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AN ERROR BOUND FOR APPROXIMATING DISCRETE TIME SERVICING BY A PROCESSOR SHARING MODIFICATION

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Abstract

An analytic *a priori* error bound is provided for comparing performance measures of discrete time queues with those of a processor sharing queue for which steady state results can be easily obtained. The error bound is of the same order as the length of the discrete time slot. This bound is applied to derive error bounds for performance measures of a time-sharing queue approximated by a processor sharing queue.

Keywords: Processor/time sharing; error bound; Markov Chain; geometric distribution.

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1. Introduction

Discrete time queueing systems are a common feature in computer networks and analyses of packetised transmissions along digitised channels in telecommunications systems. One of the most important of such discrete time queues uses the time sharing protocol in which a server processes a quantum, or segment of a job and then moves on to process a segment of the next job (see Takagi [13], for a review of work on this and related polling models). In [8] and later papers (see [9]), Kleinrock analysed time sharing systems by assuming that segments have a size that shrinks to zero and that the server shares out effort equally amongst all customers present, i.e. he approximated the system by a processor sharing queue. The investigation of processor sharing models has proved very fruitful, and has given rise to closed form expressions for steady state distributions (see [3], [7], [10], [11], [12], [17], [18], and [19]).

Such approximate modelling of time sharing systems seems justifiable, on both an experimental and intuitive basis, provided the size of the segments of service provided to each customer is sufficiently small. However, no explicit *a priori* error bound for this modelling inaccuracy seems to be available in the literature. This paper provides such a bound when focusing on the total number of segments in the queue. Since this number is independent of the actual service protocol providing that exactly one segment is processed per time slot our results actually give an error bound on the approximation of a whole class of discrete time queues by a processor sharing system.

The underlying technique based on Markov reward theory, promises to be useful in the further extension of comparisons between discretised and continuous protocols. An application of a packetised transmission system which motivated the study will be described.

Let us briefly review related literature. Most closely related in spirit are the results in [7] in which a special time sharing round robin discipline as introduced in [4] is shown to converge to a processor sharing discipline in order to derive response time results. However, no error bounds are obtained or directly concludable. Moreover, this specially devised time-sharing discipline does not correspond to the basic time-sharing model studied herein as it essentially excludes more than one arrival during a time slot by assuming a geometric input. A similar limiting result is found in [11???]. The results in [12] are of the same nature for the foreground-background processor sharing discipline. In [1] and [2] a combined processor-sharing and last-come-first-served is analysed with a comparison being made between the "to the head of the line" and "to the tail of the line" prempting rule. Such models are known to lead to geometric steady state distributions (see [17]). The references [3], [10], [17], [18] and [19], furthermore, all concern special distributions other than for the number of jobs in processor sharing systems.

2. The Model and its Approximation

Consider a central server system with jobs arriving according to a Poisson process with parameter λ. The number of segments in a job is a random variable B with probability mass function $b(\ell)$, $\ell \in Z_+$. To ensure that at least one segment arrives in a packet we assume that $b(0) = 0$. Providing at least one job is present exactly one segment of one job is served during a fixed time slot of length Δ. At the end of the time slot the job leaves the system if it is completed while it remains in the line otherwise. Assume that $\rho E(B) < 1$ where $\rho = \lambda\Delta$ thus ensuring stability of the queue.

We are interested in steady state quantities such as the probabilities and mean number of segments in the queue, In principle the steady state distribution of this number can be given in recursive expressions. These, however, involve infinite series of k-fold convolutions of the distribution of B, thus making computation most expensive.

We therefore use (as proposed in [8]) a continuous time processor sharing analogue leading to an exact explicit expression for the job and segment distribution which is much easier to compute. In this processor sharing version with probability $b(\ell)$ a job consists of ℓ successive exponential phases, each with parameter $\nu = 1/\Delta$ (i.e. an Erlang distribution of ℓ phases). Furthermore, when n jobs are present each job receives an equal amount of service $1/n$ per unit time. Let $(k_1, \ldots, k_n)$ denote the state with n jobs present with k_i segments still to be served for the job at the ith place in the line. Then, by standard partial balance arguments, or by direct substitution in the global balance equations, one can conclude that the steady state distribution $\overline{\pi}(.)$ is given by

$$\overline{\pi}((k_1 \ldots, k_n) = (1 - \rho E(B))\rho^n \prod_{i=1}^{n} r(k_i) \tag{1}$$

where

$$r(k) = \sum_{\ell=k}^{\infty} b(\ell). \tag{2}$$

Clearly the steady probabilities $\overline{\pi}(k)$ and the mean $\overline{L}$ for the total number of segments are readily obtained.

These are also the steady state probabilities for the discrete time Markov chain which is obtained by uniformisation (see Tijms [14]) and whose transition probabilities are given in equation (5) below. The major difference between this uniformised Markov chain and the original discrete time Markov chain is that multiple arrivals can occur in the latter but not in the former. In the next section we derive bounds on the error introduced when performance measures of the discrete time Markov chain are approximated by performance measures based on the equilibrium distibution (1). Thus with $\pi(.)$ and L denoting the corresponding quantities for the original discrete time queue we are interested in providing error bounds on differences such as $]\pi(.) - \overline{\pi}(.)]$ and $]L - \overline{L}]$.

3. Error Bounds

Throughout this section we denote an expression for the processor sharing model with an upper bar symbol while no symbol is used for the original discrete time queue. An expression with the symbol $\hat{}$ is to be read as applying to both models.

Let $f(.)$ be a function from Z_+ to R and N be the random variable denoting the total number of segments in the queue. We wish to compare performance measures of the form

$$\begin{aligned} g &= E(f(N)) \quad \text{and} \\ \overline{g} &= \overline{E}(f(N)) \end{aligned} \tag{3}$$

where E is the expected value with respect to the stationary distribution of the discrete time queue and $\overline{E}$ is the expected value calculated with respect to $\overline{\pi}(.)$ the stationary distribution of the processor sharing queue.

The one step transition probabilities of the discrete time model are given by

$$p(i,j) = \begin{cases} \sum_{k=1}^{\ell} \frac{e^{-\rho}\rho^k b^{k*}(\ell)}{k!} & j = i + \ell - I(i > 0) \\ e^{-\rho} & j = i - I(i > 0) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $b^{k*}(\ell)$ denotes the probability mass function of the kth convolution of the batch size mass function $b(\ell)$. Now define one-step transition probabilities $\overline{p}(i,j)$ for the Markov chain which results from applying the standard uniformisation technique (see Tijms, [14], p110) to the processor sharing model as

$$\overline{p}(i,j) = \begin{cases} \frac{\rho b(\ell)}{1+\rho} & j = i + \ell \\ \frac{1}{1+\rho} & j = i - I(i > 0) \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Further define $\widehat{V}_n(.)$ by

$$\widehat{V}_{n+1}(i) = f(i) + \sum_j \widehat{p}(i,j)\widehat{V}_n(j) \tag{6}$$

with $\widehat{V}_0(i) = 0 \ \forall \ i \geq 0$. Standard Markov reward limit arguments (see Tijms [14]) yield

$$\widehat{g} = \lim_{n \to \infty} \frac{\widehat{V}_n}{n}. \tag{7}$$

We aim to provide an error bound on $|\overline{g} - g|$ for different functions $f(.)$, i.e. for different performance measures g. The crucial step herein is the estimation of bounds on $\overline{V}_n(i+1) - \overline{V}_n(i)$ uniformly on n. These are established in the following lemma.

Lemma

Assume $f(.)$ is an arbitrary non-decreasing function and that there exists a positive constant M such that

$$0 \leq f(i+1) - f(i) \leq M \ \forall \ i \geq 0. \tag{8}$$

Then for all $i \geq 0$ and $n \geq 0$,

$$0 \leq \overline{V}_n(i+1) - \overline{V}_n(i) \leq \frac{(i+1)M(\rho+1)}{1-\rho E(B)}. \tag{9}$$

Proof

We employ induction on n. Clearly (9) holds for $n = 0$ as $V_0(i) = 0 \; \forall \; i \geq 0$. Suppose that (9) holds for all $n \leq m$. Then, by (6) and writing $h = 1/(1+\rho)$,

$$\overline{V}_{m+1}(i+1) - \overline{V}_{m+1}(i) = [f(i+1) - f(i)] + h\rho \sum_{\ell=0}^{\infty} b(\ell) \left[\overline{V}_m(i+\ell+1) - \overline{V}_m(i+\ell) \right]$$
$$+ hI(i>0) \left[\overline{V}_m(i) - \overline{V}_m(i-1) \right].$$

Substitution of the lower estimates 0 from (8) and (9) for $n = m$ gives $\overline{V}_{m+1}(i+1) - \overline{V}_{m+1}(i) \geq 0$. An upper estimate $\overline{V}_{m+1}(i+1) - \overline{V}_{m+1}(i) \leq (i+1)C$ follows similarly by substituting the upper bounds from (8) and (9) for $n = m$ and observing that

$$M + h\rho \sum_{\ell=0}^{\infty} b(\ell)(i+l+1)C + hI(i>0)iC =$$
$$[M + h\rho E(B)C] + h(\rho+1)(i+1)C - hC.$$

If C is chosen to be $M/(h(1-\rho E(B)))$ then this is equal to $(i+1)C$ and the proof is complete.

Theorem 1

Assume that there exists U such that $\sum_{j=0}^{\infty} p^r(0,j)j < U$ uniformly in r, where $p^r(i,j)$ is the (i,j)th entry of the rth power of the transition matrix, and that $f(.)$ satisfies the conditions of the Lemma. Then

$$|\overline{g} - g| \leq UC_1 + C_2 \tag{10}$$

where

$$C_1 = C\left[\frac{\rho^2}{2} + \rho e^{-\rho} + E(B)\left[\frac{3\rho^3}{2} + 2\rho^2\right]\right] \tag{11}$$

and

$$C_2 = C\left[\rho e^{-\rho} + [E(B) + E(B^2)]\left[\frac{3\rho^3}{4}\right] + E(B)^2\left[4 + 4\rho + 3\rho^2 + \frac{\rho^3}{2}\right]\right] \tag{12}$$

with

$$C = \frac{M}{h(1-\rho E(B)}.$$

Proof

From (6) we obtain for any $i \in Z_+$

$$|\overline{V}_{n+1}(i) - V_{n+1}(i)| = |\sum_{j=0}^{\infty} \overline{p}(i,j)\overline{V}_n(j) - \sum_{j=0}^{\infty} p(i,j)V_n(j)|$$
$$= |\sum_{j=0}^{\infty} [[\,\overline{p}(i,j) - p(i,j)]\,\overline{V}_n(j) + p(i,j)\,[\,\overline{V}_n(j) - V_n(j)]]\,|$$
$$\leq |\sum_{j=0}^{\infty} [\,\overline{p}(i,j) - p(i,j)]\,\overline{V}_n(j)| + |\sum_{j=0}^{\infty} p(i,j)\,[\,\overline{V}_n(j) - V_n(j)]\,|. \tag{13}$$

Now, by using equations (4), (5) and (9), and noting that $\sum_{j=0}^{\infty} p(i,j) = \sum_{j=0}^{\infty} \overline{p}(i,j) = 1$ we have

$$|\sum_{j=0}^{\infty} [\,\overline{p}(i,j) - p(i,j)]\,\overline{V}_n(j)|$$
$$= |\sum_{j=0}^{\infty} [\,\overline{p}(i,j) - p(i,j)]\,[\,\overline{V}_n(j) - \overline{V}_n(i)]\,|$$
$$\leq I(i=0)A|\overline{V}_n(i) - \overline{V}_n(i)| + I(i>0)A|\overline{V}_n(i-1) - \overline{V}_n(i)|$$
$$+ I(i=0)A\rho|\sum_{\ell=0}^{\infty} [\,\overline{V}_n(i+\ell) - \overline{V}_n(i)]\,b(\ell)|$$
$$+ I(i>0)|\rho e^{-\rho}\sum_{\ell=0}^{\infty} [\,\overline{V}_n(i+\ell-1) - \overline{V}_n(i)]\,b(\ell)$$
$$- \frac{\rho}{1+\rho}\sum_{\ell=0}^{\infty} [\,\overline{V}_n(i+\ell) - \overline{V}_n(i)]\,b(\ell)|$$
$$+ \sum_{k=2}^{\infty} \frac{\rho^k e^{-\rho}}{k!}\,[I(i=0)|\,[\,\overline{V}_n(i+\ell) - \overline{V}_n(i)]\,b^{k*}(\ell)|$$
$$+ I(i>0)|\,[\,\overline{V}_n(i+\ell-1) - \overline{V}_n(i)]\,b^{k*}(\ell)|]$$

where $A = |e^{-\rho} - 1/(1+\rho)| < \rho^2/2$. This expression is in turn bounded above by

$$I(i=0)A\rho\sum_{\ell=0}^{\infty}|\overline{V}_n(i+\ell)-\overline{V}_n(i)|b(\ell)$$
$$+\, I(i>0)\left[A|\overline{V}_n(i-1)-\overline{V}_n(i)| + \rho e^{-\rho}\sum_{\ell=0}^{\infty}|\overline{V}_n(i+\ell-1)-\overline{V}_n(i+\ell)|b(\ell)\right.$$
$$\left. +\, A\rho\sum_{\ell=0}^{\infty}|\overline{V}_n(i+\ell)-\overline{V}_n(i)|b(\ell)\right]$$
$$+\sum_{k=2}^{\infty}\frac{\rho^k e^{-\rho}}{k!}\left[I(i=0)|\left[\,\overline{V}_n(i+\ell)-\overline{V}_n(i)\right]b^{k*}(\ell)|\right.$$
$$\left. +\, I(i>0)|\left[\,\overline{V}_n(i+\ell-1)-\overline{V}_n(i)\right]b^{k*}(\ell)|\right].$$

Now, using the Lemma with $C = M/(h(1-\rho E(B)))$, and the fact that $A < \rho^2/2$, we bound this expression by

$$\frac{\rho^2}{2}iC + I(i>0)\rho e^{-\rho}\sum_{\ell=0}^{\infty}b(\ell)\left[(i+\ell)C\right]$$
$$+\frac{\rho^3}{2}\left[I(i=0)+I(i>0)\right]\sum_{\ell=0}^{\infty}b(\ell)\sum_{j=1}^{\ell}(i+j)C$$
$$+\sum_{k=2}^{\infty}\frac{\rho^k}{k!}e^{-\rho}\sum_{\ell=0}^{\infty}b^{k*}(\ell)\sum_{j=1}^{\ell}(i+j)C$$
$$\le \frac{\rho^2}{2}iC + \rho e^{-\rho}\left[i+E(B)\right]C$$
$$+\frac{\rho^3}{2}\sum_{\ell=0}^{\infty}b(\ell)\left[\frac{\ell}{2}(2i+l+1)\right]C + \sum_{k=2}^{\infty}\frac{\rho^k}{k!}e^{-\rho}\sum_{\ell=0}^{\infty}b^{k*}(\ell)\left[\frac{\ell}{2}(2i+l+1)\right]C$$
$$\le \frac{\rho^2}{2}iC + \rho e^{-\rho}\left[i+E(B)\right]C + \frac{\rho^3}{4}\left[(2i+1)E(B)+E(B^2)\right]C$$
$$+\sum_{k=2}^{\infty}\frac{\rho^k}{2k!}e^{-\rho}\left[(2i+1)kE(B)+k\left[E(B^2)-E(B)^2\right]+k^2E(B)^2\right]C$$
$$\le \frac{\rho^2}{2}iC + \rho e^{-\rho}\left[i+E(B)\right]C + \frac{\rho^3}{4}\left[(2i+1)E(B)+E(B^2)\right]C$$
$$+\frac{\rho^2\left[2+\rho\right]}{2}\left[\left[(2i+1)E(B)+E(B^2)-E(B)^2\right]\right.$$
$$\left. +\, E(B)^2\left[4+4\rho+2\rho^2\right]\right]C$$

where, in the second last inequality, we used the fact that the expected value of the square of the sum of k independent random variables, each distributed identically to B is

$$k\left[E(B^2)-E(B)^2\right]+k^2E(B)^2,$$

and in the last inequality we used the fact that

$$\begin{aligned}\sum_{k=2}^{\infty}\frac{\rho^k}{k!}e^{-\rho}k^t &< \rho^2\sum_{k=0}^{\infty}\frac{\rho^k}{k!}e^{-\rho}(k+2)^t \\ &= \begin{cases}\rho^2\left[2+\rho\right] & \text{if } t=1 \\ \rho^2\left[4+4\rho+2\rho^2\right] & \text{if } t=2.\end{cases}\end{aligned}$$

as a consequence, by collecting terms and using equations (11), (12) and (13), we get

$$|\overline{V}_r(i)-V_r(i)| \leq iC_1+C_2+\sum_{j=0}^{\infty}p(i,j)|\overline{V}_{r-1}(j)-V_{r-1}(j)|. \tag{14}$$

Repeating this equation for $r=0,\ldots,n-1$, and noting that $\overline{V}_0(.)=V_0(.)$, we obtain

$$|\overline{V}_n(i)-V_n(i)| \leq \sum_{r=0}^{n-1}\sum_{j=0}^{\infty}p^{r*}(i,j)\left[jC_1+C_2\right]. \tag{15}$$

Choosing $i=0$ in (15) and recalling the definition of U we obtain

$$|\overline{V}_n(0)-V_n(0)| \leq n\left[UC_1+C_2\right] \tag{16}$$

Applying (7) completes the proof.

4. An Application

Economou [5] discusses a model of a packet switching network in which segmentised packets arrive in a Poisson stream with parameter λ to a server who processes one segment per time slot of length Δ. Incoming packets are stored in a queueing buffer and served in a round robin fashion. Processed segments are stored in a reassembly buffer until the whole packet has been processed at which time all segments in the packet are released for further processing. In this model the parameters of interest are the total number of segments, as opposed to packets, in both the queueing and reassembly buffers.

The process describing the number of segments in the queueing buffer, measured at intervals of length Δ sufficient to process a single segment, is an example of the discrete time queueing system described in Section 2. The process describing number of segments in the reassembly buffer is more complicated and cannot be modelled by a Markov process, carrying only that number in the state description. However, both processes can be approximated by a processor sharing queue (see Economou and Taylor [6]). For the number in the queueing buffer we can use Theorem 1 to derive an upper bound on the errors in several performance measures introduced by this processor sharing approximation. Below we do this for the steady state distribution and also the mean number of segments in the queueing buffer.

The crucial step in using Theorem 1 is to find a suitable uniform upper bound U on the mean number of segments in the queue at time r conditional on the queue starting initially in the empty state. Using similar arguments to those in van Dijk [15], however, it can be shown that $\sum_j^\infty p^r(0,j)j$ converges monotone non-decreasing in r to L as $r \to \infty$ so that U can be taken equal to L.

To bound L we then use Theorem 1 with $f(i) = i$ and $M = 1$. This gives $g = L$ and $\overline{g} = \overline{L}$, and so applying Theorem 1 we get

$$|L - \overline{L}| \leq LC_1 + C_2, \tag{17}$$

where C_1 and C_2 are given by equations (11) and (12). Provided $C_1 < 1$ equation (17) yields

$$\frac{\overline{L} - C_2}{1 + C_1} \leq L \leq \frac{\overline{L} + C_2}{1 - C_1} \tag{18}$$

as bounds for L.

To get bounds on the steady state distribution of N fix $k \in Z_+$ and take $f(i) = I(i \geq k)$ with $M = 1$. Then g and $\overline{g}$ are equal to $\sum_{\ell=k}^\infty \pi(\ell)$ and $\sum_{\ell=k}^\infty \overline{\pi}(\ell)$ respectively. Thus, on account of (18)

$$|g - \overline{g}| \leq \frac{\overline{L} + C_2}{1 - C_1} C_1 + C_2. \tag{19}$$

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