows}]_{\text{ecological system}}$. Finally, using equations (23), (24) and (53) the following equality is arrived at:

$$G(N,E) = M(P,E). \quad (54)$$

This condition states that a given growth rate of renewable resources must be compensated for by an equal rate of decrease in the stock of pollution as a result of assimilation. Responsible for this condition are the material balance conditions and the assumption that no other natural material exists than that in renewable resources, pollution, and - interacting with these only through the economic system - in non-renewable resources. A fourth ecological stock variable should be introduced to allow for another relationship between G and M than the one stated in (54), when the system is closed. This additional ecological stock variable should represent material that is taken up when resource regeneration takes place.

If condition (54) is supplemented with the conditions following from sustainable use of resources and sustainable waste emission, in which cases the stocks of resources and waste do not change, then from (23) and (24) we can derive $W_{em} = R_N$. If an indirect relationship can be established which states that in the long run the average level of $W_{em}$ equals the average level of the sum of $R_N$ and $R_S$ (because, reasoning intuitively, everything that enters the economic system has to leave it once, assuming economic stocks are small compared to the total resource and waste flows and natural resource stocks), then the equality $R_N = W_{em} = R_N + R_S$ is obtained, so that for sustainable development - including sustainable use of renewable resources and sustainable assimilation of waste - the condition $R_S = 0$ has to be fulfilled in the long run.

## 6. SUSTAINABLE DEVELOPMENT SCENARIOS.

The general behaviour of this model has been analyzed by making a minimum of additional assumptions about the functional forms of relationships. Parameters were given values that comply with a world scale. The following variables are controlled for during the simulation experiment: investment, environmental policy instruments, allocation of resource use, the perception of supply of resources (resource-activity feedback: through a coefficient showing the reaction of economic activity on changing resource scarcity; see equation), and the safety margin for waste emission control.

The following scenarios can be studied with the present model and are useful for analysis of sustainable development in the system as described by our general model.:

1. Reference scenarios: no growth in closed system.
2. Growth scenarios: capital accumulation.
3. Environmental policies: waste treatment; research and development on resource efficient techniques; recycling; and environmental cleaning.
4. Resource use scenarios: a variety of allocation schedules of resource demand to non-renewable and renewable resources.
5. Feedback of resource scarcity to economic activity: supply of resources indicated by sum of all resource stocks; or the sum of the regeneration rate of renewable and the

stock of non-renewable resources; or the sum of the regeneration rate and a part of the non-renewable stock, to include much concern for future generations.

6. Feedback from waste accumulation to economic activity: to diminish the stock of pollution; or to realize sustainable waste emission; or to allow for no more than a certain positive accumulation rate of waste.

To evaluate the total systems behaviour in the context of sustainable development the dynamic pattern as a whole must be evaluated, by looking at trends, minimum values attained and fluctuation patterns. The first should not decline, the second not be too low, and the third not be too intense.

We will show the performance under a scenario of sustainable use. Investment is here much higher than depreciation, its level being chosen so that without any environmental policies finally a destruction of the system occurs. Use rates of renewable and non-renewable resources are equal so long as no limits are reached. Use of renewable stocks is sustainable, i.e. the use rate is not higher than the regeneration rate. The perceived supply of resources equals the sum of the renewable resource regeneration rate and the non-renewable stock level, so that - as explained in section 5.7 - there is only little concern for future generations. The initial level of regeneration in the ecological system exceeds the initial rate at which extraction takes place, where the latter is based on the initial stock of productive capital. The assimilative capacity is initially lower than the rate of waste emission, the latter being dependent on the resource extraction and accumulation of stocks[5]. Finally, the distribution of output is constant, and there are no environmental policy outlays.

Figure 1 shows the patterns over 50 years for output Q, environmental quality E, the regeneration rate G and the assimilation rate M (denoted by numbers 1 to 4, respectively). Output initially rises quickly until the stock of non-renewable resources is heavily depleted, which causes the feedback mechanisms described in section 5.7 to become operational. The result is a reduction in production activity level, which immediately impacts negatively on the levels of investment, consumption, and waste emission. Because also resource extraction is diminished, the impact of the feedback mechanisms slows down. It has to be understood that, as is explained in section 5.7, the slowing down of growth or even negative growth is the consequence of mechanisms that react before really damaging direct effects occur[6]. The output stabilizes at a level

[5] This can be determined on the basis of equation (51). Remember that $W_{em}$ already includes the effects of recycling.

[6] Whether this is also the case for a long time period, when indirect effects have been realized as well, is uncertain and one of the reasons to perform studies with the present model.

approximately equal to its initial value. Environmental quality starts decreasing after 8 years and reaches its minimum (equal to 0.8 in the numerical example) in the year 20. Then it starts moving upward till it reaches its optimum value after 31 years (equal to 1). Initially the renewable stock increases when the use rate is below the regeneration rate. As a result of an assumed lower regeneration rate at a higher renewable stock level and a higher use rate resulting from depletion of the non-renewable stock it stabilizes. The development shown in figure 1 can only be labelled as 'sustainable' if two conditions are satisfied. First, we have to agree that the important requirement for sustainable development is a welfare level for all generations above some minimum level (see section 4.3). Second, the initial level of consumption in the present case should exceed this minimum so that according to curve 1 welfare will always stay above it (at least for the first fifty years).

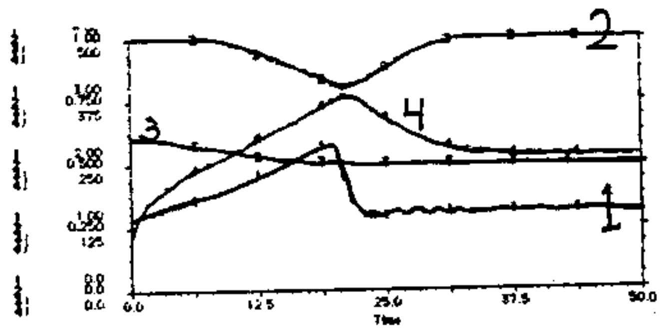


Figure 1: Sustainable Use.

## 7. CONCLUSIONS.

The purpose of the present model is not to be helpful in deciding whether there will be enough resources for long term economic activity, or whether ultimately growth is better than a steady state, or whether pollution will become a main problem eventually. The specific usefulness lies in the features of two-way economic-ecological impacts, and the inclusion of feedback mechanisms. These allow for a study of the medium and long term dynamic consequences of sustainable development conditions and concern for future generations' welfare. For instance, even if in the long term the entire system will breakdown, it is still relevant to understand what dynamic economic and ecological patterns are likely under given conditions before it actually happens. Further research regarding the general behaviour of the proposed model is necessary. This may be done both by performing simulation experiments with models based on this general structure, and by analyzing simpler dynamic economic-ecological models derived from it.

## APPENDIX: MODEL NOTATION.

**Stock variables:**

| | |
|---|---|
| K | = productive capital |
| Pop | = population level |
| $T_{rd}$ | = cumulative R&D outlays |
| $S_{wa}$ | = abated waste |
| N | = renewable resources |
| S | = non-renewable resources |
| P | = pollution |

**Flow variables:**

| | |
|---|---|
| Q | = output from production |
| C | = total consumption |
| $C_Q$ | = consumption from produced goods |
| $C_R$ | = consumption from resources |
| I | = investment in productive capital |
| J | = employment (jobs) (intermediate variable) |
| L | = labour force (intermediate variable) |
| O | = other material outlays |
| $O_a$ | = outlays for abatement |
| $O_{rec}$ | = outlays for recycling |
| $O_c$ | = outlays for environmental cleaning |
| $O_{rd}$ | = outlays for R&D |
| E | = ecological quality indicator |
| $R_{dem}$ | = total productive and consumptive demand for resources |
| $R_{sup}$ | = total extraction/supply of renewable and non-renewable resources |
| $R_{short}$ | = denotes whether resource supply equals demand |
| $R_N$ | = renewable resource extraction |
| $R_S$ | = non-renewable resource extraction |
| $R_Q$ | = total resource input for production |
| $R_{Qn}$ | = production demand for newly extracted resources |
| $R_{rec}$ | = recycled resource material input for production |
| U | = unemployment (intermediate variable) |
| W | = gross waste (before abatement and recycling) |
| $W_Q$ | = gross waste from production |
| $W_{rec}$ | = waste amenable for recycling |
| $R_{wa}$ | = abated waste |
| $W_{em}$ | = emitted waste |
| $W_{ra}$ | = abated and recycled waste |
| $W_{sup}$ | = waste emission control indicator |

**Functions:**

| | |
|---|---|
| a | = labour force |
| b | = employment |
| c | = ratio of resource input to material output in production |
| d | = relates the level of consumption of resources to that of produced goods |
| e | = determines resource shortage |
| SW | = social welfare |
| U | = a generation's welfare |
| F | = production |
| $f_a$ | = abatement |
| $f_r$ | = recycling |
| D | = depreciation of capital |
| H | = environmental quality function |

G = regeneration function of renewable resource capacity
M = assimilation function

**Parameters:**

T = time horizon
$N_{min}$ = critical resource level for growth function G
$N_{max}$ = upper bound level of renewable resource capacity (carrying capacity)
$c_{rec}$ = part of $W_{ra}$ that is recycled
r = safety margin for resource use control
m = safety margin for waste emission control
initial stock levels $K_0, Pop_0, N_0, S_0, P_0$

**REFERENCES.**


Arrow, K.J. (1973), "Rawls's principle of just saving", Swedish Journal of Economics 75, 323-335

Ayres, R.U. (1978), Resources, environment, and economics: Applications of the materials/energy balance principle, Wiley-Interscience, New York

Barbier, E.B. (1989), Economics, natural resource scarcity and development: Conventional and alternative views, Earthscan Pub., London

Braat, L.C. and W.F.J. van Lierop (1987), Economic-ecological modelling, North-Holland

Burmeister, E. and A.R. Dobell (1970), Mathematical theories of economic growth, Macmillan, New York

Clark, C.W. (1976), Mathematical bioeconomics: The optimal management of renewable Resources, Wiley-Interscience, New York

Clark, C.W. (1985), Bioeconomic Modelling and Fisheries Management, Wiley-Interscience, New York

d'Arge, R.C. and K.C. Kogiku (1973), "Economic growth and the environment", Review of Economic Studies 40, 61-77

Dasgupta, P.S. and G.M. Heal, (1979), Economic theory and exhaustible resources, Cambridge University Press, Cambridge University Press, Cambridge

Georgescu-Roegen, N. (1971), The entropy law and the economic process, Harvard U.P., Cambridge M.A.

Gross, L.S. and E.C.H. Veendorp (1990), "Growth with exhaustible resources and a materials-balance production function", Natural Resource Modeling 4, 77-94

Holling, C.S. (ed.) (1978), Adaptive environmental assessment and management, Wiley, New York

Kamien, M.I. and N.L. Schwartz (1978), "Optimal exhaustible resource depletion and endogenous technical change", Review of Economic Studies 29, 174-192

Kamien, M.I. and N.L. Schwartz (1982), "The role of common property resources in optimal planning models with exhaustible resources", in: V.K. Smith and J.V. Krutilla (eds.), Explorations in natural resource economics, Johns Hopkins University Press, Baltimore

Keeler, E., M. Spence and R. Zeckhauser (1972), "The optimal control of pollution", Journal of Economic Theory 4, 19-34

Klaassen, L.H. and T.H. Botterweg (1976), "Project evaluation and intangible effects - a shadow project approach", in P. Nijkamp (1976) (ed.), Environmental economics, Vol. 1: Theories, Martinus Nijhoff, Leiden

Kneese, A.V., R.U. Ayres, and R.C. D'Arge (1970), Economics and the environment: A materials balance approach, Johns Hopkins Press, Baltimore

Mäler, K.-G. (1974), Environmental economics: a theoretical inquiry, Johns Hopkins University Press, Baltimore

Meadows, D.H., J. Richardson and G. Bruckmann (1982), Groping in the dark: The first decade of global modelling, Wiley, NY

Odum, H.T. (1987), "Models for national, international, and global systems policy", in: L.C.

Braat and W.F.J. van Lierop (eds.), Economic-ecological modelling, North-Holland
Peterson, F.M. and A.C. Fisher (1977), "The optimal exploitation of extractive resources: A survey", The Economic Journal 87, 681-721
Pezzey, J. (1989), Economic analysis of sustainable growth and sustainable development, Environmental Department Working paper no.15 , Environmental Department, The World Bank.
Rawls, J. (1972), A theory of justice, Harvard University Press, Cambridge, Mass.
Siebert, H. (1982), "Nature as a life support system, renewable resources and environmental disruption", Journal of Economics 42, 133-142
Siebert, H. (1987), Economics of the environment: Theory and policy, Springer-Verlag, Berlin
Solow, R.M. (1974), "Intergenerational equity and exhaustible resources", Review of Economic Studies 41, 29-45
Wan, H.Y. Jr (1971), Economic growth, Harcourt Brace Javanovich, New York
Walters, C.J. (1986), Adaptive management of renewable resources, Macmillan
The World Commission on Environment and Development (WCED) (1987), Our common future, Oxford University Press