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DYNAMIC LABOUR MARKET EQUILIBRIA WITH HETEROGENEOUS UNEMPLOYMENT

by F.A.G. den Butter and J.H. Abbring*

1. Introduction

Modern search theory has recently given momentum to the so called flow approach to modelling the labour market (see Blanchard and Diamond, 1992). Whereas the traditional models of the labour market focus at explaining stocks (labour supply, labour demand and the resulting unemployment) these flow models concentrate on dynamic labour market processes, such as job creation, job destruction, job mobility, and describe labour market behaviour with respect to various flows of persons and jobs. Here stocks, such as total unemployment, total employment and the pool of vacancies result from the confrontation of gross inflow into these stocks and gross outflow from them. At the core of these models is a *matching function*, or *hiring function*, which describes the matching process between employees looking for a new job and employers who search for a proper person to fill a vacancy. In this way search theory provides the UV-curve with a micro foundation and has re-established UV-analysis as a major instrument for the description of labour market developments.

Following the literature on equilibrium unemployment theory (see e.g. Pissarides, 1990) this paper looks at concepts of equilibrium in a simple dynamic labour market model. Dynamic or steady state equilibria describe situations in which both stocks and flows described by the model grow at the same pace so that for each stock the growth of gross inflow is equal to that of gross outflow. There are two reasons to consider such equilibria. Firstly, a comparative static analysis of the equilibrium situation can be used as a yardstick for the discussion of the actual situation. The divergence of the actual situation from its equilibrium provides insight for the policy prescriptions to bring the labour market back on the right track. A second reason is that the equilibrium analysis of dynamic labour market models establishes a link with duration analysis of the labour market. Empirical microeconomic duration models often consider the escape probabilities from employment, unemployment and the probability that the vacancy becomes fulfilled in isolation from each other. The same holds true for the resulting employment, unemployment and vacancy durations[1]. Moreover, most microeconomic studies of escape probabilities and the resulting durations specify these processes by smooth distribution functions whose parameters vary with the personal characteristics of individuals on the labour market, but which are otherwise fixed and may depict an

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[1] Van den Berg and Ridder's (1992) estimation of the equilibrium search model of Burdett and Mortensen (1989), and Mortensen (1990), using micro data, constitutes a remarkable exception.

equilibrium situation. On the other hand, the dynamic labour market model gives a coherent description of the mutual relationships between job duration, unemployment duration and vacancy duration, and therefore shows the connections and restrictions of various types of duration analysis. But this link between flow models of the labour market and duration models can only be illustrated in equilibrium situations because out of equilibrium escape probabilities and durations are time-dependent and do not reconcile with smooth distribution functions.

The next section introduces a simple model of labour market flows which resembles the seminal models of Blanchard and Diamond (1989) and Jackman, Layard and Pissarides (1989). Section 3 surveys the conditions for a dynamic equilibrium in the model, when all stocks and flows remain constant, and illustrates by means of stylized numerical examples for The Netherlands how various shifts of the UV-curve described by the model depend on the model parameters. Section 4 introduces unemployment duration dependency and explains how a shift of the UV-curve in equilibrium can be the result of a change of duration dependency. Section 5 considers steady state growth dynamic equilibria according to which all stocks and flows increase at the same pace. We again consider the incidence of duration dependency in this equilibrium concept. Section 6 concludes.

2. Modelling labour market flows

The *matching function*, which describes the flow out of unemployment (F_{ue}) as a function of the stock of unemployed (U) and the stock of vacancies (V) is the central behaviourial relationship of the model. Following Van Ours (1991) this matching function is specified as a Cobb-Douglas function which is homogenous of the first degree with parameter α and constant term c as a measure of labour market efficiency:

$$F_{ue} = c\, U'^{\alpha}\, V^{1-\alpha} \qquad (1)$$

We presume vacancies to be homogenous but unemployment is assumed heterogenous and consists of k duration classes

where $U' = \sum_{k=1}^{\infty} U_k\, g(\theta, k)$

Here the weight of each duration class $g(\theta, k)$ depends on the duration dependence parameter θ and on the length of a spell of unemployment k. We assume that

$g(\theta, 1) > 0$ and $g(\theta, k) \geq 0$ for $k > 1$

We normalize the duration dependence parameter θ in such a way that when $\theta = 1$ we have no duration dependency (implying homogenous unemployment):

$g(1, k) =$ constant for all k

and, when $\theta=0$, the normalization is

$$g(0, 1) = g_1 \text{ and } g(0, k) = 0 \text{ for } k > 1, \text{ and some constant } g_1 > 0.$$

If

$$g(\theta, k) \geq g(\theta, k+1) \text{ for all } k$$

the weight is monotonously decreasing with the length of the unemployment spell and we have negative duration dependency. Positive duration dependency occurs when

$$g(\theta, k) \leq g(\theta, k+1) \text{ for all } k$$

In equilibrium the weight $g(\theta, k)$ can be identified as the escape probability from unemployment (or can be standardized to this escape probability by multiplying it with the escape probability form the first duration class p_1).

The second major mechanism described by the model is the *job destruction process*. In order to keep this model simple we make the flow from employment to unemployment dependent on total employment

$$F_{eu} = \mu_1 E \tag{2}$$

The parameter μ_1 in the above equation determines the Poisson process of job separations (see Pissarides, 1990, p.6) and is exogenous to the model. Blanchard and Diamond (1989) distinguish a quit rate (q) and an 'unproductivity' rate (π_0) where $\mu_1 = q + \pi_0$. In an extended behaviourial model of job destruction the parameter μ_1 can depend upon other characteristics of economic development, such as the state of the cycle, cy, and the wage level, w, so that

$$\mu_1 = \mu_1 (cy, w, ..)$$

It is important to note that parameter μ_1 is associated with various aspects of economic development because in equilibrium changes of this parameter lead to shifts of the UV-curve.

We now come to the *job creation mechanism*, which causes an inflow of new vacancies VI_j. In this general setup of the model we consider job creation as an autonomous process:

$$VI_j = VI_j (...) = \text{autonomous} \tag{3}$$

but we will see that in UV-analysis this job creation process is often implicitly determined by the equilibrium condition.

Labour supply constitutes the final major mechanism of the dynamic flow model. We assume that all new participants enter the labour market through unemployment. The net inflow of these new participants, F_n, depends on the flow of unemployed who leave the labour force and become non-participant, F_{un}, and the flow of non-participants who register as unemployed, F_{nu}:

$$F_n = F_{nu} - F_{un}$$

Net change of labour supply is assumed autonomous in the model:

$$F_n = F_n(\ldots) = \text{autonomous} \tag{4}$$

but may as well be implicitly determined as equilibrium condition.

The specification of the model is completed by definition equations or equations of motion for the net flows into the three stocks distinguished by the model: employment (E), unemployment (U) and vacancies (V).

The change of total employment ($\Delta E = E - E_{-1}$) is the net result of gross inflow and gross outflow:

$$\Delta E = F_{ue} - F_{eu} \tag{5}$$

The same holds true for unemployment:

$$\Delta U = F_{eu} - F_{ue} + F_{nu} - F_{un} \tag{6}$$

$$= F_{eu} - F_{ue} + F_n$$

and for the stock of vacancies:

$$\Delta V = VI - VO$$

where VI represents gross inflow of vacancies and VO gross outflow. The outflow of unemployed who obtain a job by filling a vacancy is by definition equal to the number of filled vacancies:

$$VO = F_{ue}$$

Total inflow of vacancies consists of two parts. Firstly, new vacancies emerge because of the job creation process (VI_j) and secondly, new vacancies emerge because of the outflow from employment, F_{eu}. We assume that the latter new vacancies are a fraction μ_2 of total outflow from employment:

$$VI = VI_j + \mu_2 F_{eu}$$

Following Blanchard and Diamond (1989) we may identify μ_1 $(1 - \mu_2) = \pi_0$ as the fraction of jobs that becomes unproductive so that $q = \mu_1 - \pi_0 = \mu_1\mu_2$ represents the quit rate. Van Ours (1991) analyses a simplified version of this model as he sets μ_2 equal to 1. The definition equation for the change of vacancies now reads:

$$\Delta V = VI_j + \mu_2 F_{eu} - F_{ue} \tag{7}$$

Equations (1) - (7) compose our simple model of labour market flows.

3. Stationary dynamic equilibria

This section considers a simple *stationary dynamic equilibrium*, according to which the three stocks distinguished by the model remain constant. Hence we have the equilibrium conditions:

$$\Delta E = 0; \ \Delta U = 0; \text{ and } \Delta V = 0.$$

The first equilibrium condition implies that

$$F_{ue} = F_{eu} \qquad \text{so that} \qquad c\, U'^{\alpha}\, V^{1-\alpha} = \mu_1 E$$

According to this equilibrium condition, which confronts the matching process with the job destruction process we can re-write our model in such a way that the stock of vacancies is a function of the stock of employed and the stock of unemployed

$$V = \left(\frac{\mu_1 E}{c\, U'^{\alpha}} \right)^{\frac{1}{1-\alpha}} \tag{8}$$

Hence, given total employment and given the model parameters μ_1, α and c (and in case of duration dependence θ) equation (8) represents the stationary dynamic equilibrium locus for the UV-curve. In this paper we will, as most studies on equilibrium UV-analysis do, concentrate on the above specification of the UV-curve. However, in doing so, we acknowledge that both other equilibrium conditions implicitly determine the job creation process and the labour supply process. The equilibrium condition on unemployment ($\Delta U = 0$) implies that

$$F_n = 0$$

and the vacancy equilibrium ($\Delta V = 0$) then presupposes for the job creation process that

$$VI_j = F_{ue} - \mu_2 F_{eu}$$

$= \mu_1 (1 - \mu_2) E (= \pi_0 E)$

The working of equilibrium unemployment models from the literature differs with respect to the modelling of these two equilibrium conditions. The models of Pissarides (1990) and Blanchard and Diamond (1989) both contain an additional 'UV-curve' which intersects equation (8) so that there exists one unique unemployment equilibrium[2]. In the Pissarides model this 'vacancy supply' (VS-)curve is a reduced form equation combining the job creation and wage formation (labour supply) processes. Wage formation does not play a role in the Blanchard and Diamond model and labour supply is predetermined (but net labour supply is implicitly determined by the model). Here the vacancy supply process:

$$VI_j = \pi_1 I - \pi_0 V$$

with I the number of idle jobs (K = E + V + I: total jobs is given) and π_1 the rate at which jobs become productive, determines the second UV-curve[3]. Alternatively the job supply process may determine total employment:

$$E = VI_j / \{\mu_1 (1 - \mu_2)\}$$

In that case VI_j stems from the optimizing behaviour of the firm with total (expected) output, capacity utilization and factor costs (including transaction costs) as determinants. However, this paper does not further investigate these alternatives but concentrates on the UV-relationship of equation (8).

First we consider the case of homogenous unemployment where $\theta = 1$. The UV-equation (8) now boils down to

$$V=\left(\frac{\mu_1 E}{cU^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

which gives the relationship in the initial equilibrium between U and V, when E is (pre)determined.

The literature has paid ample attention to this unemployment equilibrium specification of the UV-curve, although policy analysis often overlooks that shifts of the UV-curve may have other causes than changes of labour market efficiency. In order to recapitulate and to indicate the order of magnitude of various UV-curve shifts, we illustrate by means of numerical examples how these shifts can be explained in a comparative analysis of

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[2] Pissarides (1992) recenly investigated models with multiple unemployment equilibria.

[3] In our notation, this UV-curve is

$$(q-\pi_1)L + \pi_1 K = cU^{/\alpha}V^{1-\alpha} + (q-\pi_1)U + (\pi_0+\pi_1)V \quad where \quad L = E + U$$

dynamic equilibria. The central example gives a stylised representation of the present situation in The Netherlands as if it were in equilibrium. Total employment, E = 4 million workers (in the market sector). The parameters of the matching function (α = 0.5 and c = 0.5) are borrowed from estimates by Van Ours (1991) whereas the parameter μ_1 is set equal to 0.01. It means that in each period the job destruction is one percent of total employment. This value is taken from the calibration procedure of the model by Den Butter and Van Ours (1992), which is specified on a quarterly basis.

Figure 1 **UV-curves for different parameter values α of the Cobb-Douglas matching function**

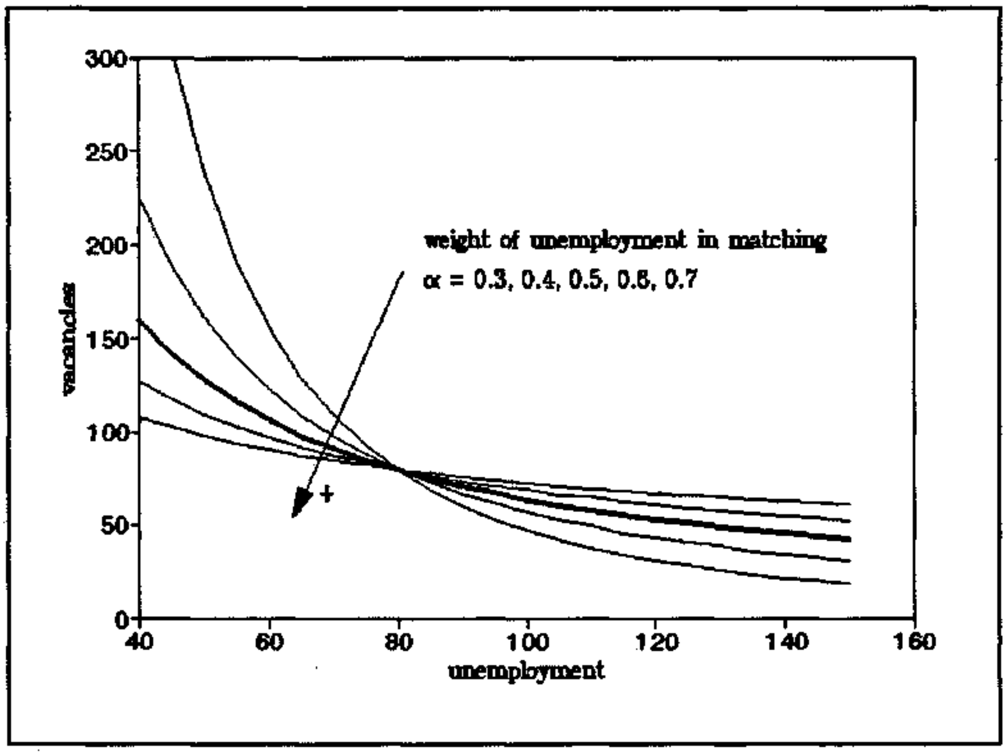


Figure 1 pictures the UV-curve for this basic numerical example together with alternative curves for different values of the parameter α. All curves intercept in the point where the total number of vacancies is equal to the total number of unemployed. In the present situation in The Netherlands, with much more unemployed than vacancies, a higher value of the parameter α appears to cause an upward shift of the UV-curve.

Figure 2 gives the UV-curve for different values of the efficiency constant c. Obviously, when the labour market becomes less efficient the UV-curve shifts upwards.

Figure 2 **UV-curves for different values for the efficiency constant c of the Cobb-Douglas matching function**

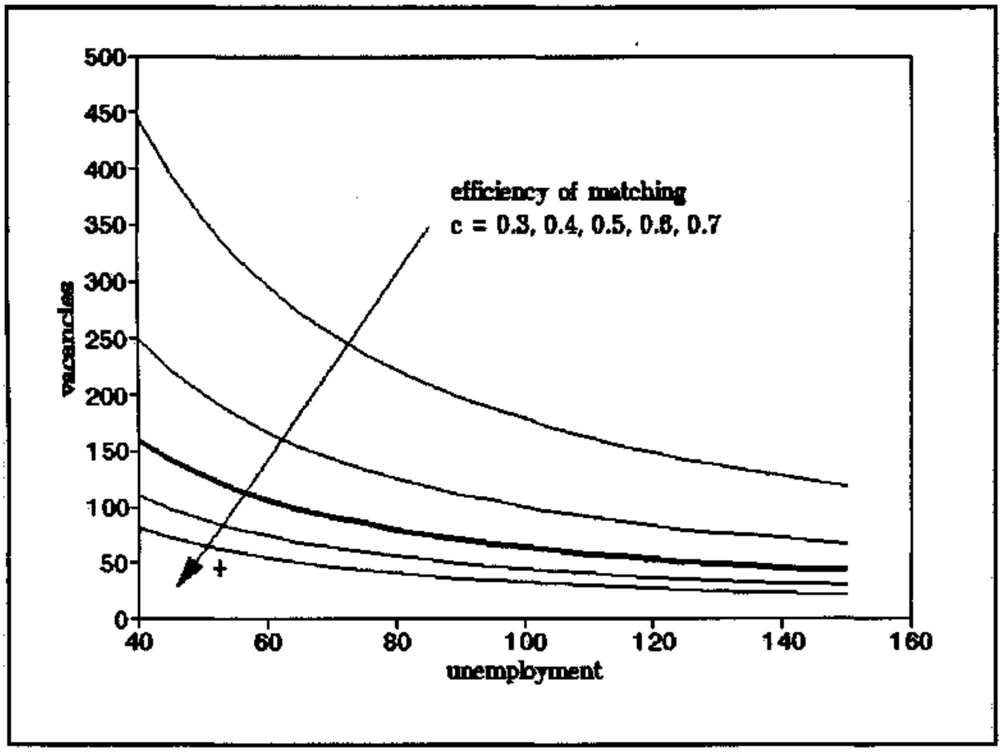


Figure 3 **UV-curves for different values for the job destruction parameter μ_1**

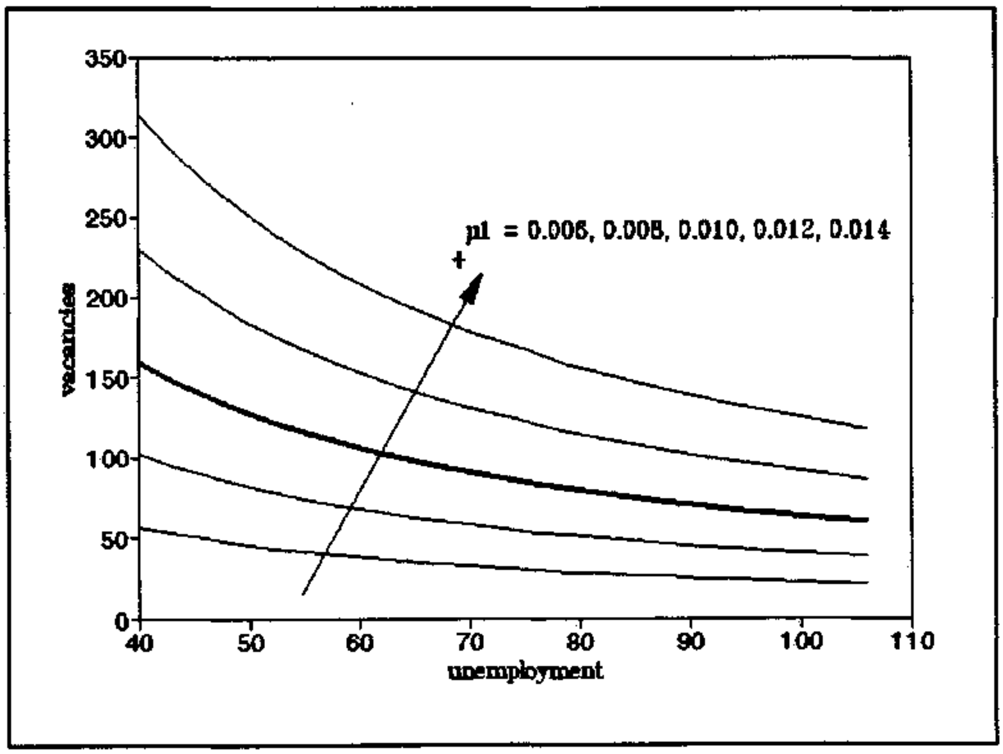


Figure 3 illustrates how the position of the UV-curve does in equilibrium depend upon the value of parameter μ_1 and hence on the job destruction process. An increase in this parameter, for instance caused by a cyclical downswing or by another mechanism which results in a larger job destruction (e.g. fast structural change, creative destruction due to a positive technology shock), makes the UV-curve shift upwards. It demonstrates that an upward shift of the UV-curve should not necessarily be associated with less labour market efficiency, as traditional UV-curve analysis does. Under constant employment such shift can also originate from parameter changes of the matching process or from changes of the job destruction process.

We note that all UV-curves of these figures are truncated to the left because our calculations use a discrete time model which has equilibrium unemployment bounded from below by the constant flow into this stock. This flow equals $\mu_1 E$, which is positive with regular employment and job destruction.

4. Duration dependency and dynamic equilibria

The previous section considered unemployment equilibria without duration dependency ($\theta=1$). Now we investigate such equilibria in case of duration dependency; that means that the escape probability from unemployment changes with the length of the spell of unemployment. The most likely case is that of negative duration dependency: the escape probability from unemployment decreases when the spell of unemployment lasts longer. We will not differentiate to the cause of this duration dependency at macro level, which can either occur because of duration dependency at micro level or because of heterogeneity at micro level.

4.1 Two classes of unemployed with one period short-term unemployment

We first distinguish two classes of unemployed, namely short-term unemployed (U_S) and long-term unemployed (U_L). Now short-term unemployment is supposed to last for one period and thereafter the unemployed becomes long-term unemployed. It implies that $U_S = U_1$ (the first duration class)

In accordance with the general specification of our weight function $g(\theta, k)$ in case of negative duration dependency we assume that the long term unemployed have a smaller escape probability from unemployment, p_L, than the short term unemployed, p_S, where for $0 \leq \theta < 1$, $p_L = \theta\, p_S$. We now have $U' = (U_S + \theta\, U_L)$. A stationary equilibrium again implies that $F_{eu} = F_{ue} = F = \mu_1 E$, and additionally that $U_S = F$ and $U_L = (U' - F)/\theta$. Given total employment E and the parameters of the flow model c, α, μ_1 and θ, and recalling that $U = U_S + U_L$, the UV-curve in equilibrium becomes

$$V = \left(\frac{\mu_1 E}{c[(1-\theta)\mu_1 E + \theta U]^{\alpha}} \right)^{\frac{1}{1-\alpha}} \qquad (9)$$

Figure 4 **Duration classes and flows into and out of unemployment**

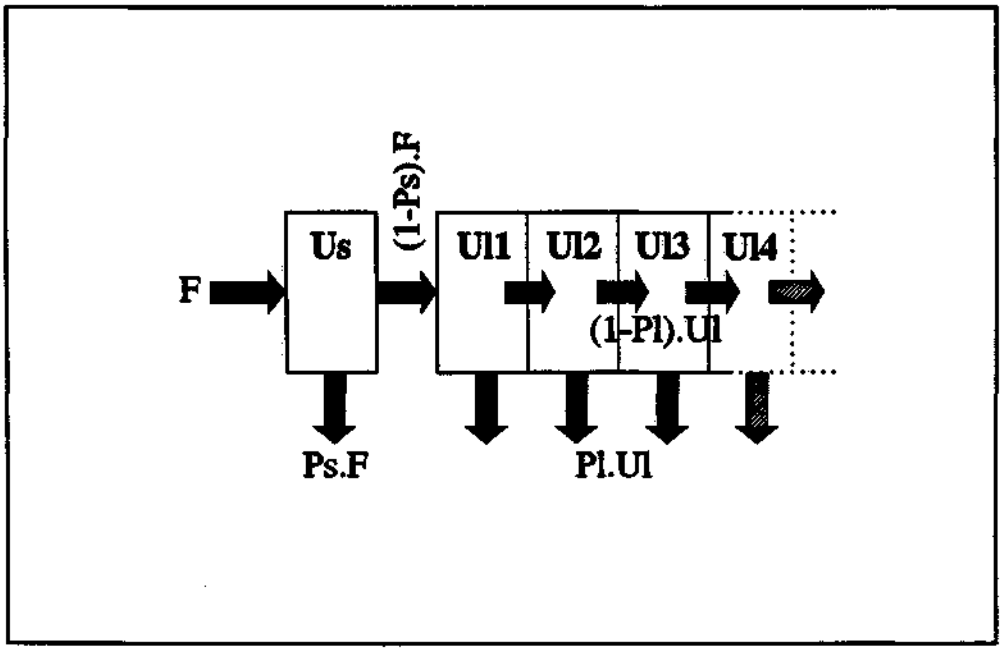


Figure 5 **UV-curves for different values for the duration dependence parameter θ**

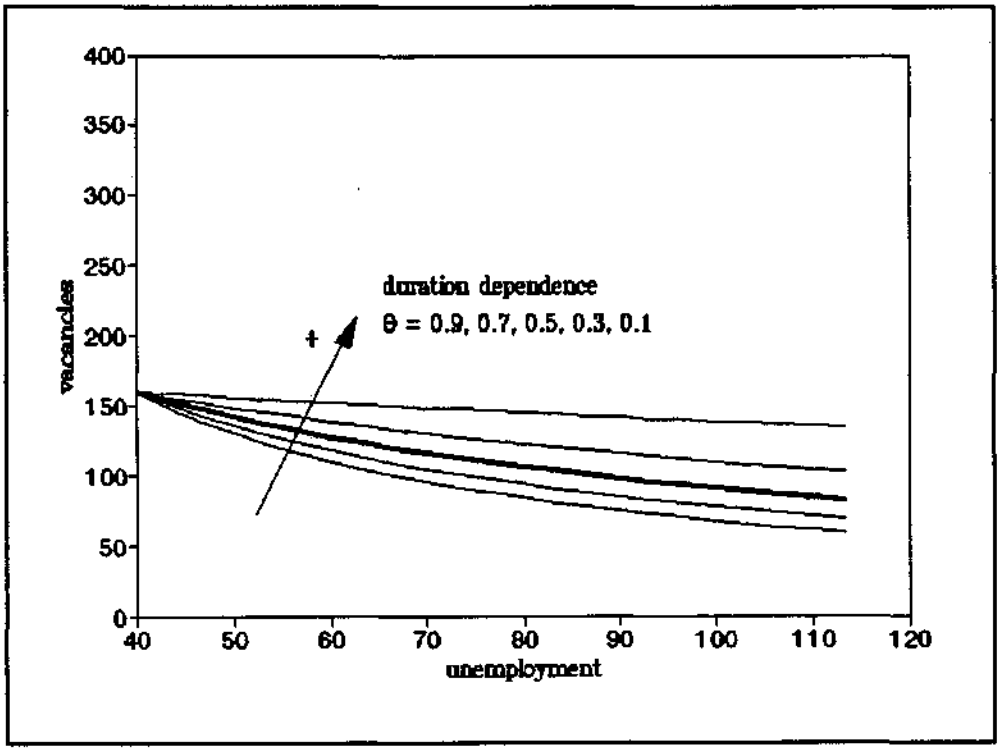


Figure 4 illustrates that the number of unemployed in all duration classes remains constant in case of a stationary dynamic equilibrium because the inflow into each class is equal to the outflow from that class. Total inflow into (short term) unemployment, and total outflow from unemployment (as result of the matching process) is equal to F. During each period $p_S F$ of the short term unemployed find a job and $(1-p_S)F$ become long term unemployed, whereas $p_L U_L = \theta p_S U_L$ of the long term unemployed find a job and $(1-p_l)U_L$ remain long term unemployed. The condition of a constant number of long term unemployed requires that $(1-p_S)F = p_L U_L$. From $p_S = F/U' = F/(U_S+\theta U_L)$ and $U_S = F$, we derive that $U_L = (1-p_S)F/(\theta p_S)$, and the above condition follows immediately: $p_L U_L = \theta p_S U_L = (1-p_S)F$.

Formula (9) allows us to draw the UV-curves for different values of the duration dependence parameter θ. Figure 5 illustrates that when duration dependency becomes smaller and hence when the parameter value of θ approximates unity the UV-curve shifts to the left. However, as compared to shifts induced by changes in other parameter values, these shifts appear to be rather small.

4.2 Two classes of unemployed with n period short-term unemployment

When we relax the assumption for short-term unemployment and have short-term unemployment last for n periods, (for instance n=4 in a quarterly model and short-term unemployment comprising all persons who are less than one year unemployed - the usual breakdown in empirical studies of insider-outsider analysis in The Netherlands) there is no essential difference from the model of the previous sub-section. We now have $U' = (U_S + \theta\ U_L)$ with U_S n periods where $p_S=F/U'$and $p_L=\theta F/U'$. A necessary condition for equilibrium is that for i=1,..,n

$$U_S = \sum_{i=1}^{n} U_{Si} \tag{10}$$

with

$$U_{Si} = (1-p_S)^{i-1} F \tag{11}$$

so that

$$U_L = \frac{U' - \sum_{i=1}^{n} (1-p_S)^{i-1} F}{\theta} \tag{12}$$

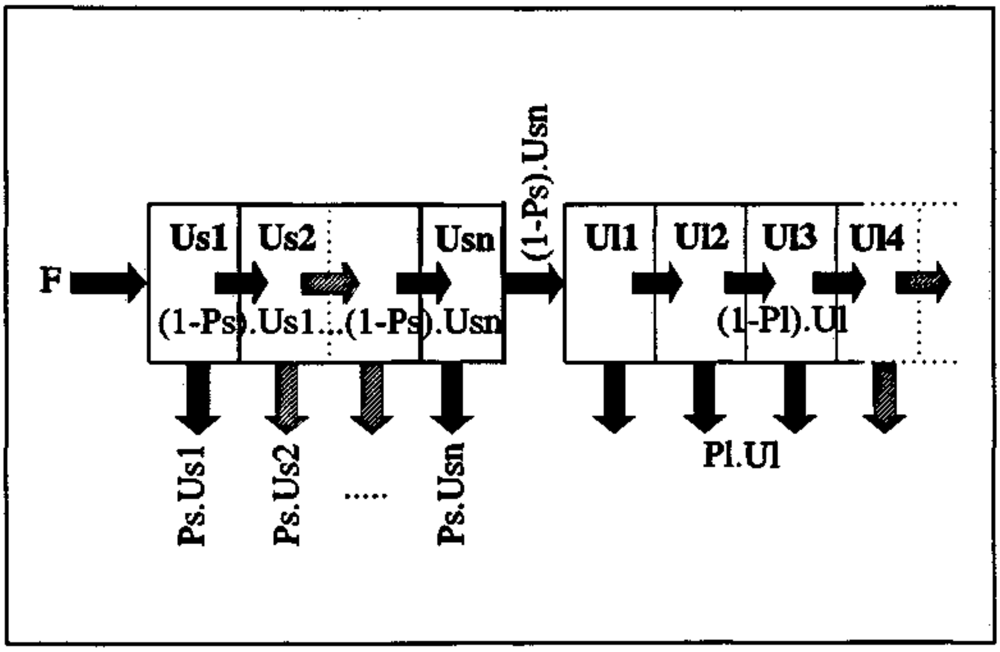
Given total employment E and the parameters of the flow model k, α, μ_1 , θ, and p_S the following equilibrium UV-curve results

$$V = \left[\frac{\mu_1 E}{c[(1-\theta)\sum_{i=1}^{n}(1-p_S)^{i-1}\mu_1 E + \theta U]^{\alpha}} \right]^{\frac{1}{1-\alpha}} \tag{13}$$

Note, however, that the equilibrium UV-locus cannot be derived analytically from (13) because p_S now depends on U'. Therefore, in our calculations, we use a numerical approximation.

Figure 6 illustrates the constant flows of unemployed through the duration classes, and into and out of unemployment in case of a dynamic equilibrium and n duration classes of short term unemployed. Total inflow into (short-term) unemployment, and total outflow from unemployment is again equal to F. $p_S F$ of the short term unemployed of the first duration class find a job and $(1-p_S)F$ move to the following duration class. This process continues until the nth duration class, which contains $(1-p_S)^{n-1}F$ short-term unemployed. $p_S(1-p_S)^{n-1}F$ of them will find a job and $(1-p_S)^n F$ become long-term unemployed. During each period $p_L U_L$ of the long-term unemployed find a job so that the condition of a constant number of long-term unemployed requires that $(1-p_S)^n F = p_L U_L$. From $p_S = F/U'$ we derive that $U_L = (F-p_S U_S)/(\theta p_S)$. Moreover we now have a total of $F \sum_{0\leq i\leq n-1}(1-p_s)^i = F[1-(1-p_S)^n]/p_S$ short-term unemployed. Therefore it follows that the flow out of long term unemployment is equal to $p_L U_L = \theta p_S U_L = F - p_S U_S = (1-p_S)^n F$, which proves that each unemployment duration class is constant over time. We note that the equilibrium of the previous subsection is a special case of the present formulation with n=1, and that there is no substantial difference with respect to a shift of the equilibrium UV-curve caused by a change of duration dependency.

Figure 6 **Duration classes and flows into and out of unemployment**



4.3 Smooth escape probabilities

Now we assume an infinite number of duration classes and a smooth function for the set of weights $g(\theta, k)$. It establishes a link with the unemloyment duration model as the escape probability defined by the weight function $g(\theta, k)$ is equal to the hazard rate implied by the probability distribution of the duration model. For instance, in case of no duration dependency with $\theta = 1$, we have a constant escape probability. Such constant hazard follows from an exponential distribution of unemployment duration.

Generally, in equilibrium we have the escape probabilities p_k, $k \epsilon N$, follow from

$$p_k = \frac{g(\theta,k)F}{U'}, \quad k=1,2,\cdots \tag{14}$$

with $F=U_1$ the inflow of unemployed into the first duration class and the weight function $g(\theta, k)$ determined by the distribution function of the duration model. Here the weight function is normalized: $g(\theta, 1) = 1$ so that $p_1 = F/U'$ and $p_k = g(\theta, k)p_1$. Given E, c, α, and μ_1 we can calculate U' from equation (8) for each number of vacancies V. Then, from

$$U_k = (1-p_k)U_{k-1} = U_1 \prod_{j=1}^{k-1} (1-p_j), \quad k=2,3,\cdots \tag{15}$$

follows that the number of unemployed in the following duration classes is constant as well and that total equilibrium unemployment is equal to

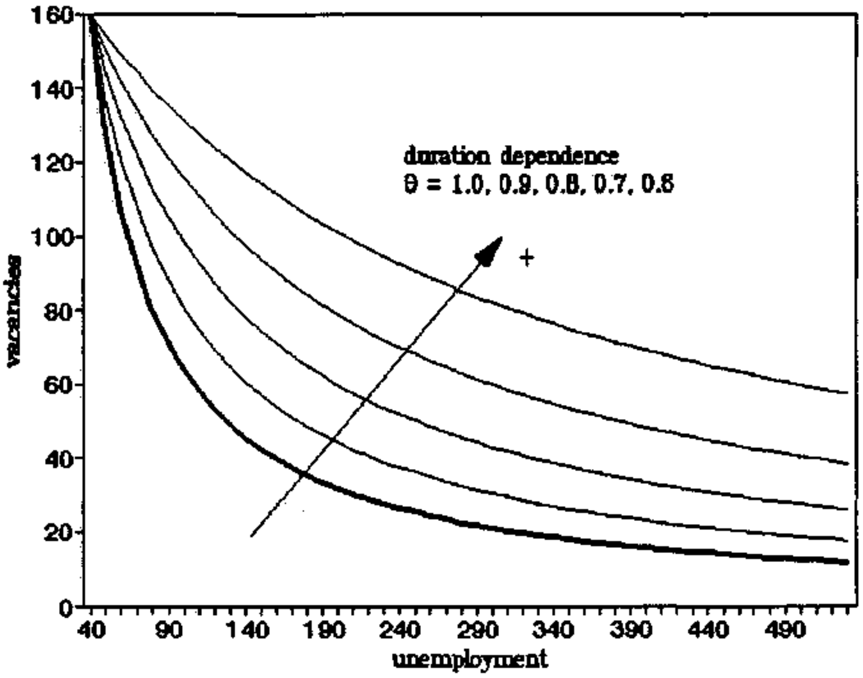
$$U = \sum_{i=1}^{\infty} U_i = U_1 \left[1 + \sum_{i=2}^{\infty} \prod_{j=1}^{i-1} (1-p_j) \right] \tag{16}$$

It proves the existence of an unemployment equilibrium UV-curve for each distribution function of unemployment duration which yields regular escape probabilities from unemployment.

Microeconomic duration models utilize a number of different specifications for the distribution function (see e.g. Lancaster, 1990). Unfortunately the *Gamma distribution* with density function $f(k) = \mu^m.k^{m-1}.e^{-\mu k}/\Gamma(m)$ generally has no explicit solution for the hazard rate. In the most simple case, however, the two step Gamma distribution with $m=2$, where the duration is described by a convolution of two exponential distributions with parameter μ, the hazard is given by $p_k = \mu^2 k/(1+\mu k)$. However, as the first derivative $p'_k = \mu^2/(1+\mu k)^2 > 0$ because $\mu > 0$, this Gamma distribution only allows for positive duration dependency which is rather unlikely from an economic point of view.

The two parameter *Weibull distribution* with density function $f(k) = \beta.\mu^\beta.k^{\beta-1}.\exp[-(\mu k)^\beta]$ appears to yield a good alternative. Here we have the hazard rate $p_k = \beta.\mu^\beta.t^{\beta-1}$, and its first derivative $p'_k = \beta.(\beta-1).\mu^\beta.k^{\beta-2}$ shows that $0<\beta<1$ describes the case of negative duration dependency, if $\beta=1$ we have no duration dependency (and the exponential distribution function), and $\beta>1$ gives positive duration dependency. We consider the likely case of negative duration dependency and note that in the macro model the specification of the weight function $g(\theta, k)$ matters which describes relative escape probabilities. Therefore we may simplify the specification of the Weibull hazard to $p_k = k^{\theta-1}$, where θ has the same interpretation as before. Figure 7 gives the UV-curves for various values of the parameter θ. The parameter values in this sensitivity analysis are selected in such a way that the escape probabilities form unemployment have, on avarage, the same values as in the case of two unemployment classes. However, comparison of figure 7 with figure 5 shows that the shifts of the UV-curve are much more pronounced with the Weibull distribution than with the 'two-step' duration dependency.

Figure 7 **UV-curves for different values for the duration dependence parameter θ in case of the Weibull distribution**



The Weibull distribution of unemployment duration describes an escape probability which decreases rather slowly along with the spell of unemployment k. As an alternative with faster decreasing escape probabilities for long term unemployed one can think of an exponential hazard, where

$$p_k = e^{(k-1)\zeta}\, p_1 \quad \text{with} \quad \zeta \equiv (\theta-1)/\theta$$

This exponential hazard can be associated with a kind of extreme value distribution for the unemployment duration. However, the Annex proves that equilibrium unemployment according to equation (16) does not converge to a finite number of unemployed - except for the trivial case of $p_1 = 1$, when each unemployed finds a job in the first duration class. Loosely explained it means that for long term unemployed escape probabilities become so small that an ever growing number is unable to escape from unemployment. From this point of view equilibrium unemployment is not compatible with a fast decay of escape probabilities from unemployment. Of course, this is a theoretical problem only as in practice we have no infinite number of duration classes but the duration distribution is truncated: individual escape probabilities always become 1 for some $k > k_0$ because unemployed finally reach the age of retirement or decease.

4.4 Relationship between average durations

Microeconomic duration analysis almost always studies unemployment duration, job duration or vacancy duration in isolation from each other. The flow approach of this paper shows how these duration data are mutually dependent. The specification of the distribution function of unemployment duration, and the specification and parameters of the rest of the flow model fully determine employment duration and vacancy duration. We have:

average unemployment duration: $u_d = U/F_{ue}$
average employment duration: $e_d = E/F_{eu}$
average vacancy duration: $v_d = V/F_{ue}$

which are constant in case of a dynamic unemployment equilibrium, when $F = F_{ue} = F_{eu}$. Then the relationship between these three duration variables for the model without unemployment duration dependency follows from equation (8):

$$v_d = \left(\frac{\mu_1 e_d}{c u_d^{\alpha}} \right)^{\frac{1}{1-\alpha}} \qquad (17)$$

whereas similar equations as (9) and (13) hold in case of unemployment duration dependency.

Table 1 gives, by way of sensitivity analysis, numerical examples of the relationship between these three average durations for selected parameter values of the model. All avarage durations are expressed in quarters. Note that average employment duration is equal to the total period that workers remain employed, maybe in different jobs, because the model does not reckon with job to job mobility.

The table shows that an increase in the pace of job destruction decreases average employment duration (compare 3. and 4. with 1. and 2.). This shift of the UV-curve also brings about a fall of the average unemployment duration and an increase of average vacancy duration. On the other hand, in equilibrium average employment duration appears not to depend upon movements along the UV-curve (compare 1.,3.,5., and 7. with 2.,4.,6., and 8., resp.), nor on shifts of the UV-curve because of a change of duration dependency (compare 1. and 2. with 5. and 6.) and because of a change in the Cobb-Douglas parameter of the matching function (compare 1. and 2. with 7. and 8.). These latter shifts only affect vacancy duration which decreases with less unemployment duration dependency and with a higher weight of unemployment in the matching function.

Table 1. Average durations with selected parameter values

	parameters				average durations		
	μ_1	θ	U (x1000)	α	u_d (quarters)	e_d	v_d
1.	0.01	0.5	500	0.5	12.5	100.0	0.5
2.	0.01	0.5	200	0.5	5.0	100.0	1.0
3.	0.05	0.5	500	0.5	2.5	20.0	1.8
4.	0.05	0.5	200	0.5	1.0	20.0	4.0
5.	0.01	1.0	500	0.5	12.5	100.0	0.3
6.	0.01	1.0	200	0.5	5.0	100.0	0.8
7.	0.01	0.5	500	0.7	12.5	100.0	0.1
8.	0.01	0.5	200	0.7	5.0	100.0	0.4

Explanatory note: E is set to 4,000,000 labour years and the efficiency parameter c to 0.5. The model from section 4.2 with 4 short term unemployment classes is used.

5. Steady state growth dynamic equilibria

We now extend our analysis of unemployment equilibria to steady state growth. Given starting values U_0, V_0, E_0 and (employment) growth rate g this steady state equilibrium imposes the following conditions to the labour market stocks

$$U_t = U_0 e^{gt};\ V_t = V_0 e^{gt} \text{ and } E_t = E_0 e^{gt}.$$

Obviously an unemployment equilibrium with constant stocks, discussed in the previous sections, is a special case of the present definition with g = 0.

5.1. Basic model without duration dependency

The generalisation of the basic model for steady state growth is rather straightforward in case for a matching function which is homogeneous of the first degree (see e.g. Blanchard and Diamond, 1989, and Van Ours, 1991). Combining the matching function and the job destruction process gives the following equilibrium UV-curve under the condition $E_t = E_0 e^{gt}$:

$$V = \left[\frac{(\mu_1 + g')E}{cU^{\alpha}}\right]^{\frac{1}{1-\alpha}} \tag{18}$$

where $g' = \left[1 - \frac{1}{e^g}\right]$

Now the equilibrium conditions $U_t = U_0\, e^{gt}$ and $V_t = V_0\, e^{gt}$ imply the following initial conditions for the job destruction process

$$F_{n0} = g'(U_0 + E_0) \tag{19}$$

and for the labour supply process

$$VI_{j0} = \mu_1(1-\mu_2)E_0 + g'(V_0 + E_0) \tag{20}$$

Alternatively these processes may imply an additional equilibrium UV-curve and hence one unique steady state dynamic equilibrium at the intersect of both curves.

Figure 8 **UV-curves for different values for the growth rate g according to the model with no duration dependency**

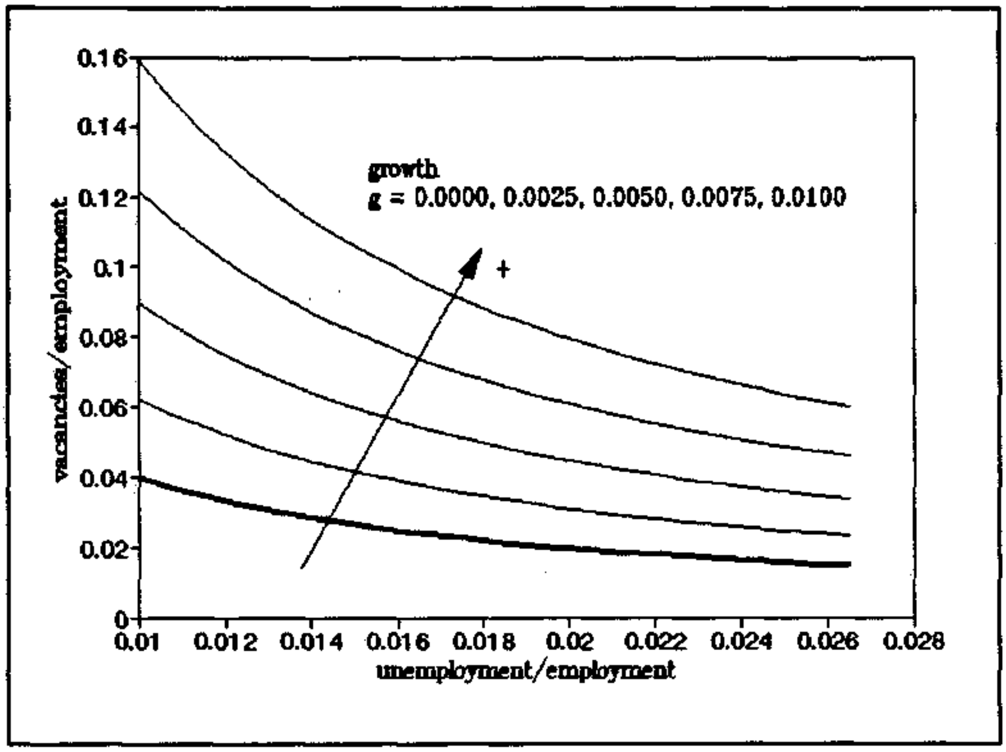


Figure 8 illustrates shifts of the UV-curve due to different (quarterly) employment growth rates of the economy according to the basic model without unemployment duration dependency. From equation (18) it is obvious that these shifts look much alike the shifts induced by changes in the job destruction parameter μ_1. We note that on the axes the stocks of vacancies and unemployed are now expressed as a ratio of total employment, because in equilibrium all stocks grow at the same pace.

5.2. Basic model with duration dependency

It is straightforward to show that a situation of equilibrium unemployment can exist in the model with steady state growth in case of unemployment duration dependency. Given the weight function $g(\theta, k)$, we have for period 0

$$U'_0 = \sum_{k=1}^{\infty} U_{0,k}\, g(\theta,k)$$

with

$$U_{0,1} = F_{-1} = F_0 e^{-g}$$

$$U_{0,2} = (1-p_1)U_{-1,1} = (1-p_1)F_{-2} = (1-p_1)F_0 e^{-2g}$$

$$U_{0,k} = \prod_{j=1}^{k-1} (1-p_j) F_0 e^{-kg}$$

so that

$$U_0 = F_0 e^{-g} + \sum_{k=2}^{\infty} F_0 e^{-kg} \prod_{j=1}^{k-1} (1-p_j)$$

This gives the initial level of the variables of interest on the equilibrium growth path. This first step of the proof shows that, given constant escape probabilities, an initial situation exists which is consistent with equilibrium unemployment. Now we have $U_t = U_0 e^{gt}$ and $U_{t,k} = U_{0,k} e^{gt}$ for each k, so that total unemployment and unemployment in all duration classes have the same growth rate g. It also holds for U' and F so that the escape probabilities p_k remain constant as well on the steady state growth path which is characterized by the initial conditions of the formulas above. Hence, the second step of the proof shows the existence of unemployment equilibrium growth with all stocks and flows growing at rate g and with constant escape probabilities. We do not perform a sensitivity analysis here because this version of the model does not differ essentially from the models of the previous sections.

6. Conclusions

Unemployment equilibria in dynamic models of the labour market provide insight into the various sources of shifts of the UV-curve. This paper has illustrated that these shifts of UV-curves should not only be associated with changes of labour market efficiency, but can also be the result of changes of weights attached to vacancies and unemployment in the matching process, of changes of the pace of the job destruction process, and of changes of the employment growth rate of the economy. Ample attention is paid to the case of duration dependent escape probabilities from unemployment. We have shown that unemployment equilibria with duration dependency do exist and we have given numerical examples of how various assumptions on the distribution of unemployment affect the equilibrium unemployment locus of the UV-curve. The link with microeconomic (unemployment) duration analysis is established by considering a number of alternative distribution functions for unemployment duration. The Weibull distribution appears to be most appropriate as it yields (for a specific range of the parameter values) escape probabilities with negative duration dependency, which is relevant from an economic point of view. Moreover, we have shown that a fast (e.g. exponential) decline of the escape probabilities is not consistent with a situation of equilibrium unemployment because in that case the expectation of unemployment duration has become indefinite. The dynamic labour market model also describes the mutual relationship on the macro level between unemployment duration, employment (or job) duration and vacancy duration. These duration variables are usually considered independent of each other in microeconomic duration analysis.

The paper has focused on unemployment equilibria without questioning whether they are stable or how the model does react on shocks which bring the model out of equilibrium (see e.g. Pissarides, 1990, for an analysis of loops around the UV-curves). In most versions of our model these equilibrating mechanisms can only be described by numerical simulations as they depend upon the specification and parameter values of the model. Such simulations involve an extensive casuistry. Therefore a good research strategy is to construct an empirical dynamic labour market model and investigate the effects of shocks (or policy measures) by means of impulse analysis. The unemployment equilibria explored in this paper can, in that case, act as long run benchmark solutions.

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List of symbols

Stocks

E	employment
U	unemployment
U_S	short term unemployment
U_L	long term unemployment
U_k	unemployment in the k-th duration class, k = 1,2,3,...
U'	weighted unemployment
V	vacancies
K	Total number of jobs (B&D)
I	Idle jobs (B&D)

Flows

F_{eu}	Workers who become unemployed by losing their job
F_{ue}	Unemployed who become employed by finding a job
F_{nu}	Non-participants who register as unemployed
F_{un}	Unemployed leaving the labour force
F_n	Net flow into the labour force of new participants
VI_j	Inflow of new vacancies
VI	Gross inflow of vacancies
VO	Gross outflow of vacancies
F	Equilibrium flow from employment to unemployment and v.v.

Parameters

c	efficiency constant in matching process
α	Weight of unemployment in matching process
θ	Duration dependence parameter
μ_1	Fraction of employed that leave their jobs
μ_2	Fraction of separated jobs that become new vacancies
q	Quit rate (B&D)
π_0	Unproductivity rate (B&D)
π_1	Rate at which jobs become productive (B&D)
n	Number of short term unemployment classes
p_S	Escape probability of short term unemployed
p_L	Escape probability of long term unemployed
p_k	Escape probability of unemployed in the k-th duration class
g	Growth rate
g'	$1 - 1/e^g$

Other symbols

g(.)	Weight function of unemployment duration
cy	State of the cycle (B&D)
w	Wages (B&D)
e_d	Average employment duration
u_d	Average unemployment duration
v_d	Average vacancy duration

Note: B&D: notation of Blanchard and Diamond (1989)

<u>Annex</u>

Proposition

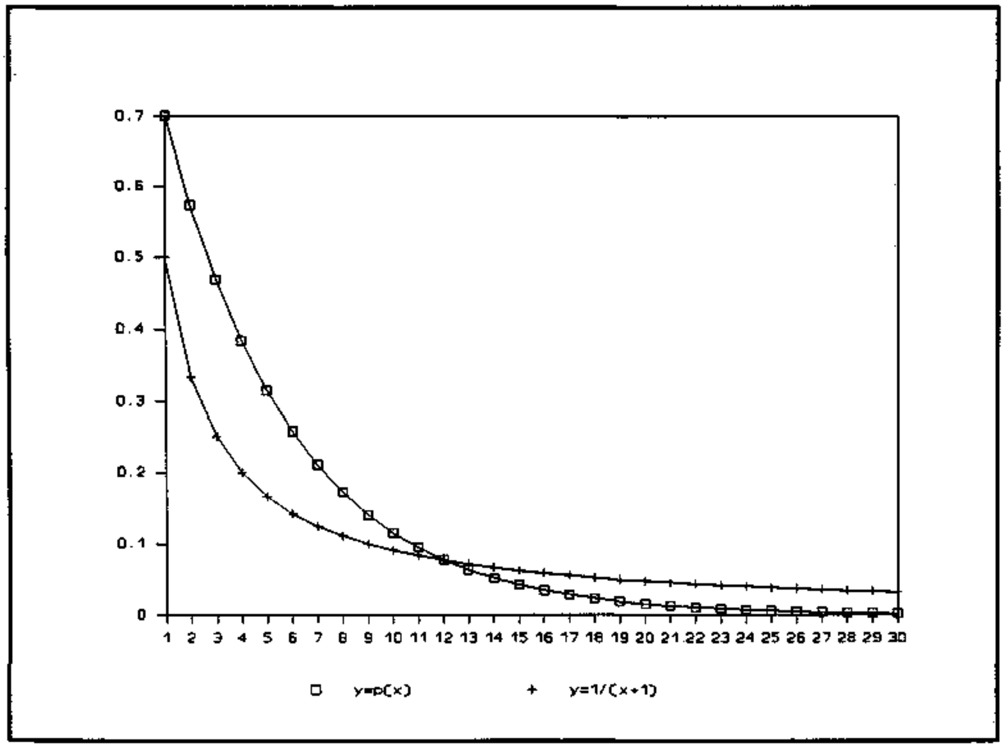
$$S_n = \sum_{k=1}^{n} \prod_{j=1}^{k} (1 - p_j) \text{ diverges when } n \to \infty \text{ for } p_1 < 1', \quad \text{where } p_j = p_1 e^{(1-j)\theta} \tag{A1}$$

Proof

Figure A1 shows that

$$\exists\, n_0 \in \mathrm{N}: j \geq n_0 \Rightarrow p_j \leq \frac{1}{j+1} \Leftrightarrow 1 - p_j \geq \frac{j}{j+1} \tag{A2}$$

Figure A1 **Intersection of p_j (ζ=0.2;p_1=0.7) and p_a = 1/(j+1)**



Write S_n, $n \geq n_0$, as the sum of S_1 and S_2, where

$$S_1 \equiv \sum_{k=1}^{n_0-1} \prod_{j=1}^{k} (1-p_j) \text{ if } n_0 > 1 \text{ and } S_1 \equiv 0 \text{ if } n_0 = 1$$

and

$$S_2 \equiv \sum_{k=n_0}^{n} \prod_{j=1}^{k} (1-p_j)$$

We will prove that S_2, and therefore S_n, diverges. Using (A2) we can derive lower bounds for the terms in S_2,

$$\prod_{j=1}^{k} (1-p_j) = P_{n_0} \prod_{j=n_0}^{k} (1-p_j) \geq P_{n_0} \prod_{j=n_0}^{k} \frac{j}{j+1} = P_{n_0} \frac{n_0}{n_0+1} \frac{n_0+1}{n_0+2} \cdots \frac{k}{k+1} = P_{n_0} \frac{n_0}{k+1},$$

where $P_{n_0} \equiv \prod_{j=1}^{n_0-1} (1-p_j)$ if $n_0 > 1$ and $P_{n_0} \equiv 1$ if $n_0 = 1$,

$k = n_0, n_0+1, \cdots,$

which implies

$$S_2 \geq P_{n_0} n_0 \sum_{j=n_0}^{n} \frac{1}{j+1}$$

The sum in this expression is well known to diverge when n approaches infinity[4]. Moreover, $p_1 < 1$ ensures that $P_{n0}n_0 > 0$. Thus, S_2, and therefore S_n, diverges. Q.E.D.

Summary

The dynamic labour market equilibria of this paper show that shifts of UV-curves should not solely be associated with changes of labour market efficiency. A shift of the UV-curve may also occur because of other changes in the matching process or job destruction process. The position and shape of the UV-curve appears to depend as well on the cyclical situation and on the rate of employment growth. The paper pays ample attention to the effects of duration dependent escape probabilities form unemployment on dynamic labour market equilibria. It establishes a link between microeconomic duration analysis and the mutual dependency of unemployment duration, employment duration and vacancy duration on the macro level.

[4] Incidentally this sets the limiting case for the Weibull distribution with $\theta=0$.

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