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An Interactive Procedure to Assess Value Functions for Environmental Quality

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An Interactive Procedure to Assess Value Functions for Environmental Quality

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In this paper a new procedure to assess value functions for environmental quality is presented. It has being particulary designed for eliciting expert-based value functions for pollutant substances. The main features of the model are the possibility to integrate direct and indirect assessment techniques, the possibility to make explicit assessment uncertainties and the ability to avoid forced quantitative judgements. The main aim of the procedure is to improve the reliability of the assessment but also to make the expert's task easier by using qualitative judgements more frequently. The procedure is here presented with an application for soil pollution.

1. Introduction

Among the many available multicriteria techniques [see Nijkamp and Rietveld (1986), Nijkamp et al. (1990), Bogetoft and Pruzan (1991)] the value function approach is still one of the most common and well known methods. These functions are a mathematical representation of the decision-maker's preferences and are used to rank multicriteria alternatives. The technique is based on a strong theoretical framework [Keeney and Raiffa (1976),French (1988)] and many methods to assess value functions have been used and tested . For practical reasons the additive representation of value functions is the most frequently used technique and two main assessment strategies are available: direct and indirect assessment [see Zeleny (1982) for a broader discussion on these approaches].

Direct assessment implies stating unidimensional value functions and a combination of weights to aggregate them leading to a multidimensional value model. Unidimensional value functions are evaluated by stating the value of some criterion scores, provided numerical values for reference scores are defined. These techniques are widely known and many assessment procedures are available; the reader is referred to Fishburn (1967) and von Winterfeld and Edwards (1986) for a taxonomy of methods.

Indirect techniques reverse this approach: given a ranking among a set of alternatives and assuming an additive decision model (an underlying decision model, in general) a regression or other optimization technique calculates the set of value functions which better represent the order according to some measure of optimum fitting. To this approach belong, among others, the so-called UTA and PREFCALC methods [Jacquet-Lagreze (1990), Jacquet-Lagreze and Siskos (1982)] and, with some extensions, the HOPIE technique [Weber (1985)].

When using these methods for assessing value functions for pollutant substances, some serious problems arise. In direct assessment a major point is the great difficulty encountered in assessing values for contaminant concentrations. The experts are usually toxicologists, biologists, chemists etc. used to the notion of dose-effect functions. These functions state a relationship between a pollutant concentration and a precise effect for specific species, provided models of absorbtion are available [Aiking et al. (1989)]. A large number of effect data and complex models of absorbtion are needed to estimate dose-effect functions, which explains why they are not available for a large part of known industrial substances. Value functions, on the other hand, are often regarded as rough and imprecise tools and consequently experts feel very uncomfortable in assessing single values [see Brown et al. (1972) for this "psychological barrier"]. The attitude towards this task can vary from an almost complete refusal to state any quantitative measure of value to an uncertain attempt to assess some basic values for well known situations (concentrations). In general, the shape of the function can be guessed in advance because of the behaviour of the pollutant; features such as monotonicity, existence of thresholds etc. are usually known but seldom the analysis goes further with direct assessments.

With indirect assessment techniques a different problem arises. The basic information needed is a ranking among a set of real or artificial alternatives. If the model is based on n evaluation criteria, then we need to order a set of n-dimensional alternatives. For example, in the case of soil pollution, we can imagine a number of sites polluted with n contaminants in various combinations of concentrations. A rank based on remediation urgency, for instance, provides the necessary information for an indirect assessment. If the number of substances is high, the experts face the problem of comparing multidimensional alternatives: the higher the number of criteria (pollutants), the more difficult the task. As a result, the experts feel that such approaches are providing very crude and unreliable responses which are also far away from their own expertise. It is clear that the lower the dimension of alternatives, the easier and reliable the responses. For practical reasons, therefore, it is advisable to avoid comparisons among complex multidimensional alternatives and to stick to simple cases, for example bi-criteria alternatives, to make the ordering task sound in the expert's view.

A final remark concerns the connection between direct and indirect assessment: they are often seen as alternative ways of stating someone's preferences, although in an integrated procedure they should lead to a more reliable assessments. On a practical base this need is clearly highlighted: firstly, there is a need to connect the assessment phase with the decision phase to make experts aware of the possible selections based on the functions they provide. Secondly, there is the need to maximise the expert's contribution by widening the assessment boundary including indirect assessments, leading to an increased reliability of outcomes.

Related to the first point is the fact that the experts are not the decision makers; they just provide information which is not otherwise available. Although they may feel concerned with the outcomes of a decision based on the information they provide, they do not have a precise idea on the decision that could be made according to their assessments. Even assuming that they are accustomed to multicriteria value functions, by using direct assessment techniques they do not have the opportunity to link future decisions to current elicitations. As concerns the second point, it can be observed that experts tend to stick to a particular definition of value by considering it in a strictly toxicological or biological sense; in indirect assessments they partly have to abandon this requirement since trading off among alternatives means considering them in a composite fashion and not only according to a very specific point of view; in other words, they are driven towards responses based on values.

All these remarks make clear the need to support the assessment of expert based value functions with some specific tools. The assessment procedure should take into account the difficulties in assessing direct values by avoiding forcing expert responses; it should be based on an integrated direct-indirect assessment procedure while the indirect phase should be based on ranks of simple alternatives. Moreover, the elicitation process should have an interactive character by providing the analysis with indications of which kind of elicitations need to be checked or refined.

2. A Procedure to Assess Value Functions for Environmental Quality

Let $X_1,...,X_n$ be the evaluation criteria (pollutants) and $R_i=[x_i^*,x_{i*}]$, $\forall i=1,..,n$, the criterion ranges such that $x_i \varepsilon R_i$, $\forall i=1,..,n$.

The model is based on the assumption of an additive value function structure. Therefore, let v_i: $[x_i^*, x_{i*}] \rightarrow [100,0]$, $\forall i=1,...,n$, be the monotonically decreasing unidimensional value functions and w_i, $\forall i=1,...,n$, the weight of the i-th criterion scaled to add up to one: $w_1+...+w_n=1$. An n-dimensional alternative is a combination of n criterion scores $(x_1,...,x_n)$, $x_i \varepsilon R_i$, while the overall value is the weighted sum of the unidimensional values:

$$v(x_1,...,x_n)=\sum_{i=1}^{n} w_i \cdot v_i(x_i) \qquad (1)$$

The structure of the assessment procedure is shown in Fig. 1; the blocks correspond to the steps the method is based upon. Some of these are based on question-answer sessions between analyst and expert and others are computational steps. Apart from block A, which aims to provide the necessary information before starting the assessment, the procedure starts eliciting three kinds of information: unidimensional value functions (Block B1), criterion weights (Block B2) and rankings on sets of alternatives (Block B3). A linear regression provides the most consistent set of functions and weights according to all information collected (Block C). The results are confronted with initial elicitations (Block D); if they are satisfactory, the procedure ends while, if refinements are necessary, the procedure starts again on a more precise data base (Block E). The next sections explains the modules in greater detail.

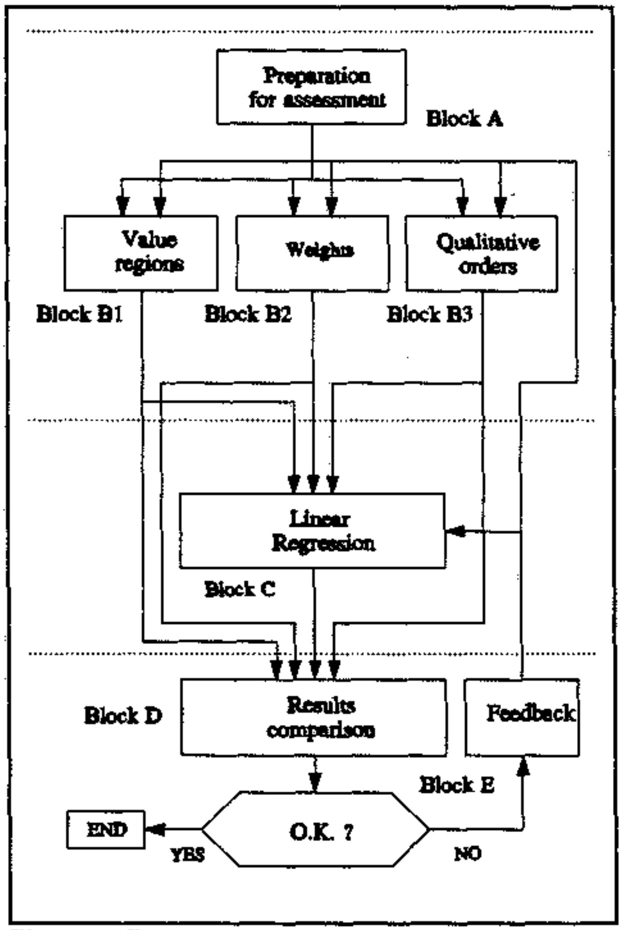

Figure 1: Flow chart of the assessment procedure.

3. Unidimensional Value Function Assessment

As stated before, experts feel uncomfortable in assessing single values with direct assessment techniques. Better results can be obtained by making uncertainty explicit and avoiding forcing responses. Instead of assessing single values, experts are asked to state in which value interval the real value is likely to be included. Basing the analysis on some reference criterion scores[1], they are asked to assess ranges of reliable values. This leads to a definition of "region of values" which can be interpreted as an uncertainty effect but also as an attempt to consider simultaneously various curves for various species and effects. The region of values can be rather broad but this seems to be more reliable and easier to assess than a single curve: we can assume that the real curve falls within the extreme curves v_{i*} and v_i^* (Fig. 2).

An empirical example is shown in Fig. 3: the value intervals for three reference concentrations (B, C and S1) and

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[1] Dutch Legislation [Soil Clean-up Guideline (1983)] provides a list of common pollutants and some reference concentrations to evaluate the state of polluted sites. Since the effects for these concentrations are rather well known, they can be used as reference points for the assessment.

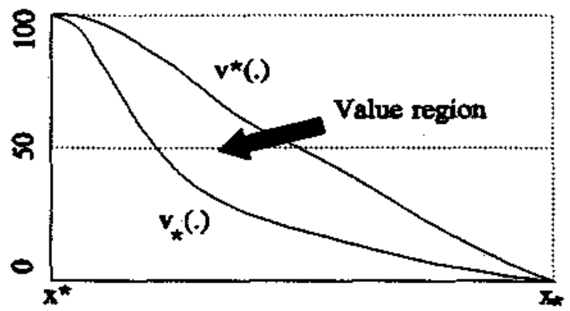

Figure 2: Example of value region and extreme curves.

the extreme curves are reported for lead pollution in soil. The first elicitation is a preliminary result, which can usually be polished and refined. At the very beginning experts tend to provide very safe elicitations leading to very large value regions. By checking some values it is often possible to reduce the region of values to narrower limits. Fig. 4 shows how the value region in Fig. 3 has been reduced: the dashed line at the top is the first elicitation. As it can be seen, the value v=50 could correspond to a set of scores between x_a and x_c. In expert's opinion the x_c concentration was too high and a new value x_b was provided. In a similar fashion other values have been checked leading to the final region of values.

The aim of block B1 is that of assessing n value regions; this phase provides only little information on the real value functions but fixes some robust boundaries for the final curves. Accepting such an incomplete information from the experts clearly makes the assessment easier. Sometimes this is the only way to obtain judgements and, in general, the elicitation is regarded as a more reasonable task compared to single value assessments.

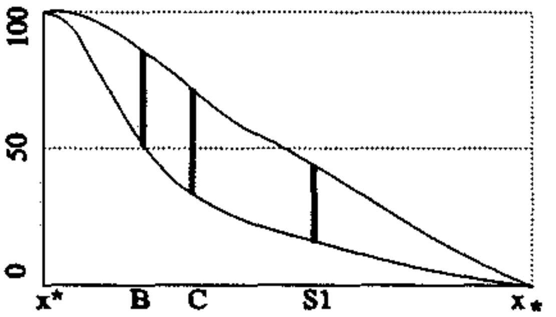

Figure 3. Lead initial value region.

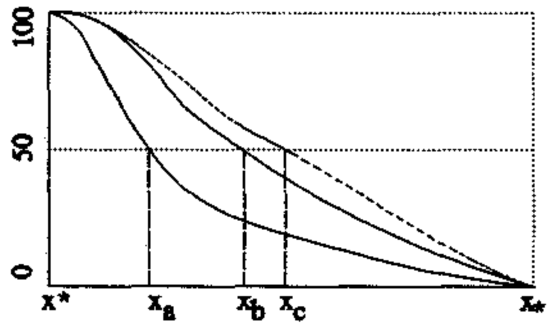

Figure 4. Lead final value region.

4. Weight assessment

In order to build a multidimensional value function model a set of weights is needed. Among the various techniques to assess weights [see von Winterfeld and Edwards (1986), Janssen (1991)] the swing technique proves useful and easy to understand [name according to von Winterfeld and Edwards (1986)]. In Fig. 5 a four criteria example is shown for the case of soil pollution: the numbers displayed are the high (x_i^*) and low (x_{i*}) range limits. By swinging one criterion at a time from the worst level to the best level we design four extreme alternatives. Ranking these alternatives implicitly means to assess a weight order; it is also possible to use the technique to assess numerical estimation of weights by

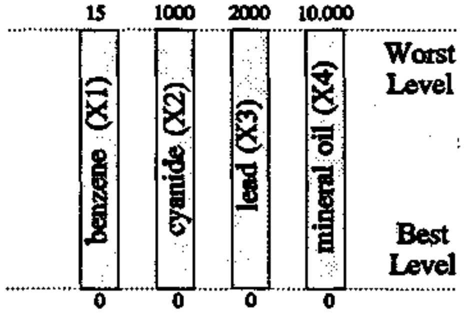

Figure 5: Swing technique for assessing weights: four criteria example.

scoring the achievement of every step but in this procedure it is used only for ranks.

It is always useful to provide some practical cues for the assessment: in this case the expert has been given the possibility to clean one substance at the time from a soil in which the highest pollutant concentrations (x_{i*}) were detected. The order depends on the intrinsic risk of the pollutant and on the range of concentrations taken in account.

The result is a weight order: it can be a simple rank order, but also indifference information, incomparability or strength of preference can be elicited. In general, given n criteria, they can be grouped into p weight classes, $C_1,C_2,...,C_p$, where every class can be an indifference or incomparability[2] class and between every class a preference relation can be established. In the case of Fig. 5 the order assessed was $w_1>w_2\approx w_3>w_4$ which corresponds to three classes $C_1=\{w_1\}$, $C_2=\{w_2,w_3\}$ and $C_3=\{w_4\}$, where $C_1 > C_2 > C_3$.

Due to the simplicity of the approach and due to the strict connection between weights and ranges, this step of the assessment does not present particular difficulties and the results are usually very reliable compared to other elicitations of the expert.

5. Qualitative Orders

Qualitative orders represent the part of assessment related to the indirect approach. As mentioned above, the indirect approach is based on the information given by a subjective ranking on a set of alternatives (real or artificial). The alternatives used in this procedure are simple bicriteria artificial alternatives (2c-alternatives): a 2c-alternative on X_i and X_j is defined as:

$$(x_i,x_j)_{2c} = (x_1^*,...,x_{i-1}^*,x_i,x_{i+1}^*,...,x_{j-1}^*,x_j,x_{j+1}^*,...,x_n^*);\ x_i \varepsilon R_i,\ x_j \varepsilon R_j \qquad (2)$$

It is an n dimensional vector in which all scores but two take on the best level. Given a set of two criteria alternatives, the experts are asked to provide a rank order among these simple alternatives; an example is shown in Fig. 6 for the criteria X_i and X_j; to every point corresponds a score combination, i.e., a 2c-alternative. As it will be shown in the next section, the linear regression is based on 2c-alternatives rank orders and the outcomes are the value estimations of the points composing 2c-alternatives; in Fig. 6, for example, the values $v_i(x_{i,k})$ and $v_j(x_{j,k})$, k=1,2,3.

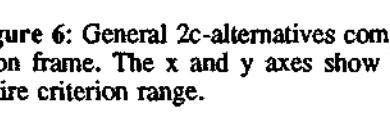

Figure 6: General 2c-alternatives comparison frame. The x and y axes show the entire criterion range.

The procedure, therefore, requires the selection of the pairs of criteria on which to base the 2c-alternatives and the points to generate these alternatives.

As concerns the pairs of criteria, the frame in Fig. 6 contains information on X_i and X_j only. With n criteria we would need at least n-1 of these comparison frames, each based on different pairs of criteria. The basic requirement is to provide each criterion with the same amount of information and to link all single criterion information to an n-dimensional context. In fact, by using 2c-alternatives instead of n-dimensional alternatives the regression provides information only on the pair of criteria involved and not on the complete set. Fig. 7 shows one way to link the various pairs in an n-dimensional context. For every pair of connected criteria a set of

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[2] Incomparability means that the expert is not able to assess reliable weight differences and prefers to defer the judgement.

2c-alternatives can be designed (and subsequently ordered). The set of pairs of criteria in Fig. 7 is defined as a Pairwise Group (PWG). In this case:

$$PWG = \{(X_1,X_2), (X_2,X_3), (X_3,X_4),\dots,(X_{n-1},X_n), (X_n,X_1)\} \tag{3}$$

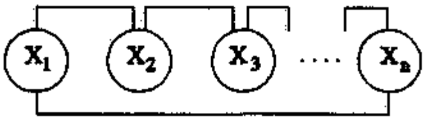

Figure 7: Connection of pairs of criteria for qualitative orders.

With n criteria (n>2) many different pairwise groups are possible. For reasons of symmetry, every possible PWG consists in n pairs of criteria and every criterion is used twice with two different partner criteria. Apart from that, the particular PWG we choose for the assessment is basically a matter of practical convenience and the reader is referred to Beinat (1992) for more details.

Besides that it is necessary to select which points to use as coordinates for 2c-alternatives. In Fig. 6 for every criterion three points have been selected: a different number of points could be selected although this is a reasonable compromise between the advantage of having a numerous set of alternatives and the length and difficulty of the ordering task.

The selection of the points is based on the need to distribute the estimations on both the value and score axis. With three points for every criterion, for example, a possible solution is the selection of three scores which partition the corresponding value region in three equal areas. In the following exposition we assume that for each criterion three points have been selected as 2c-alternative coordinates; in general, n_i points can be selected for the criterion X_i. They define n sets marked with SP_i: $SP_i=\{x_{i.1},x_{i.2},\dots,x_{i.ni}\}$, $\forall i=1,\dots,n$. For every pair of criteria $(X_i,X_j)\varepsilon PWG$ a set of 2c-alternatives, SA_{ij}, is defined: $SA_{ij}=\{\forall x_t \varepsilon SP_i, \forall x_s \varepsilon SP_j, (x_t,x_s)_{2c}\}$ and the output of Block B3 is a rank order of these alternatives.

6. Linear Regression

The set of data available at this point consists of n value regions, a weight order and n rank orders of 2c-alternatives. The aim of the linear regression is to calculate the set of weights and the set of value functions (point estimations) which are most consistent with all data collected; the regression is performed by a linear programme optimization procedure (LP). The objective function of the LP model is the sum of three kinds of errors (to be minimized): the deviation of value functions from value regions (er), the departure of weights from weight order (ew) and the departure of computed orders from assessed orders[3] (eo). The basic formulation of the linear optimization model[4] is shown in (4). The constants δ_w and δ_o are thresholds variables which can be used to stress a minimum difference between weights or qualitative orders. This model can be further extended to take into consideration more detailed information such as indifference among weights, indifference among 2c-alternatives, strength of preference in weight order, and strength of preference for 2c-alternatives orders [Beinat (1992)].

The L constants included in the objective function have a twofold function: firstly, they are used to rescale errors on weights (measured in a 0-1 range) and errors on values (measured

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[3] The assessed orders are those assessed by the expert, the computed orders are those calculated with value functions and weights.

[4] For the sake of the notation the set of order constraints is presented as non linear; to linearize the model a set of normalized variables is introduced. They are defined as $z_i(.)=w_i \cdot v_i(.)$.

in a 0-100 range). Secondly, they serve to modulate results according to the reliability of elicitations. The pieces of information collected in blocks B1, B2 and B3 may have a different reliability, while usually the experts feel more confident in some part of the elicitations and less in others. By increasing one particular L_i, the LP model tends to reduce the corresponding errors with higher priority and this feature can be used to modulate the LP solution according to data reliability. We now have the following model:

$$\text{Min} \ \left(L_1 \cdot \sum_{\forall (X_i,X_j) \varepsilon PWG} eo_{ij} + L_2 \cdot \sum_{\substack{w_i \varepsilon C_k \\ w_j \varepsilon C_r \\ r>k}} ew_{ij} + L_3 \cdot \sum_{i=1}^{n} er_i \right) \tag{4}$$

subject to:

(value functions and weights respect the order of alternatives)

$\forall (X_i,X_j) \varepsilon PWG,\ \forall (x_k,x_h)_{2c},\ (x_t,x_s)_{2c}\ \varepsilon\ SA_{ij}$ such that $(x_k,x_h)_{2c} > (x_t,x_s)_{2c}$:
$[w_i \cdot v_i(x_k) + w_j \cdot v_j(x_h)] - [w_i \cdot v_i(x_t) + w_j \cdot v_j(x_s)] > \delta_o - eo_{ij}$

(weights respect weight order)

$\forall w_i \varepsilon C_k,\ \forall w_j \varepsilon C_r$ such that $w_i > w_j$: $w_i - w_j > \delta_w - ew_{ij}$

(point estimations of value functions fall within value regions)

$\forall i=1,\dots,n,\ \forall x_t \varepsilon SP_i : v_{i*}(x_t) - er_i < v_i(x_t) < v_i^*(x_t) + er_i$

(monotonicity and scaling constraints)

$\forall i=1,\dots n;\ \forall x_t,x_{t+1} \varepsilon SP_i$ such that $x_t < x_{t+1}$: $v_i(x_t) > v_i(x_{t+1})$
$w_1 + w_2 + \dots + w_n = 1$

The number of constraints of the LP model can be considerable[5] and large LP models are rather common in practical applications; this is not a severe problem since commercial LP software can easily handle several hundreds of constraints and variables, which covers almost all practical needs.

To summarize, the outcomes of the regression are the point estimations of value functions ($\forall i=1,\dots,n$ and $\forall x_t \varepsilon SP_i$: $v_i(x_t)$), a set of weights and a set of errors on weights, value regions and orders. The complete value functions can be obtained via interpolation of point estimations.

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[5] Due to the monotonicity assumption, some of the order constraints for 2c-alternatives can be dropped as they are automatically satisfied. The number of constraints in the LP, therefore, depends on the particular rank order assessed; empirical tests show that the total number of constraints of the LP model is about 20 to 25 times the number of criteria.

The regression errors can be seen as a measure of consistency of elicitations: if they are zero, the LP solution is consistent with all kind of elicitations, whereas some positive errors indicate some inconsistency in the assessment. The amount and distribution of errors can be used in evaluating and refining the results, as will be shown in the next section.

7. Analysis of Results and Feedback

The complete set of results is next submitted to the experts for the final evaluation. If the results are not accepted, the original elicitations are updated and a new LP model is defined. This second round provides a new set of results and the same steps are repeated until a satisfactory set of value functions and weights is obtained.

The analysis of results consists in comparing the LP outcomes with the original elicitations. For the value functions the analysis concerns the whole curve or some representative parts, rather than the single values. The refinements, when needed, concern value regions rather than value functions and this phase may lead to a more refined set of value regions or to some new regions of values.

Similar conclusions can be drawn for 2c-alternative orders. The original orders are compared against the orders defined by value functions and weights and the refinements concern the orders rather than the overall value of alternatives.

Some more precise results seem to be possible for weight evaluation. Since the numerical estimation of weights can be closely related to danger of substances and criterion ranges, it is often possible to make some more precise statements on their numerical estimations. The refinements can lead to a more reliable weight order but also to a new numerical estimation of weights. This is not always easy and feasible but, while the interpretation of numerical results seems to be extremely difficult for value functions and orders, it seems to be relatively simple for weights.

In the refinement phase an important guide is offered by the regression errors. They are a measure of concordance between LP results and original elicitations and suggest which piece of information needs to be checked with higher priority, as this likely needs refinements. The refinement phase ends with a new set of assessments; on this base a new formulation of the LP problem is defined and the new optimization procedure provides a new set of results. In general terms, the results are refined step by step and every new regression phase is more and more constrained around the final results.

8. An Assessment Example for Soil Pollutants

This example concerns the assessment of value functions for four common soil contaminants: Benzene (X_1), Cyanide (X_2), Lead (X_3) and Mineral Oil (X_4), all measured in mg[contaminant] per kg[soil]. The test took place with an expert of soil contamination of the Dutch National Institute of Environmental Management (RIVM).

This assessment took two rounds: after the first, the results were considered almost satisfactory but some marginal refinements were required. This led to a new set of elicitations and a new LP model which provided the final results.

After an introductory phase and the range assessment for each criterion, the session started by assessing the value regions. In Fig. 8 the Cyanide value region is shown; similar results were obtained for the other three substances. Afterwards, a weight ranking was assessed by using the swing technique. Fig. 5 was used as a graphical aid and the swing order led to the following weight order: $w_1>w_2=w_3>w_4$, where the equal sign, at this stage of the assessment, has to be interpreted as an approximate equivalence.

The third set of elicitations concerned the rank orders for 2c-alternatives. The pairs of criteria selected were: $PWG=\{(X_1,X_2),(X_1,X_3),(X_3,X_4),(X_4,X_2)\}$; Fig. 9 shows the frame for X_1-

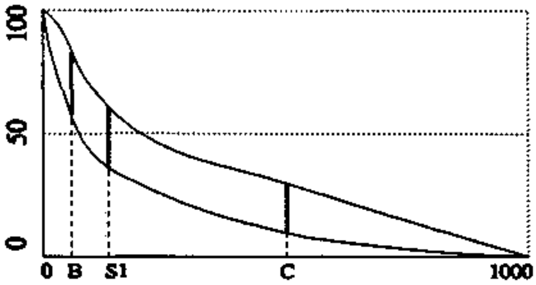

Figure 8: Cyanide value region.

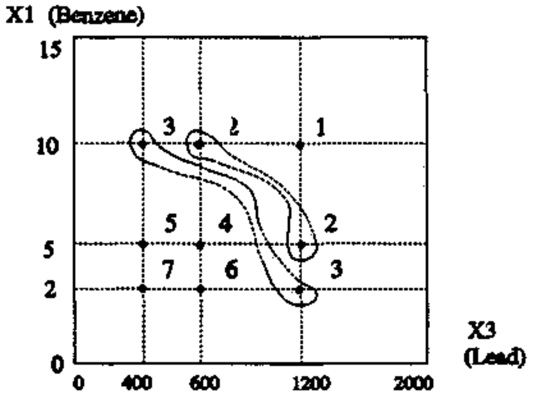

Figure 9: Set of alternatives for X_1-X_3: the numbers represent the urgency of cleaning-up ranking.

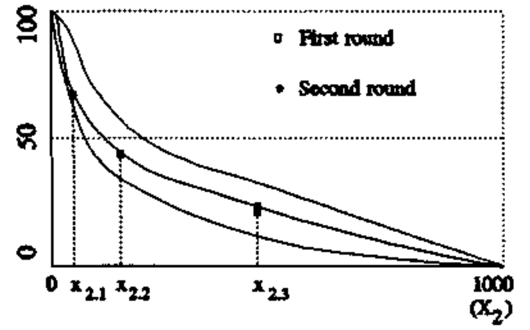

Figure 10: Cyanide value function: first and second round results.

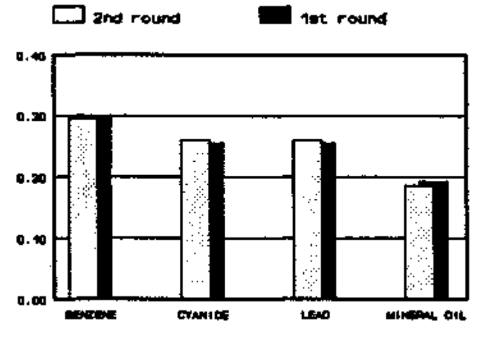

Figure 11: Weights after first and second round.

X_3. The expert was then asked to consider these alternatives as polluted soils to be cleaned and to provide a rank order based on cleaning-up urgency; the numbers shown in Fig. 9 represent the sanitation order. Three other frames of the same structure were used in the assessment and a total of 36 alternatives was ordered.

On this base of elicitation an LP model was built, which provided the first set of outcomes. As said, they required some small refinements which led to a second regression phase. In Fig. 10 the cyanide value function is shown after the first and second round; in Fig. 11 the weights are presented for the two rounds and in Fig. 12 the overall values for the alternatives of Fig. 9 are displayed.

As it can be seen, the second round led to very marginal changes. This is because the first set of results were already considered as very satisfactory by the expert, and the refinements concerned those 2c-alternatives which were ranked different but, after the first round, had an equal overall value; the expert asked then to emphasize a difference in value. This was obtained by imposing a δ_o=0.5 in the order constraints of the second round LP model.

The major difficulties in this assessment related to value region elicitation and value functions evaluation. In the expert's view, the reliability of this set of data has always been considered lower compared to the rest of the elicitations. It is also worth mentioning the expert's attitude towards the complete procedure: as far as results were provided and the link between choices and functions was made clear, the confidence in the procedure significantly increased making the assessment faster and of higher quality.

9. Conclusions

The assessment of expert-based value functions for environmental pollutants highlights the need of new elicitation procedures. The main inconveniences of traditional procedures concern the direct quantitative assessment of values and the lack of connection between the elicitation phase and the selection phase, where the value functions are used to discriminate among alternatives. The procedure presented in this paper

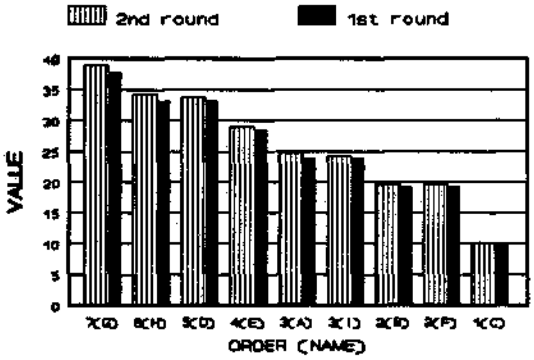

Figure 12: Overall values for X_1-X_3 alternatives. The numbers represent the rank order; the letters are the conventional name of the alternatives (Fig. 6).

provides tools to overcome these difficulties by extensively using qualitative data and by making explicit assessment uncertainties. The effect is that the expert's confidence in the assessment significantly increases and the results are very robust. It should be mentioned, however, that the assessment can be rather time consuming and effort taking: this can be seen as an unavoidable tradeoff between simplicity of assessment and reliability and robustness of results.

References

Aiking, H., H.A.J. Govers, R.W.Keulen, P.A.H. Hermens, R. Jeltes (1989). A general approach to environmental health impact assessment in The Netherlands, in: D.D.Hemphill ed., Trace substances in environmental health-XXII. University of Missouri, 98-108.

Beinat, E., (1992). An interactive procedure to assess value functions for environmental quality. Institute for Environmental Studies, Free University, Amsterdam.

Bogetoft, P., P. Pruzan (1991). Planning with multiple criteria. Amsterdam: North Holland.

Brown, R.M., N.I. McClelland, R.A. Deininger, M.F. O'Connor (1972). A water quality index : crashing the psychological barrier, in: W. Thomson, ed., Indicators of Environmental Quality. New York: Plenum Press, 173-182.

Chankong, V., Y.Y. Haimes (1983). Multiobjective decision making: theory and methodology. Amsterdam: North Holland.

Fishburn, P.C. (1967). Methods of estimating additive values. Management science. 13: 435-453.

French, S. (1988). Decision theory: an introduction to the mathematics of rationality. Chichester: Ellis Horwood.

Goicoechea, A., D.Hansen, L.Duckstein (1982). Multiobjective decision analysis with engineering and business applications. New York: John Wiley & Sons.

Jacquet-Lagreze, E. (1990). Interactive Assessment of preferences using holistic judgements: the Prefcalc system, in: Bana e Costa ed., Readings in Multiple Criteria Decision Aid. Berlin: Springer, 335-350.

Jacquet-Lagreze, E., J.Siskos (1982). Assessing a set of utility functions for multicriteria decision-making, the UTA method', European Journal of Operational Research. 10:151-164.

Janssen, R. (1991). Multicriteria Decision Support for Environmental Problems, Ph.D. dissertation, Free University of Amsterdam, The Netherlands.

Keeney, R.L., H.Raiffa (1976). Decision with multiple objective: preferences and value tradeoffs. New York:John Wiley & Sons.

Nijkamp, P., P. Rietveld (1986). Multiobjective decision analysis in regional economics, in: P. Nijkamp ed., Handbook of regional and urban economics. Amsterdam: North Holland, 493-541.

Nijkamp, P., P. Rietveld, H. Voogd (1990). Multicriteria evaluation in physical planning. Amsterdam: North Holland.

Soil Clean-Up Guideline (1983). Leidraad Bodemsanering (in Dutch), The Hague: Staatsuitgeverij.

Weber, M. (1985). A method of multiattribute decision making with incomplete information. Management Science, 11:1365-1371.

von Winterfeld, D., W. Edwards (1986). Decision analysis and behaviourial research. Cambridge: Cambridge University Press.

Zeleny, M. (1982). Multiple criteria decision making, New York: McGraw Hill.