

VU Research Portal

Static and dynamic spatial interaction models

Nijkamp, P.; Reggiani, A.

1992

document version

Publisher's PDF, also known as Version of record

[Link to publication in VU Research Portal](#)

citation for published version (APA)

Nijkamp, P., & Reggiani, A. (1992). *Static and dynamic spatial interaction models*. (Serie Research Memoranda; No. 1992-52). Faculty of Economics and Business Administration, Vrije Universiteit Amsterdam.

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

E-mail address:

vuresearchportal.ub@vu.nl

05348

Serie Research Memoranda

Static and Dynamic Spatial Interaction Models: an Integrating Perspective

P. Nijkamp
A. Reggiani

Research-Memorandum 1992-52
December 1992







1. Introduction

The evolution of sciences is based on the rise and decline of scientific 'mental constructs'. For many decades scholars from various disciplines have been intrigued by the question whether there are unifying principles or models that have a validity in different disciplines. The building of such analytical framework bridging the gaps between different scientific traditions is a very ambitious task and has not been very successful up till now.

In the past - in a static context - several such principles have been defined and advocated at the edge of the natural sciences on the one hand and social sciences (in particular, economics and geography) on the other hand, mainly based on the paradigm of 'social physics'. Some important contributions to the integration of the spatial systems sciences and physics can be found in gravity theory and entropy theory, which have formed the corner stones of interaction models in space. The present paper is about spatial interaction models. It summarizes the correspondence of such models from a 'social physics' perspective. It is noteworthy that such models need a behavioural underpinning as a sine qua non for a valid use in spatial systems analysis. This view also explains the use of micro-based disaggregate choice models as a tool for analyzing spatial systems. This is mainly analyzed in Section 2 of the present paper.

In recent years much attention has been devoted to qualitative (structural) changes in dynamic systems, evolutionary theory, morphogenesis, bio-social science and the like. Also here the question emerges as to the validity of such approaches in social sciences in general and in spatial systems in particular. Is it, for instance, possible to design models that describe indigenous behavioural shocks in spatial systems models?

In this context, non-linear dynamics has many important lessons to offer to the analysis of the dynamic behaviour of spatial systems. Especially modern chaos theory, which has gained much popularity in recent years, presents fascinating new analytical departures. At the same time the need for a behavioural explanation in such qualitative structural change models has to be emphasized. Therefore, this paper

contains in Section 3 a critical overview of non-linear types of models, with a particular emphasis on the applicability in dynamic spatial systems.

We postulate here that spatial interaction models - interpreted in a broad sense from a logistic perspective - may offer a general framework for many (static and dynamic) phenomena in interconnected spatial systems (Section 4). They appear to be compatible with 'social physics' and chaos principles. Clearly, a major challenge is to generate a more solid empirical basis for such models. For the time being we have to take resort to simulation experiments. The past development of spatial interaction models in a dynamic environment suggests also various items for a research agenda, and therefore an outline of such a research ambition is sketched in the concluding section of this paper.

2. Relevance of Spatial Interaction Models in a Static Context

The relevance of Spatial Interaction Models (SIMs) was clearly and significantly outlined by Olsson (1970, p. 233) in the following statement: "The concept of spatial interaction is central for everyone concerned with theoretical geography and regional science... Under the umbrella of spatial interaction and distance decay, it has been possible to accommodate most model work in transportation, migration, commuting, and diffusion, as well as significant aspects of location theory."

Similar observations were also made by Fotheringham and O'Kelly (1989, p. 1), who claimed: "Spatial interaction can be broadly defined as movement or communication over space that results from a decision process. The term thus encompass such diverse behaviour as migration, shopping, travel-to-work, the choice of health-care services, recreation, the movement of goods, telephone calls, the choice of a university by students, airline passenger traffic, and even attendance at events such as conferences, theatre and football matches. In each case, an individual trades off in some manner the benefit of the interaction with the costs that are necessary in overcoming the spatial separation between the individual and his/her possible desti-

nation.... It is the pervasiveness of this type of trade-off in spatial behaviour that has made spatial interaction modelling so important and the subject of such intense investigation".

In this paper we will proceed along these lines by showing how SIMs play nowadays a fundamental role as the common basis for many other models widely used in transportation studies and related applications, in both a static and dynamic context.

To this purpose we will briefly give here a presentation of SIMs, which may also be useful for a better understanding of our subsequent considerations. We will start from the general formulation of a doubly-constrained SIM, whose role is more predictive than explanatory, since it takes the propulsiveness of origins and the attractiveness of destinations as exogenously given and allocates a known number of outflows and inflows to links between these origins and destinations (see Fotheringham and O'Kelly, 1989).

Consequently, a doubly-constrained SIM models the flow T_{ij} (flows of people, goods, messages, etc.) from some origin i to some destination j as follows:

$$T_{ij} = A_i B_j O_i D_j \exp(-\beta c_{ij}) \quad (2.1)$$

with:

$$A_i = [\sum_j B_j D_j \exp(-\beta c_{ij})]^{-1} \quad (2.2)$$

and

$$B_j = [\sum_i A_i O_i \exp(-\beta c_{ij})]^{-1} \quad (2.3)$$

where:

T_{ij} = the (unknown) trips from origin zone i to destination j

O_i = the (given) trips generated from origin zone i

D_j = the (given) trips attracted to destination zone j

c_{ij} = a measure of the (given) unit cost of transport between zone i and zone j

$\exp(-\beta c_{ij})$ = a (non-linear) function of transport costs, where β is a parameter to be determined by means of a calibration

process for the whole model.

A_i and B_j are so-called balancing factors ensuring the additivity condition that the total activity leaving origin zone i is equal to O_i , and the total activity attracted to destination j is equal to D_j , respectively.

To calibrate model (2.1) at some base year, with data on a given set of origin zones O_i and destination zones D_j , a parameter value for the transport cost function $f(c_{ij})$ has to be found that 'best' reproduces the known base year trip pattern T_{ij} (see, e.g., Foot, 1986). Since the balancing factors A_i and B_j form a set of simultaneous non-linear equations (see equations (2.2) and (2.3)), an iterative procedure is necessary to solve the model. For this purpose Hyman's method (Hyman, 1969) is usually applied.

Particular cases of model (2.1) are the singly-constrained SIM and the unconstrained SIM. The former class provides information only on the destination characteristics (the so-called production-constrained SIM) or on the origin characteristics (the so-called attraction-constrained SIM), while the latter class (the unconstrained SIM) provides information on the attributes of both the origins and the destinations of the interactions. Thus these three types of models are more 'explanatory' in nature, since they determine, by means of model calibration, the attributes of locations giving rise to the flows of people, goods, etc. (see again Fotheringham and O'Kelly, 1989). In particular the production-constrained SIM can be represented by the following equation:

$$T_{ij} = A_i O_i D_j \exp(-\beta c_{ij}) \quad (2.4)$$

where the balancing factor A_i now has the following specification:

$$A_i = [\sum_j D_j \exp(-\beta c_{ij})]^{-1} \quad (2.5)$$

The value of A_i serves to ensure that model (2.4) reproduces the volume of flows originating from zone i , so that:

$$O_i = \sum_j T_{ij} \quad (2.6)$$

The inverse of A_i in (2.5) is often interpreted as a measure of the accessibility of zone i (see, among others, Weibull, 1976).

It is interesting to recall that in a static context SIMs have various origins:

- The most common specification of a SIM originates from gravity theory (see e.g., Ravenstein, 1885, and for a review, Isard and Maclaren, 1982).

- A second, macro-oriented approach from which SIMs can emerge is the entropy concept. Even though the entropy model has its roots in statistical mechanics (see Wilson, 1967, 1970), the entropy concept can be interpreted in terms of a generalized cost function for transport behaviour, thus offering a macro-behavioural context to SIMs (see Nijkamp and Reggiani, 1992).

- A further utility background of entropy is offered by programming (or optimization) models, since entropy can be regarded as a specific type of the latter models. Consequently, the family of SIMs can be derived from different formulations of an entropy (or utility) maximizing macro-approach and hence viewed as an optimum system's solution. Once again, the entropy concept results are compatible with a macro-behavioural interpretation of spatial interaction (see Nijkamp and Reggiani, 1992). It should be noted that in this framework SIMs are essentially considered as aggregate models describing the macro-state of a system and based on macro data.

- A further, very interesting derivation of SIMs, is also the one emerging from random utility theory in economics (see, for example Anas, 1983). In particular the formal analogy between Multinomial Logit (MNL) models belonging to the class of Discrete Choice Models (DCMs) and SIMs has often been stressed in recent years (see, for a review, Nijkamp and Reggiani, 1989).

In fact, if we examine the structure of a production-constrained SIM in its probabilistic form, i.e.:

$$P_{ij} = T_{ij} / O_i = A_i D_j \exp(-\beta c_{ij}) \quad (2.7)$$

where P_{ij} represents now the probability of moving from origin i to destination j , it is clear that, if we introduce in equation (2.7) the

expression of the balancing factor A_i like in (2.2), the following equivalence results:

$$P_{ij} = D_j \exp(\beta u_{ij}) / \sum_j D_j \exp(\beta u_{ij}) \quad (2.8)$$

where D_j can be interpreted as a weighting factor reflecting the attractiveness of a point of destination j , and where $u_{ij} = -c_{ij}$ can be considered as the deterministic part of the individual random utility underlying an MNL model (see, for the derivation of an MNL model, the seminal work of McFadden, 1974 and Domencich and McFadden, 1975).

Expression (2.8) is clearly an MNL model. For the sake of convenience we suppose here that the reader is familiar with the basic literature on MNL models and DCMs. Consequently, we will summarize here some basic considerations related to the equivalence between (2.7) and (2.8):

- If a singly-constrained SIM can be considered equivalent to an MNL model, it also means that SIMs embed the limits inherent in MNL models such as the so-called IIA property.
- Secondly, since << the same model without any aggregation error may be derived in disaggregate form from both entropy and utility maximization >> (see Batten and Boyce, 1986, p. 378), it follows that entropy and SIMs are not inherently less behavioural than stochastic utility models of DCMs (and MNL models in particular). Therefore, in this perspective, SIMs can be considered as aggregate models of human behaviour.
- SIMs can also emerge from the maximization of a micro-economic (deterministic or random) utility function. In particular, the latter analysis illustrates that SIMs are formally analogous to DCMs; consequently, depending on the type of available data (aggregate or disaggregate) SIMs can provide a similar behavioural background as DCMs. In particular, a singly-constrained SIM can be considered equivalent to an MNL model thus embedding also the limits inherent in MNL models such as the IIA property. Only a doubly-constrained SIM - and more generally an Alonso model - with a sequential choice process may be associated with a nested MNL model, thus overcoming the IIA axiom. It then follows that entropy can also be interpreted as a measure of interaction between

economic individuals and consequently as a social/collective utility function (including cost elements) (see Nijkamp and Reggiani, 1992).

It should be noted that the above mentioned SIMs mainly deal with the demand side in a transportation system (or more generally in a spatial interaction system). However, it has also been pointed out (see Florian and Gaudry, 1983) that a phenomenon designated as supply at one particular level for a transportation system may become demand at another level. Consequently, relationships among different levels can be considered as input-output interactions, so that a precise distinction between supply and demand side can be made only at one particular level of the system. It turns out that the same SIM can be used in both demand and supply side analysis (see, for example, the concept of accessibility derived from spatial interaction analysis and applied to infrastructure systems; see Rietveld, 1989).

However, most models developed in the sixties and seventies and ending up with a SIM structure are still static/deterministic equilibrium models and do not consider the time paths followed by the transportation system components as well as the uncertainty of the system and its network (see the special issue of Transportation Research; Boyce, 1985).

In this context it is interesting to emphasize the formal connections between SIMs and MNL models, since this link can place more emphasis on the analysis of individual motives and on the impact of micro random behaviour upon the functioning of transport and spatial interaction systems (in view of the need to a better understanding of the stochasticity of systems).

A further connection is also offered by the integration of SIMs with rational screening methods related to risky alternatives (e.g., the stochastic dominance approach). This unifying approach, which also shows the possibility of linking models of choice behaviour under certainty and models of choice behaviour under uncertainty (see Reggiani and Stefani, 1986), has recently also been applied to modal choice problems in a transportation system (see Reggiani and Stefani, 1989) by considering the attributes of the alternatives according to different states of nature.

Having now briefly reviewed the connections between SIMs and behavioural models, the next step will be to draw our attention to the broad category of dynamic models (DCMs) developed more recently in transportation systems, in order to take into account newly emerging relevant aspects of system dynamics, such as slow and fast dynamics, uncertainty, bifurcations, catastrophic changes, chaotic behaviour, fractal structures, etc. In particular we will point out a common similarity in these DCMs - despite their different theoretical sources - viz. their close association, under particular conditions, with dynamic SIMs.

3. Relevance of Spatial Interaction Models in a Dynamic Context

3.1 Dynamic Properties of SIMs

3.1.1 Prologue

It has recently been shown (see Nijkamp and Reggiani, 1992) that also in a dynamic framework SIMs emerge from various roots. In particular a dynamic analysis of SIMs has led to the following main results:

- The formal parallel between SIMs and DCMs can be extended also in a dynamic and stochastic context. Moreover, a SIM can be shown to be the optimal solution of a dynamic spatial interaction problem. Consequently, the interrelations between entropy and SIMs exist also in a dynamic framework. This indicates that dynamic SIMs are able to capture also the evolutionary patterns of a dynamic interaction system and that a dynamic entropy can be viewed as a cumulative utility function concerned with generalized cost minimization (see Nijkamp and Reggiani, 1992).
- SIMs also have the possibility of explaining the interaction activities in a stochastic dynamic framework. In fact, a stochastic (doubly-constrained) SIM can be shown to emerge as an optimum system's solution to a dynamic entropy optimal control problem subject to random disturbances. Furthermore, a singly-constrained SIM shows a structural stability even in the presence

of such small perturbations. This has been shown in an illustration of catastrophe behaviour in the field of mobility (see Nijkamp and Reggiani, 1992).

The relevance of stochastic (exogenous) fluctuations (according to which the usual form of a SIM varies) then leads to the issue of analyzing SIMs in the context of states of disequilibrium, causality and unpredictability; in other words, in the context of potentially chaotic behaviour.

The discovery of 'chaos' seems to have created a new paradigm in scientific modelling. However, chaos models in economic and social sciences are often theoretical or illustrative rather than empirical, given the lack of available data. In particular, SIMs in the framework of chaos theory show that irregular dynamic behaviour is a possibility depending on initial conditions and on critical parameter values. More specifically (see Nijkamp and Reggiani, 1990a) SIMs have the possibility of showing chaotic features in their MNL formulation, in particular for high values of the marginal utility function. Consequently, a great deal of attention has to be paid to the speed of change of the utility function (or to the distance decay), since some critical parameter values can lead to a chaotic movement with unfeasible values for the socio-economic variables considered. This interesting result essentially derives from the mathematical interrelationships between SIMs and logistic structures, as will be shown in the next subsection.

3.1.2 Relationships between SIMs and logistic function

If we consider, in a dynamic framework, the equivalence between SIMs and MNL models, we get the following logit form:

$$P_j = \exp(u_j) / \sum_l \exp(u_l) \quad (3.1)$$

which is equivalent to a dynamic SIM and which represents the probability of choosing alternative j at time t .

In this context we may recall a recent result, i.e., that the rate of changes of P_j (formulated in 3.1) with respect to time (i.e., \dot{P}_j) can be expressed by a structure of the Volterra-Lotka type (see Sonis,

1988, and Nijkamp and Reggiani, 1990a), as follows:

$$\dot{P}_j = \dot{u}_j P_j (1 - P_j) - P_j \sum_{\ell \neq j} \dot{u}_\ell P_\ell \quad (3.2)$$

It is also interesting to note that a particular case of the above dynamic MNL, i.e. a binary case, obtained by deleting the last term in (3.2) (i.e. the interaction term), is a logistic growth model. It is also well-known that the difference version of the standard logistic growth model has been thoroughly discussed in the literature for its capability of generating bifurcations and chaos for certain critical values of the growth parameter (see, e.g., the seminal work by May, 1976). Therefore, a 'binary' logit model, belonging to the family of May models, shows the same 'chaotic' properties, i.e. a cascade of bifurcations leading to chaos for certain values of its dynamic utility function \dot{u}_j which represents the growth parameter of the variable P_j (see Figure 1).

Given the above mentioned link between logistic functions, dynamic SIMs and dynamic MNL models, we will give in the next section a more thorough overview of the most frequently used dynamic interaction models in order to show their connection with a logistic structure and hence with SIMs.

3.2 Relevance of the Logistic Form in Dynamic Spatial Models

3.2.1 Introduction

As noted above the last decade has shown a boom in the interest in the development of both behavioural and dynamic models, as it is generally expected that such models are capable to describe and represent the behavioural mechanisms underlying the evolutionary changes in complex transportation and network systems (see, e.g., Ben-Akiva, 1985). Consequently, a wide variety of multi-temporal or dynamic transportation models has arisen in the past decade with the aim of providing a stronger and more useful analytical support to planning processes than conventional static tools (such as static

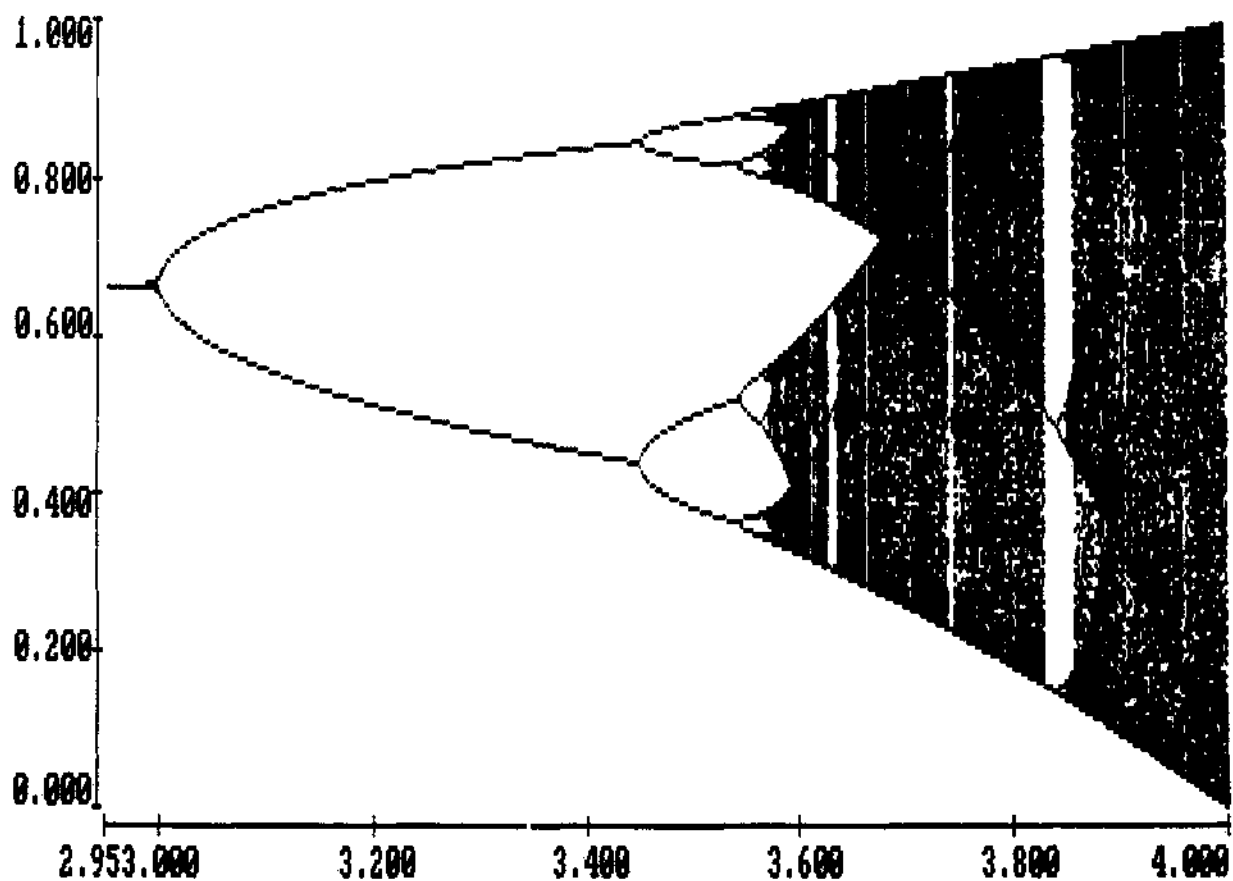


Figure 1. Bifurcation Diagram for a Dynamic - Binary - Logit Model

y - axis = P_j

x - axis = u_j

spatial interaction models, linear programming models, etc.). In this context it is noteworthy that SIMs tend to become again a focal point of analysis, since they can deal with the complicated and interwoven pattern of human activities in space and time.

However, despite all progress made in this new research direction (mostly from a theoretical viewpoint), still some important research questions remain, which largely concern the applicability of these dynamic models in relation to the scale of analysis at which various operational developments of these models are taking place. In particular, this important field of reflection concerns the advantages and disadvantages related to the use of macro-meso-micro approaches.

On the one hand, it is evident that aggregate representations may become extremely cumbersome and inefficient when it is necessary to represent complex systems, especially where there is considerable heterogeneity amongst the actors in those systems (see Clarke and Wilson, 1986). On the other hand, it is clear that the problems of data availability and computational processing requirements are often in contrast with the need to use a micro-oriented approach. Moreover, the response of population in aggregated models does not always correspond to an aggregation of the individual responses obtained from a micro model, so that it seems evident that the phenomena being studied require a careful consideration as regards the nature of their level of analysis (see again Clarke and Wilson, 1986). This problem has also been treated in analytical attempts focusing attention on the interdependencies between micro- and macro-responses, which also depend on the interaction between demand and supply.

3.2.2 Logistic forms in macro-dynamic approaches

Several dynamic models of spatial structure have recently been developed at a macro level. We will give here a few illustrations. An example is the model developed by Allen et al. (1978), in which the evolutionary growth of zonal activities is assumed to follow a logistic pattern. Allen et al.'s model is a comprehensive model incorporat-

ing urban activities such as employment and residential population. A major finding in this model is that random fluctuations (e.g., changes due to infrastructure constructions) may alter the related urban evolution.

Another dynamic model of the logistic type is the one developed by Harris and Wilson (1978) and Wilson (1981). In this case the standard static spatial interaction model for activity allocation has been embedded into a dynamic evolutionary framework, again of a logistic type. Bifurcations and catastrophe behaviour emerge from this model, depending on particular values of the parameters. Obviously, owing to this logistic structure also oscillations and cycles may occur.

These two important models have induced a wide spread production of related models both from theoretical and empirical viewpoints, also in a stochastic framework (see also Nijkamp and Reggiani, 1988a). However, it should also be noted that the above mentioned two models primarily focus on the supply side, without clear dynamic equations at the demand side.

Another stream of research at the macro-level is the series of models based on ecological dynamics of the Volterra-Lotka type (see Dendrinos and Mullally, 1981); in this formulation of interacting biological species, each species is characterized by a birth-death process of the logistic type. Recent papers on this topic show the integration between ecological models and optimal control models (Nijkamp and Reggiani, 1990b), between ecological models and random fluctuations of a white noise type (Campisi, 1986) or between ecological models, SIMs of a gravity type and turbulence (Dendrinos, 1988).

Obviously, since a Lotka-Volterra system is a system of interrelated equations, we get by necessity here interaction mechanisms of supply and demand. Furthermore, given the related logistic form, it is also here again possible to get - for critical parameter values - oscillations and complex behaviour.

The last group of macro approaches in the area of SIMs are represented by models based on optimal control approaches or dynamic

programming analysis (see, for example, Nijkamp and Reggiani, 1988b). Also here different forms of equilibrium/disequilibrium may emerge (e.g., saddle points, borderline stability), which show the possibility of unstable motions.

As a synthesis we may conclude that a common trend in these groups of macro-approaches is the development of models that are able to exhibit (under certain conditions) complex or chaotic behaviour and hence outcomes which are hardly foreseeable by modellers and planners. This lack of predictability of future events is clearly also a major concern in transportation planning and reflects analytically - in terms of a scientific paradigm - essentially the beginning of a new phase of a research life cycle. Here another important research problem is emerging, i.e., the relevance of critical parameter values, such as their speed of change in a geographic or planning context in order to understand whether the system at hand is moving towards a predictable or complex (unpredictable) evolution.

3.3.3 Logistic forms in micro-meso dynamic approaches

In this subsection we will briefly pay attention to the considerable body of literature based on micro simulation models (see Clarke and Wilson, 1986). Given the above mentioned drawbacks related to a macro-approach, a mixture of aggregate dynamic models in conjunction with micro-simulation (the micro-meso approach) has recently been advocated and adopted for various spatial applications (see also Birkin and Clarke, 1983). In this way also an integration between demand and supply results is possible. In other words, micro-meso dynamic approaches utilize individual data in conjunction with aggregate equations.

Another interesting micro-meso approach is the well-known logit model, based on a micro-economic foundation. In Section 3.1.2 it has been shown that the growth over time of people choosing such alternatives as travel choice mode, destination, etc. according to a logit procedure follows again a logistic pattern. Thus development can also

lead to a chaotic or irregular behaviour for particular values of the underlying utility function. The most important consequence of this result is the link between DCMs - and hence the equivalent SIMs - and logistic formulation of these models. Since most of the models referred to end up with a logistic shape, it is clear that SIMs can be considered as a comprehensive framework incorporating many advanced models also at a dynamic level.

In this area also the master equation/mean value equation models (see Haag and Weidlich, 1984; Haag, 1989) have to be mentioned. These models have been used extensively in spatial flow analysis. This framework models the uncertainties in the decision process of the individuals via the master equation approach. The mean values are then obtained from the master equation by an aggregation of the individual probability distributions. Thus this approach provides the link between micro-economic aspects and the macro-economic equation of motions for aggregate mean values.

In this context also compartmental analysis should be mentioned (see De Palma and Lefevre, 1984) which consists of equations which are the approximate mean-value equations. It has recently been shown (see Reiner et al., 1986) that also these meanvalue equations may display chaotic behaviour with strange attractors, given particular conditions for the group interaction. On the other hand, this result is not surprising, since the mean value equations are strongly related to logit models. Hence it is plausible that interrelated logistic functions cause the emerging chaotic motions.

After this brief review based on a typology of dynamic models and their potential in transportation planning, we will now incorporate them in the framework of a life-cycle (evolutionary) pattern.

4. Evolution of Spatial Interaction Models

The models discussed in the previous section can be unified in the broad area of DMs and then compared, in their evolution, to the

groups of static SIMs and DCMs, in the light of life cycle concepts. As a synthesis, we can represent the series of the transportation models studied and adopted so far according to the scheme of overlapping generations illustrated in Figure 2.

In Figure 2 we have essentially depicted the generation and diffusion of the three main families of models which have received a great deal of attention in the last century, i.e., Spatial Interaction Models (SIMs), Discrete Choice Models (DCMs) and Dynamic Models (DMs).

From Figure 2 we can see that - while SIMs present a smooth development at the beginning of the century with more emphasis after the sixties - DCMs and DMs exhibit a rapid growth (from both a theoretical and empirical point of view). It should be noted that in DMs we have included the whole group of dynamic models treated so far, so that we can observe that after the mid seventies a large number of mathematical models emerged.

Altogether we observe an overlapping generation of models: in particular we can unify all these models in a unique general logistic shape where the points A and B mean the theoretical conjunction of DCMs and DMs with SIMs, respectively. However, it seems plausible that a saturation level of the development of the above mentioned models will be approached. Probably, from this upper level, new tools will emerge in the next century, in response to new activity patterns. This upper level can likely be linked to the analytical structure of the models, since there are inevitable constraints in their formulation, so that also from a mathematical point of view the potential of such models will certainly reach a limit.

Since it has been underlined in the previous section that most of these models can be reformulated in terms of a logit-logistic formulation - and hence can be interpreted in a SIM structure - the broad potential of SIMs can be considered as such a limit.

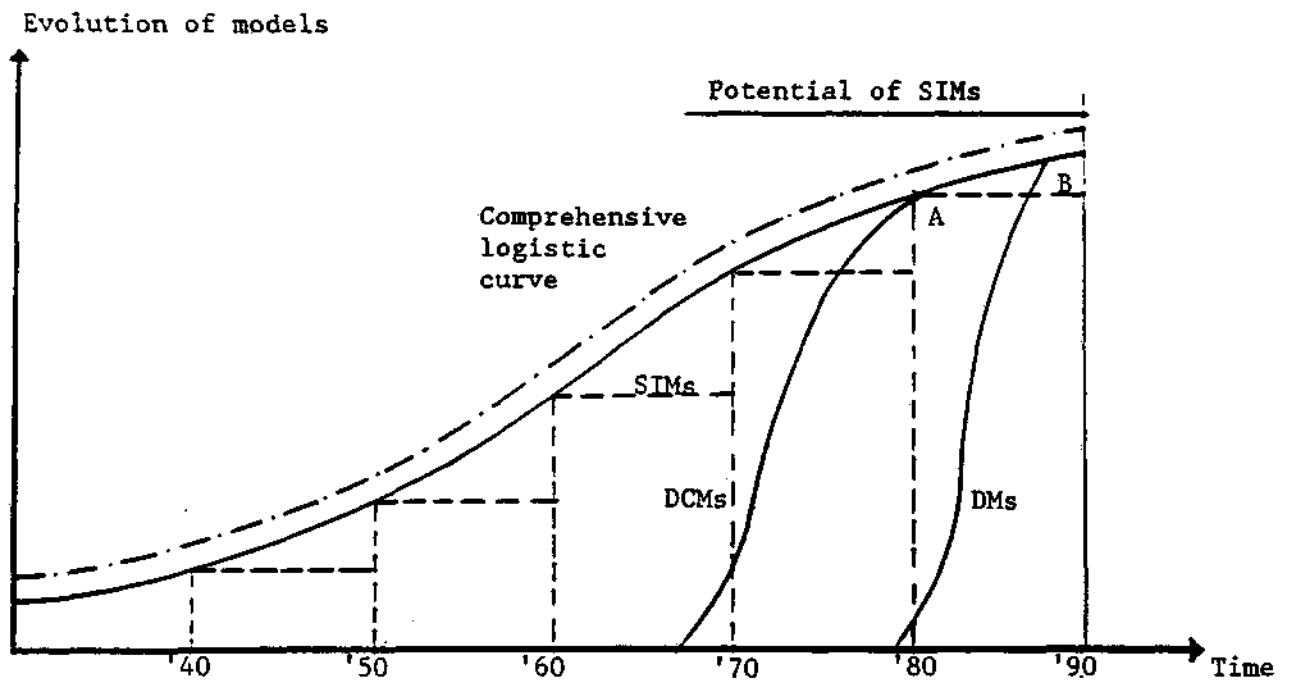


Figure 2 Overlapping Generations of Models

Legend: SIMs - Spatial Interaction Models
 DCMs - Discrete Choice Models
 DMs - Dynamic Models

5. Towards a Research Agenda for Dynamic Spatial Interaction Systems

It is clear from the previous arguments that the field of transportation and communication research is increasingly dominated by dynamic systems considerations. Bifurcation, catastrophe and chaos theory have become critical components of a new framework for investigating the long term evolution of spatial interaction flows.

At the same time it is also evident that still a long trajectory has to be followed before the 'new dynamics' movement will have led to operational and testable analytical propositions which can also be used in empirical research. For the time being, various new research directions are necessary in order to complement the tools developed in the past decades.

A systematic listing of such new tools leads directly to the design of a research agenda for dynamic spatial interaction systems. The following items are central in such a research agenda.

1. **Specification theory.** The formulation of dynamic spatial systems models which are compatible with plausible behavioural hypotheses on the one hand and which lend themselves for empirical testing on the other hand is a difficult methodological task which so far has not yet been very successful. The extent to which a dynamic SIM is a satisfactory mapping of highly dynamic real world processes is a formidable research effort, mainly since empirical tests are lacking.
2. **Verification analysis.** The question whether (theoretical) model results are in agreement - in a qualitative or quantitative sense - with non-linear patterns incorporated by an underlying data set is another important research challenge. So far it has been very difficult to find statistically satisfactory parameter estimates in non-linear models whose value at the outset is falling in the chaotic domain. Besides, the statistical tools for identifying non-linear dynamic (and possibly chaotic) patterns are not very well developed, although the value of the Liapunov exponents, the Brock-Dechert-Scheinkman test, and the use of recurrence plots may provide analytical support.
3. **Behavioural analysis.** The identification of chaos behaviour in the decisions of actors is to a large extent dependent on the degree of

aggregation of observed time series. In a very short time span (e.g., on an hourly basis) the possibility of chaotic patterns in behaviour is much higher than in a longer time span (e.g., on a yearly basis), as in the latter case a smoothing amendment has taken place. Furthermore, in the longer run rational expectations of actors would generate negative feedback reactions, so that wild fluctuations would be prevented. Altogether, it is difficult to separate random shocks, measurement errors, impacts of time series and behavioral feedbacks in a given data set.

4. **Impacts of time delays.** Although it was sometimes assumed in the past that the inclusion of more time lags would destabilize a growth trajectory, it has recently been recognized that this is not necessarily true (e.g., in the rational expectations model the probability of occurrence of chaos diminishes if the weight of the past increases). In recent publications it has been demonstrated that an increase in time lags may increase the probability of chaos, but on a much smaller domain. Two research directions might be interesting in this context, viz. the relevance of fractal theory (which takes for granted that phenomena at a given scale level are replicated at lower levels) and of percolation theory (which analyzes the time trajectory of a dynamic phenomenon in case of unstructured barriers).

5. **Impacts of chaos modules.** This question focuses on the overall stability of a non-linear dynamic systems model, if this model incorporates one smaller module which may exhibit chaotic behaviour. This leads to the intriguing research question whether lower order chaos may affect overall stability and vice versa. In this context there is much scope for innovative research strategies, viz. niche theory (which deals with partly overlapping and interwoven sets of populations in a dynamic system) and autopoiesis (which addresses the issue of self-organization in dynamic social-cybernetic or self-organizing systems).

It is clear that in all above research suggestions the behavioural aspects of actors are of decisive importance. This once more emphasizes the need for an integration of behavioural modelling along the lines of discrete choice theory with meso/macro dynamic spatial interaction modelling. In this context - and given the usual lack of

appropriate time series data - the recent trend towards experimental social science research is undoubtedly an important step forward.

Acknowledgement

The second author gratefully acknowledges the NECTAR grant from the European Science Foundation (ESF) as well as CNR grant No.91.02288. CT11.

References

- Allen, P.M., J.L. Deneubourg, M. Sanglier, F. Boon and A. De Palma, The Dynamics of Urban Evolution, Final Report of the U.S. Department of Transportation, Washington D.C., 1978.
- Anas, A., Discrete Choice Theory, Information Theory and the Multinomial Logit and Gravity Models, Transportation Research, vol. 17B, 1983, pp. 13-23.
- Batten, D.F. and D.E. Boyce, Spatial Interaction, Transportation and Interregional Commodity Flow Models, Handbook of Regional and Urban Economics, (P. Nijkamp, ed.), North Holland, Amsterdam, 1986, pp. 357-406.
- Ben-Akiva, M. and A. De Palma, Analysis of a Dynamic Residential Location Choice Model with Transaction Costs, Journal of Regional Science, vol. 26, 1986, pp. 321-341.
- Birkin, M. and M. Clarke, Comprehensive Dynamic Urban Models: Integrating Macro and Micro Approaches, Evolving Geographical Structures (D.A. Griffith and A.C. Lea, eds., 1983, pp. 164-191).
- Boyce D.E. (ed.), Transportation Research: the State of the Art and Research Opportunities, Transportation Research: Special Issue, vol. 19A, 5/6, 1985, pp. 349-542.
- Campisi, D., Lotka-Volterra Models with Random Fluctuations for the Analysis of Oscillations in Urban and Metropolitan Areas, Sistemi Urbani, 2/3, 1986, pp. 309-321.
- Clarke, M. and A.G. Wilson, A Framework for Dynamic Comprehensive Urban Models: The Integration of Accounting and Microsimulation Approaches, Sistemi Urbani, no. 2/3, 1986, pp. 145-177.
- Dendrinis, D.S., Volterra-Lotka Ecological Dynamics, Gravitational Interaction and Turbulent Transportation: An Integration, Paper presented at the 35th North American Meeting of the Regional Science Association, Toronto, Canada, 1988.
- Dendrinis, D.S., and H. Mullally, Evolutionary Patterns of Urban Populations, Geographical Analysis, vol. 13, no. 4, 1981, pp. 328-344.
- De Palma, A. and C. Lefèvre, The Theory of Deterministic and Stochastic Compartmental Models and its Applications: the State of the Art, Sistemi Urbani, no. 3, 1984, pp. 281-323.
- Domencich, T.A. and D. McFadden, Urban Travel Demand: A Behavioural Analysis, North Holland, Amsterdam, 1975.
- Florian M. and M. Gaudry, Transportation Systems Analysis: Illustrations and Extension of a Conceptual Framework, Transportation Research, vol. 17B, no. 2, 1983, pp. 147-153.

- Foot, D., Spatial Interaction Modelling, Models in Urban Geography, (C.S. Yadav, ed.), Concept Publ., New Delhi, 1986, pp. 51-69.
- Fotheringham, A.S. and M.E. O'Kelly, Spatial Interaction Models: Formulations and Applications, Kluwer Academic Publishers, Dordrecht, 1989.
- Haag G., Spatial Interaction Models and Their Micro Foundation, Advances in Spatial Theory and Dynamics, Andersson A.E., D.F. Batten, B. Johansson and P. Nijkamp (eds.), North-Holland, Amsterdam, 1989, pp. 165-174.
- Haag, G. and W. Weidlich, A Stochastic Theory of Interregional Migration, Geographical Analysis, vol. 16, 1984, pp. 331-357.
- Harris, B. and A.G. Wilson, Equilibrium Values and Dynamics of Attractiveness Terms in Production-Constrained Spatial Interaction Models, Environment and Planning A, vol. 10, 1978, pp. 371-338.
- Hyman, G.M., The Calibration of Trip Distribution Models, Environment and Planning A, vol. 1, 1969, pp. 105-112.
- Isard, W. and V.W. Maclaren, Storia e Stato Attuale delle Ricerche nella Scienza Regionale, Problematiche dei Livelli Subregionali di Programmazione (Bielli, M. and A. La Bella, eds.) Franco Angeli, Milano, 1982, pp. 19-35.
- May, R., Simple Mathematical Models with Very Complicated Dynamics, Nature, vol. 261, 1976, pp. 459-467.
- McFadden, D., Conditional Logit Analysis of Qualitative Choice Behaviour, Frontiers in Econometrics (P. Zarembka, ed.), Academic Press, New York, 1974, pp. 105-142.
- Nijkamp, P. and A. Reggiani, Dynamic Spatial Interaction Models: New Directions, Environment and Planning A, vol. 20, 1988a, pp. 1449-1460.
- Nijkamp, P. and A. Reggiani, Analysis of Dynamic Spatial Interaction Models by Means of Optimal Control, Geographical Analysis, vol. 20, no. 1, 1988b, pp. 18-29.
- Nijkamp, P. and A. Reggiani, Spatial Interaction and Discrete Choice: Statics and Dynamics, Urban Dynamics and Spatial Choice Behaviour, (Hauer, J., H. Timmermans and N. Wrigley, eds.), Kluwer Academic Publishers, Dordrecht, 1989, pp. 125-151.
- Nijkamp, P. and A. Reggiani, Logit Models and Chaotic Behaviour: A New Perspective, Environment and Planning A, vol. 22, 1990a, pp. 1455-1467.
- Nijkamp, P. and A. Reggiani, An Evolutionary Approach to the Analysis of Dynamic Systems, with Special Reference to Spatial Interaction Models, Sistemi Urbani, vol. 1, 1990b, pp. 95-112.

Nijkamp, P. and A. Reggiani, Interaction, Evolution and Chaos in Space, Springer-Verlag, Berlin, 1992 (Forthcoming).

Olsson, G., Explanation, Prediction and Meaning Variance: an Assessment of Distance Interaction Models, Economic Geography, vol. 46, 1970, pp. 223-233.

Ravenstein, E.G., The Laws of Migration, Journal of the Royal Statistical Society, vol. 43, 1885, pp. 167-235.

Reggiani, A. and S. Stefani, Aggregation in Decisionmaking: A Unifying Approach, Environment and Planning A, vol. 18, 1986, pp. 1115-1125.

Reggiani, A. and S. Stefani, A New Approach to Model Split Analysis: Some Empirical Results, Transportation Research B, vol. 23, no. 1, 1989, pp. 75-82.

Reiner, R., M. Munz, G. Haag and W. Weidlich, Chaotic Evolution of Migratory Systems, Sistemi Urbani, vol. 2/3, 1986, pp. 285-308.

Rietveld, P., Infrastructure and Regional Development. A Survey of Multiregional Economic Models, The Annals of Regional Science, vol. 23, no. 4, 1989, pp. 255-274

Weibull, J.W., An Axiomatic Approach to the Measurement of Accessibility, Journal of Regional Science and Urban Economics, vol. 6, 1976, pp. 357-379.

Wilson, A.G., A Statistical Theory of Spatial Distribution Models, Transportation Research, vol. 1, 1967, pp. 253-269.

Wilson, G., Entropy in Urban and Regional Modelling, Pion, London, 1970.

Wilson, A.G., Catastrophe Theory and Bifurcation, Croom Helm, London, 1981.

- 1991-1 N.M. van Dijk On the Effect of Small Loss Probabilities in Input/Output Transmission Delay Systems
- 1991-2 N.M. van Dijk Letters to the Editor: On a Simple Proof of Uniformization for Continuous and Discrete-State Continuous-Time Markov Chains
- 1991-3 N.M. van Dijk
P.G. Taylor An Error Bound for Approximating Discrete Time Servicing by a Processor Sharing Modification
- 1991-4 W. Henderson
C.E.M. Pearce
P.G. Taylor
N.M. van Dijk Insensitivity in Discrete Time Generalized Semi-Markov Processes
- 1991-5 N.M. van Dijk On Error Bound Analysis for Transient Continuous-Time Markov Reward Structures
- 1991-6 N.M. van Dijk On Uniformization for Nonhomogeneous Markov Chains
- 1991-7 N.M. van Dijk Product Forms for Metropolitan Area Networks
- 1991-8 N.M. van Dijk A Product Form Extension for Discrete-Time Communication Protocols
- 1991-9 N.M. van Dijk A Note on Monotonicity in Multicasting
- 1991-10 N.M. van Dijk An Exact Solution for a Finite Slotted Server Model
- 1991-11 N.M. van Dijk On Product Form Approximations for Communication Networks with Losses: Error Bounds
- 1991-12 N.M. van Dijk Simple Performability Bounds for Communication Networks
- 1991-13 N.M. van Dijk Product Forms for Queuing Networks with Limited Clusters
- 1991-14 F.A.G. den Butter Technische Ontwikkeling, Groei en Arbeidsproductiviteit
- 1991-15 J.C.J.M. van den Bergh, P. Nijkamp Operationalizing Sustainable Development: Dynamic Economic-Ecological Models
- 1991-16 J.C.J.M. van den Bergh Sustainable Economic Development: An Overview
- 1991-17 J. Barendregt Het mededingingsbeleid in Nederland: Konjunctuurgevoeligheid en effectiviteit
- 1991-18 B. Hanzon On the Closure of Several Sets of ARMA and Linear State Space Models with a given Structure
- 1991-19 S. Eijffinger
A. van Ristel The Japanese Financial System and Monetary Policy: a Descriptive Review
- 1991-20 L.J.G. van Wissen
F. Bonnerman A Dynamic Model of Simultaneous Migration and Labour Market Behaviour
- 1991-21 J.M. Sneek On the Approximation of the Durbin-Watson Statistic in $O(n)$ Operations
- 1991-22 J.M. Sneek Approximating the Distribution of Sample Autocorrelations of Some Arima Processes in $O(n)$ Operations
- 1991-23 B. Hanzon
R. Hut New Results on the Projection Filter
- 1991-24 R.J. Veldwijk
E.R.K. Spoor
M. Boogaard
M.V. van Dijk On Data Models as Meta Models, An Application Designers Point of View
- 1991-25 C. Camfferman Some aspects of voluntary disclosure
- 1991-26 D. van der Wal Monetary Policy Credibility: The Experience of the Netherlands
- 1991-27 J.A. Vijlbrief Unemployment Insurance in a Disequilibrium Model for The Netherlands
- 1991-28 H.L.M. Kox The "Non-Polluter gets paid" Principle for Third World Commodity Exports
- 1991-29 H. Tijms A New Heuristic for the Overflow Probability in Finite-Buffer Queues
- 1991-30 B. Hanzon On the Estimation of Stochastic Linear Relations
- 1991-31 R.L.M. Peeters Comments on Determining the Number of Zeros of a Complex Polynomial in a Half-Plane
- 1991-32 A.A.M. Boons
H.J.E. Roberts
F.A. Roozen The Use of Activity-Based Costing Systems in a European Setting: a case study analysis
- 1991-33 J.C. van Ours Union Growth in the Netherlands 1961-1989
- 1991-34 R. van Zijp The Methodology of the Neo-Austrian Research Programme
- 1991-35 R.M. de Jong
H.J. Bierens On the Limit Behaviour of a Chi-Square Type Test if the Number of Conditional Moments Testes Approaches Infinity Preliminary Version
- 1991-36 K. Burger
J.W. Gunning Gender Issues in African Agriculture: Evidence from Kenya, Tanzania and Côte d'Ivoire
- 1991-37 M. Boogaard
R.J. Veldwijk
E.R.K. Spoor
M.V. van Dijk On Generalization in the Relational Model
- 1991-38 R. Dekker
E. Smeitink Preventive Maintenance at Opportunities of Restricted Duration