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Nijkamp, P.

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SOFT ECONOMETRIC MODELS

An Analysis of Regional Income Determinants

Peter Nijkamp

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1. Soft Econometric Models : General[1)]

Soft econometric models are econometric models based on and estimated by soft information (for example, qualitative or ordinal information on the explanatory variables). Soft information means in general that the observations on certain variables are not measured in a metric sense (such as a cardinal or interval scale). Hence, non-metric measurement units are crucial in soft information analysis.

The major part of modern econometric models is based on metric information. The development of advanced models, however, has not kept pace with the data base required to estimate such models. Despite the progress in data collection and the improvement of information systems, much information is still non-metric in nature. This gives often rise to the problem of omitted variables and false specifications. Even many metric variables are essentially pseudo-metric because of significant uncertainties and measurement errors, so that cardinal conclusions can hardly be inferred from such variables.

Several phenomena can hardly be measured in metric terms (for example, happiness, welfare, quality of life, etc.). Clearly, one may introduce certain proxy variables which assign a cardinal value to such non-metric variables, but this operation may easily lead to a biased view of reality (cf. Adelman and Morris [1974]).

In the history of statistical and econometric analysis several ways have been chosen to deal with soft information.

A first way to treat soft data (especially ordinal information) is the use of correlation analysis from non-parametric statistics (cf. Siegel [1956]). By regarding ordinal values as pseudo-cardinal information, one may calculate among others correlation coefficients for series of ordinal data (for example, the Spearman rank correlation coefficient). Despite the popularity of this approach, it has to be noticed that this correlation analysis rests in general on non-permissable numerical operations on ordinal data. Although the results are normally used to test the existence of rank correlation among ordinal items, one should be aware of the severe limitations and stringent assumptions inherent in this approach.

Another way to treat soft data (especially qualitative data) is the use of dummy variables. Such dummy variables are often used in econometric modelling to take account of non-metric explanatory variables such as occupational status, marital condition, sex, etc. (see Johnston [1972] and Theil [1971], e.g.). Dummy variables are natural numbers 0 and 1, which indicate whether or not a certain variable belongs to a nominal class. Dummy variables have

1) The author is indebted to Leen Hordijk, Franz Palm and Piet Rietveld for their useful comments on a first draft of this paper.

often been applied in econometrics to integrate sociological and demographic variables (see, for example, Orcutt et al [1961]). A drawback of the use of dummy variables is that a situation of many distinct classes leads to a high number of dummy variables. In addition, a zero-one indicator for certain class characteristics does normally not make use of the available information in the most efficient way : usually more than purely zero-one information is available, for instance, in a qualitative sense such as 'good, better, best'. In the latter case, adjusted methods may be more fruitful (see later).

A third way of dealing with soft information in econometric modelling is the use of path models. Path models are based on the assumption that in many theoretical constructs the variables of direct interest cannot be observed directly, so that proxy variables have to be used which reflect a certain quantitative characteristic of the original variables (see Blalock [1964]). Path models are not primarily based on the traditional assumption of causal or functional relationships between variables mutually, but on causal or functional relationships (correlations) between qualitatively-oriented clusters of variables (for example, economic growth efforts, political conditions etc.). Each cluster is made up by a set of underlying variables. These clusters can among others be constructed by means of principal component techniques. Path models attempt to find significant correlations in terms of causal relationships between such clusters. Two multivariate techniques can be used to analyse further such correlations between clusters, viz. Lisrel (see Jöreskog [1977]) and Partial Least Squares (see Wold [1975, 1979]).

The strength of a path model is its ability to deal with latent (indirectly observed) variables. Such latent variables have usually only a soft qualitative meaning, but they can be represented by a vector profile of indirect indicators. The techniques to deal with latent variables in path models differ substantially. The Lisrel technique is based on a maximum likelihood approach and needs precise information about the distribution of the observed variables and the specification of the theoretical model. Application of Lisrel can be found among others in Folmer [1979], Jöreskog [1977] and Jöreskog and Sörbom [1977].

The Partial Least Squares method attempts to identify a block structure for the latent variables and their indicators as well as between the latent variables (the 'inner' structure) on the basis of multivariate techniques (especially iterative regression analysis). Applications of this techniques can be found among others in Apel[1978], Hui [1978], and Wold [1977].

The path models with latent variables are a specific example of models with non-directly observable information, because the latent variables are ultimately reflected by a multidimensional set of relevant observable indicators. A more difficult problem, however, arises if (part of) the explanatory variables are only measured in ordinal terms. In that case, the Lisrel and Partial Least Squares technique cannot be applied. This paper is an attempt to address the problem of constructing an econometric model based on purely soft (i.e., ordinal) information. It will be shown that multidimensional scaling techniques may provide a useful tool to deal with soft information in econometric model building. This will also be illustrated by means of an empirical example.

2. Soft Econometric Models

Soft econometric models use ordinal information as input. Ordinal data have seldom been used in econometric modelling (see Van Setten en Voogd [1979]). It is clear that ordinal observations cannot be directly used in traditional estimation procedures for econometric models, although it has to be noticed that rather frequently ordinal data have been treated as cardinal units. Suppose now a model which incorporates both cardinal and ordinal data as explanatory variables :

(2.1.) $$y = f(\underline{x}, \underline{z}),$$

where : y = endogenous variable

$\underline{x}$ = (J x 1) vector of cardinal explanatory variables

$\underline{z}$ = (K x 1) vector of ordinal explanatory variables

f = functional relationship describing the impacts of $\underline{x}$ and $\underline{z}$ on y

When this model is written as a linear regression model, one obtains :

(2.2.) $$\underline{y} = X\underline{\beta} + Z\underline{\gamma} + \underline{\varepsilon},$$

with : $\underline{y}$: I x 1 vector of observed endogenous variables

X : I x J matrix of observed cardinal exogenous variables

Z : I x K matrix of observed ordinal exogenous variables

$\underline{\varepsilon}$: I x 1 vector of disturbance terms

$\underline{\beta}$: J x 1 vector of unknown parameters

$\underline{\gamma}$: K x 1 vector of unknown parameters

Normal regression procedures aim at estimating a parameter set $\underline{\beta}$ and $\underline{\gamma}$ which reflects the quantitative impact structure of $\underline{x}$ and $\underline{z}$ with respect to $\underline{y}$. In the case of ordinal information, however, a traditional least squares procedure cannot be applied, because this would require unpermissable numerical operations.

A meaningful approach in the case of ordinal data is to transform first the ordinal data matrix Z into metric data and, next, to apply a regression analysis to the transformed model. It is clear, however, that a metric transformation of Z is only possible if either additional information is available or the transformed (metric) matrix has a lower dimensionality. In the framework of the present paper no additional information will be assumed, so that an attempt has to be made to construct a matrix Z^* of order $I \times K^*$ $(K^* < K)$, such that the metric values of Z^* form a best representation of the original ordinal matrix Z. In that case, the transformed regression model becomes :

$$(2.3.) \qquad \underline{y} = X\,\underline{\beta} + Z^*\,\underline{\gamma}^* + \underline{\varepsilon}^* ,$$

where $\underline{\gamma}^*$ is the new parameter vector of order $K^* \times 1$.

Given the usual assumptions concerning the linear model, the unknown parameters in (2.3.) can then be estimated by means of standard regression techniques. When this regression analysis leads to satisfactory results, (2.3.) may then also be used as a prediction model for the endogenous variables $\underline{y}$.

A major problem, however, is the question whether the transformation from Z to Z^* is a possible and permissable operation. It will be shown in section 3 that the transformation of an ordinal matrix Z to a cardinal matrix Z^* of a lower dimensionality can be carried out in a meaningful way by means of recently developed multidimensional scaling techniques. Therefore, the following steps have to be undertaken to estimate soft econometric models :

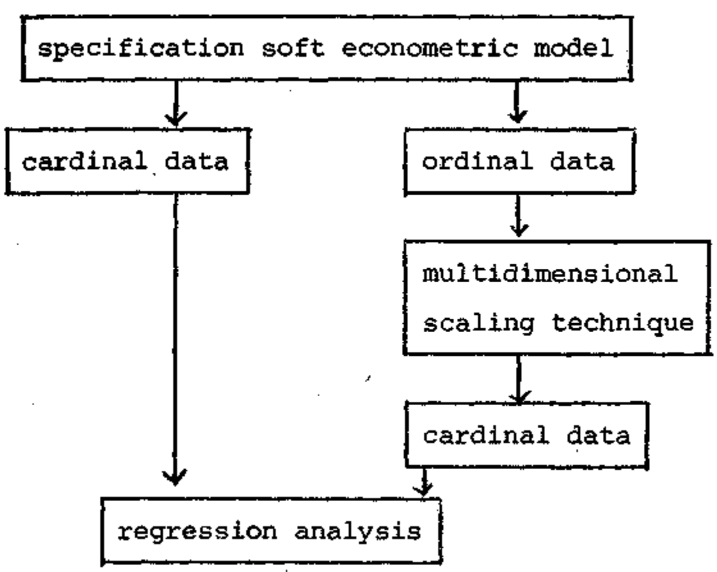

Fig. 2.a. Estimation procedure of soft econometric models.

3. Multidimensional Scaling Analysis

Multidimensional scaling analysis aims at deriving quantitative, metric information from qualitative, ordinal input data. This analysis, originally developed in mathematical psychology, has recently become very popular in many scientific fields (for example, geography, economics, marketing theory, physical planning, regional science and operations research). Multidimensional scaling analysis, sometimes also called polynomial conjoint analysis or ordinal geometric scaling, is a rather complicated mathematical technique which will not be described at length in this paper; mathematical expositions about this technique are contained among others in Nijkamp [1979], Nijkamp and Van Veenendaal [1979] and Nijkamp and Voogd [1979]. Only a brief introduction to multidimensional scaling analysis will be given here.

Multidimensional scaling methods take for granted that the aspects of a certain phenomenon can be described by means of a set of K ordinal indicators. The variation among these aspects is reflected by differences in ordinal numbers. One of the aims of multidimensional scaling analysis is then to transform the ordinal data input into cardinal information of a lower dimensionality. This requires the identification of elements in a K^*-dimensional geometric space ($K^* < K$) such that the aspects or the attributes of the phenomenon concerned can be meaningfully depicted in a Euclidean space, so that the configuration of the elements in this Euclidean space does not contradict the orginal ordinal input data. The criterion to determine this configuration in a K^*-dimensional Euclidean space is that the interpoint distances between the coordinates of the attributes in this K^*-dimensional space show a maximum resemblance to the orginal ordinal ranking of the attributes. This means, that the Euclidean distances derived from the metric configuration should be consistent (in the sense of a monotone relationship) with the observed ordinal rankings (see later).

There are various techniques in the field of multidimensional scaling analysis which aim at finding a maximum correspondence between the I ordinal rankings of K attributes and the distances between I points in a K^*-dimensional geometric space. The various steps to be undertaken in the majority of these algorithms can be illustrated as follows (cf. Nijkamp and Voogd [1979]).

Suppose (without loss of generality) a symmetric N x N paired comparison table Δ in which the (dis)similarities between N objects (regions, e.g.) are expressed in ordinal numbers. The elements of Δ are $\delta_{nn'}$, and represent ordinal rank numbers for the (dis)similarity between any two points n and n'. Assume next that the N objects can be represented as a configuration of N points in a K-dimensional Euclidean space ($K \leq \frac{1}{2}(N-1)$), so that each object can be characterized by a K-dimensional vector. When the co-ordinates of each k^{th} point are denoted by x_{nk} ($k = 1, \ldots, K$), the interpoint distances $d_{nn'}$ between each pair of points n and n' are:

$$(3.1.) \qquad d_{nn'} = \{\sum_{k=1}^{K} (x_{nk} - x_{n'k})^2\}^{\frac{1}{2}}$$

The co-ordinates x_{nk} and the distances $d_{nn'}$ can be included in an N x K configuration matrix X and an N x N distance matrix D, respectively. The latter distance relationship has metric properties and should have a maximum correspondence with respect to $\delta_{nn'}$; in other words, the geometric configuration x_{nk} ($n = 1, \ldots, N$; $k = 1, \ldots k$) should be such that the distances from (3.1.) do not violate the (dis)similarity conditions from the matrix Δ. A best fit would be achieved by specifying a goodness-of-fit (or loss) function Φ which minimizes the residual variance between all distances $d_{nn'}$ and $\delta_{nn'}$, but the ordinal nature of $\delta_{nn'}$ precludes the application of arithmetic operations to $\delta_{nn'}$.

Therefore, instead of introducing $\delta_{nn'}$ itself into the goodness-of-fit function, a metric auxiliary variable $\hat{d}_{nn'}$ is calculated which is in agreement with $\delta_{nn'}$. This implies a monotonic relationship between $\delta_{nn'}$ and $\hat{d}_{nn'}$, so that the following condition is satisfied : $\hat{d}_{nn'} \leq \hat{d}_{nn''}$, whenever $\delta_{nn'} \leq \delta_{nn''}$. The auxiliary variables $\hat{d}_{nn'}$ will be called here order-isomorph values or disparities. They can be incorporated in an N x N matrix $\hat{D}$. The diagonal elements of Δ, D and $\hat{D}$ are equal to zero.

Therefore, the essential ideas of an ordinal multidimensional scaling method can be formalized in the following concise manner :

$$(3.2.) \qquad \begin{cases} \min\limits_{X} \phi = f\ (D - \hat{D}) \\ \text{subject to} \\ \hat{D} \overset{m}{=} \Delta \\ D = g(X) \end{cases}$$

where the symbol $\overset{m}{=}$ denotes the above mentioned monotonicity condition between $\hat{d}_{nn'}$ and $\delta_{nn'}$. The distance function g(X) is already defined in (3.1.). The determination of the matrix $\hat{D}$ with order-isomorph values can be based on various approaches, for example, a monotone regression (Kruskal [1964]).

This approach implies a constrained minimization problem, written as :

$$(3.3.)\quad \begin{cases} \min \psi = \sum_{n} \sum_{n'} (d_{nn'} - \hat{d}_{nn'})^2 \\ \text{subject to} \\ \delta_{nn'} \geq \hat{\delta}_{nn''} \rightarrow \hat{d}_{nn'} \geq \hat{d}_{nn''} \end{cases}$$

So, in fact, monotone regression consists of finding a set of values $\hat{d}_{nn'}$, which are monotone with the imputed rankings and also match the distances among all points. An alternative approach to arrive at order-isomorph values is the rank-image procedure (Guttman [1968] and Golledge and Rushton [1972]).

The goodness-of-fit function ϕ may have the following specification :

$$(3.4.)\quad \phi = \left(\frac{\sum_{n,n'} (d_{nn'} - \hat{d}_{nn'})^2}{\sum_{n,n'} d^2_{nn'}} \right)^{\frac{1}{2}}$$

Alternative specifications of ϕ can be found in Voogd [1978].

The foregoing scaling technique is not only relevant for square paired comparison matrices (complete matrices), but can also be adjusted for rectangular ordinal rank order matrices (conditional matrices). Expositions of these alternative approaches and of problems of ties can be found among others in Roskam [1968]. It is clear that the conditional approach is particularly important for our problem of soft econometric models.

After this brief presentation of the ingredients of ordinal multidimensional scaling techniques the steps of the algorithmic procedure will be described.

The purpose of the algorithm is to find the co-ordinates of N points - representing N objects (regions,e.g.) - in a K-dimensional geometric space such that the distances among these points are in approximately the same rank order as the imputed rankings of the (dis)similarities among N objects.

The first step of the algorithm is the assessment of an initial configuration X_1 in a K-dimensional space. This initial configuration can be determined in an arbitrary way, but frequently a principal component procedure with K components is applied to the ordinal input data in order to speed up the convergence of the algorithm (cf. Guttman [1968]).[1)]

The second step is to calculate the resulting matrix of interpoint distances D_1 from X_1.

The next step is to determine the initial set of order-isomorph values

1) See for a further discussion also Van Setten en Voogd [1978].

$\hat{D}_1$ on the basis of a monotonicity condition with respect to Δ, so that the disparities are in accordance with the (dis)similarities.

Then the goodness-of-fit has to be maximized (i.e., the loss function ϕ has to be minimized). This procedure implies that the values of $\hat{D}_1$ are substituted into (3.3.), while the formal expressions for the elements of D (see 3.1.) are also substituted into (3.3.). Consequently, ϕ is a nonlinear function with N x K unknown arguments x_{nk}, which has to be minimized by means of a numerical solution procedure (a gradient procedure, e.g.). The resulting values of x_{nk} are in fact the elements of a new configuration X_2. Then again a new distance matrix D_2 and a new order-isomorph matrix $\hat{D}_2$ can be calculated, so that ϕ can again be minimized, etc., until finally a convergent solution is attained, for which the goodness-of-fit is at a maximum (see for a discussion of convergence properties Van Setten and Voogd [1978]).

The above mentioned steps are briefly summarized for a fixed number of dimensions K in the flow chart of Fig. 3.a.

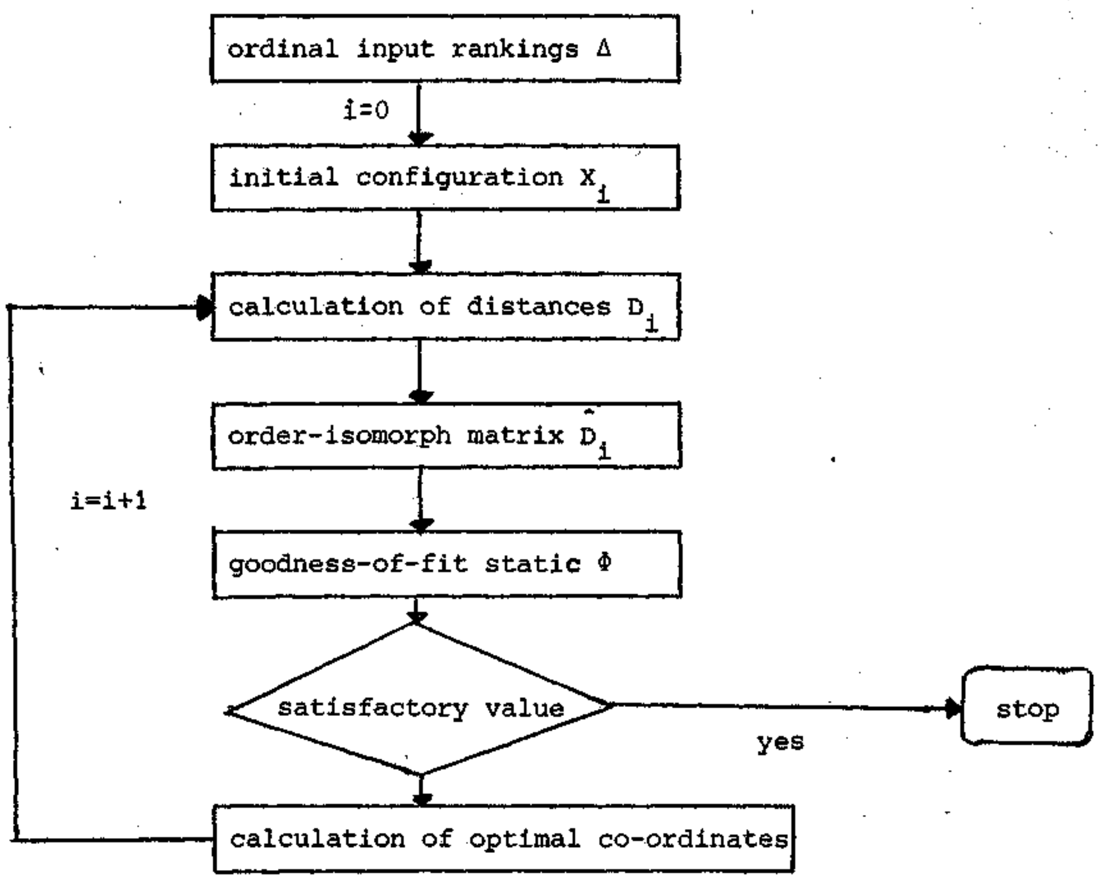

Fig. 3.a. A brief flow chart of the algorithmic structure of an ordinal multidimensional scaling method.

The ultimate configuration depicts the N objects in a K-dimensional space. The interpretation of this configuration -more precisely : the interpretation of the axes of the K-dimensional Euclidean space- is to a certain extent a matter of personel inventiveness of the researcher (just like in factor analysis). This ex post identification of the K attributes of N objects can to a certain extent be tested (or at least justified) by confronting the K attributes with observable metric data on the characteristics of these attributes via a least squares procedure or via a two-stage multi-dimensional scaling procedure (see Nijkamp and Van Veenendaal [1978]). The latter approach is a prerequisite to arrive at an verifiable multidimensional scaling procedure.

It is easily seen that the previous steps can also be applied to transform the ordinal matrix Z from (2.2) into a cardinal matrix Z* (from (3.2.), so that next Z* can be used as a matrix of observations in a regression analysis (this step is to a certain extent analogous to the use of principal components results in a regression analysis). In this way, ordinal data can be integrated in traditional econometric estimation procedures.

4. Empirical Illustration [1)]

The empirical application of soft econometric models is based here on an explanatory model for regional incomes in the Netherlands. The total number of regions (provinces) taken into account in this study is equal to 11. This number is fairly small and it certainly affects the results of a soft regression analysis, but for the sake of illustration this empirical example is satisfactory.

The assumption underlying the model used in this section is that average regional income (y) can be explained from two categories of income determinants, viz. an economic profile ($\underline{e}$) and a socio-geographical profile $\underline{g}$ (see for a general discussion of profile analyses Nijkamp [1979]). In other words :

$$(4.1.) \qquad y = f\,(\underline{e}\,,\,\underline{g}) + \varepsilon\ .$$

The economic profile $\underline{e}$ is composed of 3 elements, viz. the average regional unemployment level u (percentage of unemployed male and female persons), the average regional wealth w (measured in Dutch guilders) and gross regional product p (also measured in Dutch guilders), i.e.,

1) The author is indebted to Wouter van Veenendaal for his computational assistance.

(4.2.) $\underline{e}' = (u \vdots w \vdots p)$

The socio-geographical profile $\underline{g}$ includes 4 elements, viz. regional agglomeration phenomena a (for instance, population density measured as the number of persons per square kilometer), regional accessibility c (for instance, density of transportation network measured as regional road length divided by the regional area), educational opportunities s (for instance, measured as the weighted number of schools of different categories) and the regional attractiveness t (for instance, measured via the average distance to the economic heartland of the Netherlands). This implies that the socio-geographical profile is composed as follows :

(4.3.) $\underline{g} = (a \vdots c \vdots s \vdots t)$

Next, model (4.1.) had to be estimated by means of a regional cross-section for the year 1970. Data for the regression analysis were mainly derived from Blommestein et al [1978].

The information concerning the economic profile could easily be obtained from available statistics, so that the values of the elements of the economic profile could be measured in cardinal units (see Table 1). However, the data about the socio-geographical profile are much harder to obtain in a cardinal sense. As indicated above, some variables such as agglomeration factors or accessibility can be approximated by means of cardinal auxiliary variables, but it is clear that such proxy variables may not be regarded as precise cardinal indicators of agglomeration advantages or accessibility. Therefore, it is more plausible to use ordinal indicators for such variables. These ordinal indicators varying from 1 to 11 may be derived from the auxiliary proxy variables suggested above.[1)]

Consequently, the socio-geographical profile $\underline{g}$ is composed of 4 x 11 ordinal indicators (see Table 2). Such data cannot be used directly in a normal regression analysis. Therefore, first a multidimensional scaling procedure has to be applied in order to extract metric information from the ordinal input data. The results of this scaling procedure are included as a one-dimensional configuration g in Table 3 (the stress value or badness-of-fit appeared to be low enough to justify only a one-dimensional representation of the ordinal input data).

Next, a normal regression analysis was applied to model (4.1.), in which the four-dimensional socio-geographical profile was reduced to the one-dimensional configuration g from Table 3. The results of this regression analysis are included in Table 4.

1) Of course, one might also apply a multidimensional scaling prodedure to these proxy variables.

Clearly, the estimation of 4 parameters from a cross-section with 11 observations is a heroic task and the outcomes have only a tentative and indicative value.

The results give rise to the following comments. The squared multiple correlation coefficient appears to be rather high (0.928). The values of the t-statistic indicate that the intercept is statistically significantly different from zero. The same holds true for w (regional wealth) and p (gross regional product). The parameter for the socio-geographical profile appears to be only marginally significant as its t-value is 1.29, but given the small number of degrees of freedom this result is not unplausible. Finally, the unemployment variable appeared to provide no significant explanation to the regional income levels (this may be explained from the good social security system in the Netherlands). In general, the residual terms appear to be fairly low: the calculated values of regional incomes are in rather close agreement with the observed values (see Table 5).

Another advantage of the use of multidimensional scaling techniques in econometric models is that it leads to a rise in the number of degrees of freedom. For example, the original model (4.1.) based on (4.2.) and (4.3.) had only 3 degrees of freedom, whereas the same model has 6 degrees of freedom after the application of the scaling technique.

Clearly, the statistical properties of the parameter estimates are hard to investigate, as the multidimensional scaling prcedure is a non-parametric statistical technique. When, however, the scaled outcomes (see Table 3) would have a certain statistical distribution (for example, a normal one), one may analyse the statistical characteristics of the parameter values according to the usual econometric-statistical methods.

Finally, the results of this multiregional income model can be examined as fas as the presence of spatial autocorrelation is concerned. The value of the Moran coefficient (see Hordijk and Nijkamp [1978]), however, clearly shows that spatial autocorrelation among the disturbances of the linear model does not exist at the spatial (provincial) scale chosen in the analysis.

5. Conclusion

Soft econometric modelling appears to provide a useful approach in the case of inaccurate data (ordinal data, proxy data, large measurement errors etc.). In such situations the use of multidimensional scaling techniques is worth while to tackle uncertainties in the data input for econometric models. The numerical exercises carried out in the previous section show that soft econometric techniques may be appropriate tools to deal with ordinal or qualitative information.

variable \ province		1	2	3	4	5	6	7	8	9	10	11
	y	4652.0	4353.0	4419.0	4501.0	4752.0	5438.0	5846.0	5850.0	4990.0	4556.0	4422.0
e	u	3.2	3.3	4.3	2.3	1.7	1.2	1.8	1.5	2.3	2.2	2.8
e	w	5978.0	4825.0	4954.0	4837.0	5355.0	7134.0	6563.0	6559.0	7440.0	4316.0	3370.0
e	p	8994.0	5202.0	3866.0	10938.0	17910.0	10242.0	30574.0	42300.0	4440.0	22305.0	10924.0

Table 1. Regional economic profile of the Netherlands (1970)[1)]

variable \ province		1	2	3	4	5	6	7	8	9	10	11
g	a	4	2	1	5	6	9	10	11	3	7	8
g	c	10½	10½	9	8	6½	4½	3	1½	4½	6½	1½
g	s	2½	1	2½	5	7½	7½	10½	10½	4	7½	7½
g	t	11	8½	8½	6	4½	1	2½	4½	8½	2½	8½

Table 2. Regional socio-geographical profile of the Netherlands (1970)

variable \ province	1	2	3	4	5	6	7	8	9	10	11
g	-2.04	-1.25	-1.02	0.25	0.25	0.28	1.38	1.38	0.24	0.26	0.27

Table 3. One dimensional metric representation of ordinal socio-geographical profile

1) The provinces are: Groningen (1), Friesland (2), Drente (3), Overijssel (4), Gelderland (5), Utrecht (6), North-Holland (7), South-Holland (8), Zeeland (9), North-Brabant (10) and Limburg (11) .

	intercept	e			g
		u	w	p	
estimated coefficient	2947.30	32.98	0.28	0.02	137.07
t-statistic	5.46	0.28	4.84	2.90	1.29

$R^2 = 0.928$

Table 4. Parameter values associated with e and g.

variable \ province	1	2	3	4	5	6	7	8	9	10	11
observed	4652.0	4353.0	4419.0	4501.0	4752.0	5438.0	5846.0	5850.0	4990.0	4556.0	4422.0
calculated	4611.9	4324.9	4397.1	4613.5	4897.4	5209.6	5660.3	5901.1	5200.8	4723.1	4239.1

Table 5. Values observed and calculated income.

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