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Rietveld, P.; Janssen, R.

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SENSITIVITY ANALYSIS IN DISCRETE MULTIPLE
CRITERIA DECISION PROBLEMS:
ON THE SITING OF NUCLEAR POWER PLANTS

P. Rietveld
R. Janssen

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VRIJE UNIVERSITEIT
FACULTEIT DER ECONOMISCHE WETENSCHAPPEN
EN ECONOMETRIE
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ABSTRACT

Inclusion of evaluation methods in decision support systems gives way to extensive sensitivity analysis. In this article new methods for sensitivity analysis are developed and applied to the siting of nuclear power plants in the Netherlands.

1. INTRODUCTION

The use of decision support systems in public planning is slowly increasing following developments in private enterprise. This development results in a renewed interest in formal evaluation methods such as multicriteria analysis and cost benefit analysis. A major advantage of the integration of evaluation methods in a decision support system are the increased opportunities for sensitivity analysis. The availability of a wide range of procedures for sensitivity analysis allows the decision maker to investigate the limits of a decision problem (see figure 1).

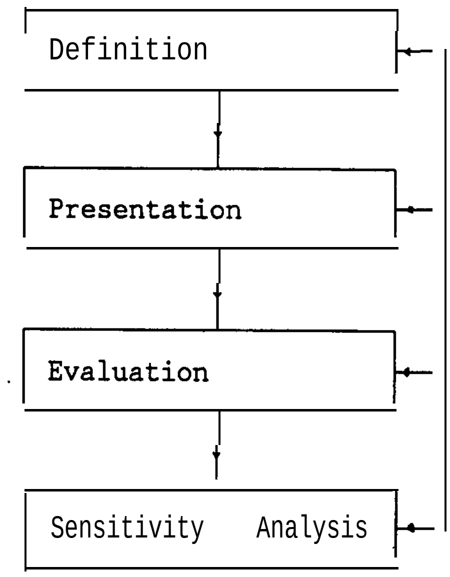

Figure 1. Feed back loops in a decision procedure.

The main focus of this article is the use of procedures for sensitivity analysis on results from discrete evaluation problems. This type of evaluation can be performed by a wide range of evaluation methods such as cost benefit analysis and multicriteria methods.

In the first three steps of an evaluation procedure scores are assigned to all alternatives, weights to all criteria and a ranking of the alternatives is produced. Especially in decisions that involve negotiations or public debate it is useful to know within which limits the derived rankings hold. This results in the following types of questions:

- to what extent can these scores of weights increase or decrease without changing this ranking (calculation of robustness intervals).
- how similar is the set of weights that produces the first rank reversal.

We will describe procedures to deal with these questions followed by an application to the highly controversial decision on the location of two nuclear power plants in the Netherlands.

The procedures described in this article are included in our decision support system DEFINITE. This system is developed to support DEcisions based on a FINITE set of alternatives. This system contains a wide range of procedures to assist all steps in the evaluation procedure. This makes it possible to feed back results of sensivitity analysis directly to problem definition and evaluation. (See Herwijnen and Janssen, 1988, for a description of DEFINITE, and Rietveld, 1988, for a complete description of the procedures for sensitivity analysis in DEFINITE.)

Sections 2 and 3 of this article are devoted to a forma1 introduction of discrete multicriteria methods and procedures for sensitivity analysis respectively. In section 4 the multicriteria methods are applied to rank locations for nuclear plants in the Netherlands. The derived rankings are analyzed using various procedures for sensitivity analysis. Finally, the usefulness of this type of approach is discussed in section 5.

2. DISCRETE MULTIPLE CRITERIA METHODS

An important aim of discrete multiple criteria analysis is to provide a rational basis for ranking a number of alternatives on the basis of multiple criteria. There are many different discrete multiple criteria methods currently in use (see, e.g. Nijkamp, 1979; Rietveld, 1980, Voogd, 1983).

A major step in these methods is the construction of an impact (or evaluation) matrix representing the effect of a certain alternative on a decision criterion. In order to aggregate the information of the evaluation matrix usually a weighting scheme is necessary which expresses the relative importance of the various criteria. The impact matrix will be denoted by the symbol P. This matrix has elements pij which represent the impact of alternative i ($i = 1, \ldots, I$) on the value of criterion j ($j = 1, \ldots, J$). The vector of weights is denoted as $\underline{\lambda} = (\lambda_1, \ldots, \lambda_J)$. It is often assumed that the criteria have been defined in such a way that all weights are positive. In addition, one may impose the restriction that the weights add up to unity. Thus, the set S of feasible weights can be defined as:

$$S = \{ \underline{\lambda} \mid 0 \leq \lambda_j \leq 1 \text{ for all } j = 1, \ldots, J, \text{ and } \sum_j \lambda_j = 1\} \qquad (1)$$

In many applications, part of the information on P and $\underline{\lambda}$ is soft. For example, for some criteria, no precise quantitative values of impacts may be available. At best one may have a ranking of alternatives in such a case. Similarly, one may only have a ranking of criteria to indicate their relative importance. Therefore the development of multicriteria methods which can deal with these types of problems is important. A survey of such methods is contained in Nijkamp, Rietveld and Voogd (1989). In this paper we will only shortly discuss those approaches which will be used in the empirical application.

A relatively easy way of dealing with ordinal data is by interpreting them as unknown quantitative data which satisfy certain inequalities. For example, if J criteria are ranked in increasing order, one arrives at J unknown cardinal weights satisfying:

$$\begin{cases} 0 \leq \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_J \\ \sum_j \lambda_j = 1 \end{cases} \tag{2}$$

Every cardinal $\underline{\lambda}$ satisfying (2) is consistent with the original ranking. When one assumes that all vectors $\underline{\lambda}$ satisfying (2) are equally probable, i.e. $\underline{\lambda}$ is uniformly distributed on (2), one can derive expected values of the weights in a relatively straightforward way. As shown in Rietveld (1984), the expected values of weights are:

$$\begin{cases} E(\lambda_1) = 1/J^2 \\ E(\lambda_2) = 1/J^2 + 1/[J.(J-1)] \\ E(\lambda_3) = 1/J^2 + 1/[J.(J-1)] + 1/[J.(J-2)] \\ \text{etc.} \end{cases} \tag{3}$$

Along similar lines one may deal with ordinal data on criterion scores p_{ij}. A ranking of alternatives in increasing order of attractiveness according to a certain criterion j, combined with the assumption of a uniform distribution leads to the following expected values:

$$E(p_{ij}) = i/I \quad \text{for } i = 1, \ldots, 1 \text{ and a certain } j \tag{4}$$

where the highest outcome for pij has been set equal to 1 by way of standardization[1]).

This stochastic approach provides a basis for translating ordinal data into cardinal ones. Of course its relevance depends on the appropriateness of the assumption of uniformly distributed variables. By using this cardinalization step, one can employ standard multicriteria methods for cardinal data even if (part of) P or $\underline{\lambda}$ are cardinal. An example of a standard multicriteria method is weighted summation which is based on an additive utility structure.

Another class of multicriteria methods has been specifically designed to deal with evaluation problems where qualitative data are used. Examples of such methods are EVAMIX (Voogd, 19831, QUALIFLEX (Paelinck, 1976), and the regime method (Hinloopen, Nijkamp and Rietveld, 1983).

A multicriteria method frequently used is ELECTRE (also called concordance analysis, cf. Roy, 1974, Crama and Hansen, 1983). This method is based on a pairwise comparison of alternatives, thus using only the metric interval characteristics of the various scores in the evaluation of the impact matrix. The basic idea is to measure the degree to which the scores and their associated weights conform or contradict the dominant pairwise relationships among alternatives. The differences in weights and the differences in evaluation scores are usually analyzed separately. The centra1 concept in ELECTRE is the so-called concordance index $c_{ii'}$. This index represents the extent to which alternative i is better than alternative i'. This index may be defined as the sum of weights attached to the criteria included in the so-called concordance set $C_{ii'}$; this is the set of all evaluation criteria for which alternative i in the impact matrtix P is at least equally attractive as alternative i'. Clearly,'this set can be determined irrespective of the degree of information on the impact matrix. Hence, the concordance index can be defined as follows:

$$c_{ii'} = \sum_{j \in C_{ii'}} \lambda_j \qquad (5)$$

A dominating alternative can now be found by employing threshold values, relative dominance indicators, or other concepts from graph theory.

In an analogous way, one may define a discordance index. This index reflects the extent to which alternative i is worse than i'. Instead of using weights in this index, the corresponding relative pairwise differences from the impact matrix are then taken into consideration. By combining the results from the concordance and discordance approach, final inferences on the ranking of alternatives may be made (see e.g. Nijkamp, 1979). Most of the methods presented in this section will be applied in section 4.

3. SENSITIVITY ANALYSIS

3.1. Introduction

Results of multicriteria analysis depend on various factors such as the choice for a particular multicriteria method, the choice of weights, methods for standardizing criteria or methods for dealing with uncertainty on effects of alternatives. In this section, we will pay attention to methods for investigating the sensitivity of evaluation results for the choice of criterion weights.

One way of dealing with the problem of uncertainty on weights is the Monte Carlo approach. In this case a random generator is used to produce a large number of weight vectors centered around a given weight vector. For each vector a multicriteria evaluation is carried out, after which the result is compared with the result for the given weight vector (see e.g. Nijkamp, 1979).

Another way is to formulate a limited number of policy views, each of which is represented by a certain weight vector. Then, in a second step, multicriteria analysis is carried out to find out whether the views lead to different options (see e.g. Voogd, 1983).

In the present paper another approach is proposed which gives a more detailed account of the sensitivity of, results of multicriteria analysis for the coice of weights. Our point of departure is a given weight vector $\underline{\lambda}^0$. For a certain multicriteria method, this leads to a ranking of alternatives, for example $A_1 \geq A_2 \geq A_3 \geq$ .... where Ai ≥ Ak means that alternative Ai performs equal to or better than Ak. Depending on the multicriteria method chosen, such a ranking is not necessarily complete, i.e., certain pairs of alternatives may be incomparable.

The method we will discuss is on the sensitivity of the result for an arbitrary pair of alternatives (e.g., $A_1 \geq A_2$) for changes in $\underline{\lambda}^0$. The question addressed is: "how far must $\underline{\lambda}$ be from $\underline{\lambda}^0$ before $A_1 \geq A_2$ does no longer hold true"

This question can be approached in various ways. One way is to focus on the weight for one particular criterion and assume that the ratios between other weights remain unaltered. Another approach would be that the weights of all criteria are allowed to change freely (the only condition being that they add up to 1). Both methods will be discussed below.

3.2. The nearest tuming point: focus on one criterion

For the ease of presentation we start with te assumption that the multicriteria method yields complete rankings. Let Sik be the set of

weights for which Ai ≥ Ak. Similarly, let Tik be the set of weights according to which the alternatives Ai and Ak perform equally well ($T_{ik} = S_{ik} \cap S_{ki}$).
Suppose that $\underline{\lambda}^o$ is an element of S_{ik}. We want to know how much a particular criterion weight (for example: λ_1) has to change in order to make the weights vector an element of T_{ik}. Since we impose the restriction that the sum of the weights equals one, a change in λ_1 implies that other weights will also change. We will assume that their ratios remain unchanged:

$$\lambda_j/\lambda_2 = \lambda_j^o/\lambda_2^o \quad \text{for } j = 3, \ldots, J \tag{6}$$

In Figure 2 an example is given for the case that J = 3. The turning point $\underline{\lambda}^A$ is found by extrapolating $\underline{\lambda}^o$, using (1, 0, 0) as a reference point. In this case, one finds one turning point. It is not difficult to see that other examples could be given where there is no turning point at all. Also, the occurence of multiple turning points cannot be excluded. In that case the nearest turning point is the most relevant one.

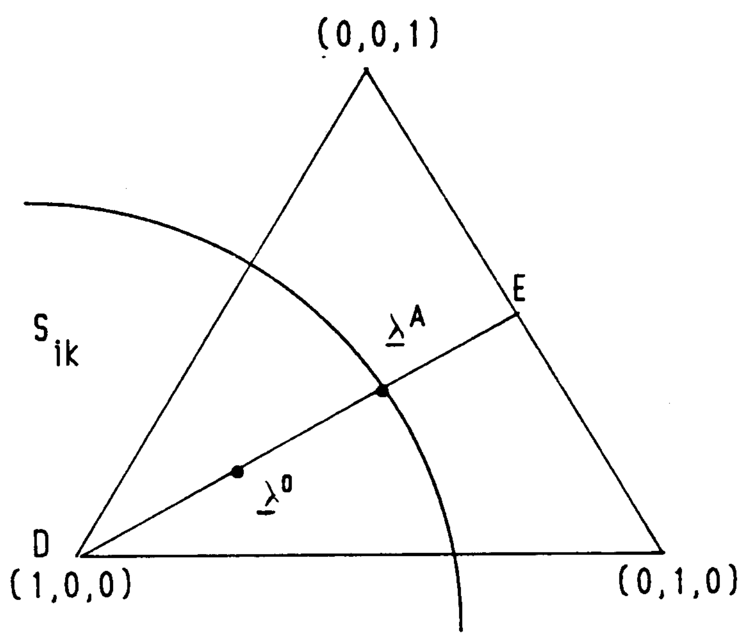

Figure 2. Turning point in the rank order of two alternatives.

How can $\underline{\lambda}^A$ be found (if it exists)? If a utility based multicriteria method is used, the set Tik is defined by $U(\underline{\lambda}, \underline{p_i}) = U(\underline{\lambda}, \underline{p_k})$. In that case it is not difficult to determine the element of Tik which satisfies condition (6). However, if a method for multicriteria analysis is used which is not based on a utility concept, as is the case for example with concordance analysis, no such definition can be given of the set T_{ik}. In that case, the turning point $\underline{\lambda}^A$ has to be found by a systematic inspection of the set of weights satisfying:

$$\begin{cases} 0 \le \lambda_1 < 1 \\ \lambda_j = \lambda_j^o(1-\lambda_1)/(\sum_{j=2}^{J} \lambda_j^o) \qquad j = 2, \ldots, J \end{cases} \tag{7}$$

It is not difficult to check that weights satisfying (7) are non-negative and add up to 1.

In order to find the turning point $\underline{\lambda}^A$, we propose the method of halving. In terms of Figure 2, we first investigate whether a point exists on the line between D and E for which the original ranking Ai $\geq$ Ak does not hold. If such a point appears to exist, an additional point is investigated which is in the middle of two points which are on different sides of the unknown line Tik. After a sufficient number of halvings one obtains a point which is very near to the turning point $\underline{\lambda}^A$.

The first three steps of the following.algorithm are carried out to investigate whether a turning point exists. In addition, these steps aim at solving the problem of multiple turning points. If more than one turning point exists, it is the one nearest to $\underline{\lambda}^o$ which has to be found. In septs 4 to 7 halving iterations are carried out.

In the algorithm, t stands for iteration. Further, $a(\underline{\lambda})$ is used to indicate whether $\underline{\lambda}$ is an element of the set S_{ik}:

$$\begin{cases} a(\underline{\lambda}) = 1 \text{ if Ai} \geq \text{Ak} \\ a(\underline{\lambda}) = 0 \text{ in all other cases.} \end{cases}$$

The algorithm consists of the following steps:

1. t = 0
   A,(0) = λ_1^o
   Compute $a(\underline{\lambda}(0))$ by means of multicriteria method, where $\underline{\lambda}(0) = \underline{\lambda}^o$.

2. t = t + 1
   If t = 12, stop: no turning point found.
   $\lambda_1(t) = c_1(t)$, where $c_1(t)$ is defined below.
   Compute $\underline{\lambda}(t)$, by means of (7).
   Compute $a(\underline{\lambda}(t))$ by means of multicriteria method.
   (For t = 1, ... 13 $c_1(t)$ assumes the following values[2]:

λ_1^o	$.8\lambda_1^o + .2$	$.6\lambda_1^o + .4$	$.4\lambda_1^o + .6$	$.2\lambda_1^o + .8$	1.0
	$.8\lambda_1^o$	$.6\lambda_1^o$	$.4\lambda_1^o$	$.2\lambda_1^o$	.0)

3. If $a(\underline{\lambda}(t)) - a(\underline{\lambda}(t-1)) = 0$, return to 2.
   If $a(\underline{\lambda}(t)) - a(\underline{\lambda}(t-1)) \neq 0$, go to 4.

4. $\underline{y} = \underline{\lambda}(t-1)$ $\qquad a(\underline{y}) = a(\underline{\lambda}(t-1))$
   $\underline{z} = \underline{\lambda}(t)$ $\qquad a(\underline{z}) = a(\underline{\lambda}(t))$

5. $\underline{v} = (\underline{y} + \underline{z})/2$
   Compute $a(\underline{v})$ by means of multicriteria method.

6. If $a(\underline{y}) - a(\underline{v}) = 0$, then $\underline{y} = \underline{v}$ and $a(\underline{y}) = a(\underline{v})$, go to 7.
   If $a(\underline{z}) - a(\underline{v}) = 0$, then $\underline{z} = \underline{v}$ and $a(\underline{z}) = a(\underline{v})$, go to 7.

7. If $|y_j - z_j| \leq \epsilon z_j$ for all j where ϵ is a certain small value, stop: turning point found;
   otherwise: return to 5.

The algorithm needs some straightforward extentions when multicriteria methods are used which may yield incomplete rankings. As illustrated in Figure 4, the indifference line T_{ik} is replaced by a band of weights leading to the result that two alternatives are incomparable. In this case two turning points are found ($\underline{\lambda}^B$ and $\underline{\lambda}^C$).
In a strict sense, the algorithm does not guarantee that a turning point is found if there exists one, even if T_{ik} is continuous. As shown in Figure 3, a turning point may be overlooked when the points investigated in Step 2 are too far removed from each other. The probability that this occurs can be made very small by increasing the number of iterations in Step 2.

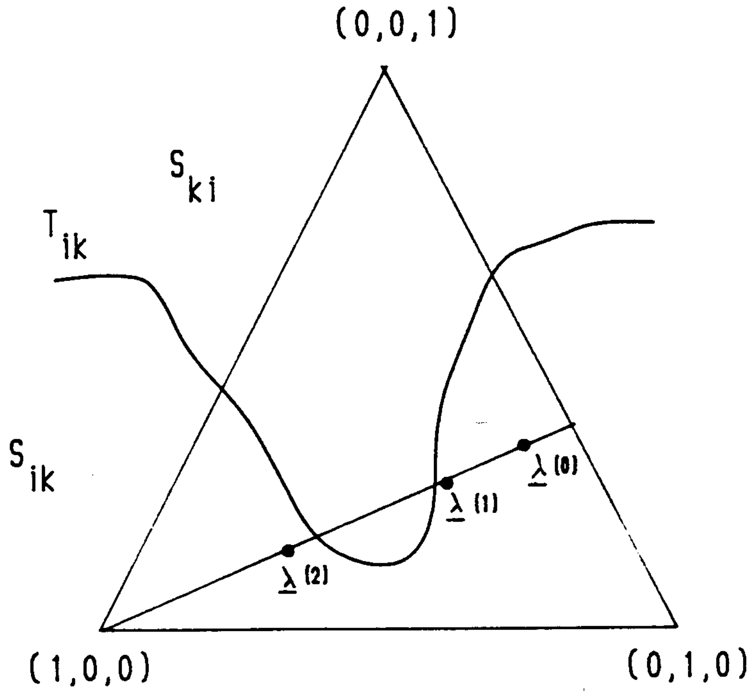

Figure 3. Possibility of overlooking a turning point.

The method of **halving provides** a decision **making** unit (DMU) with an indication of the degree of sensitivity of a certain outcome Ai > Ak for changes in the value of a certain weight. For a DMU, the degree of uncertainty about the original weight vector $\underline{\lambda}^o$ is usually **considerable.** By comparing $\underline{\lambda}^o$ with $\underline{\lambda}^A$, the DMU is informed on **how** sensitive the results of multicriteria analysis are for changes in the weights vector. The method of **halving can** be used for **any** multicriteria method.

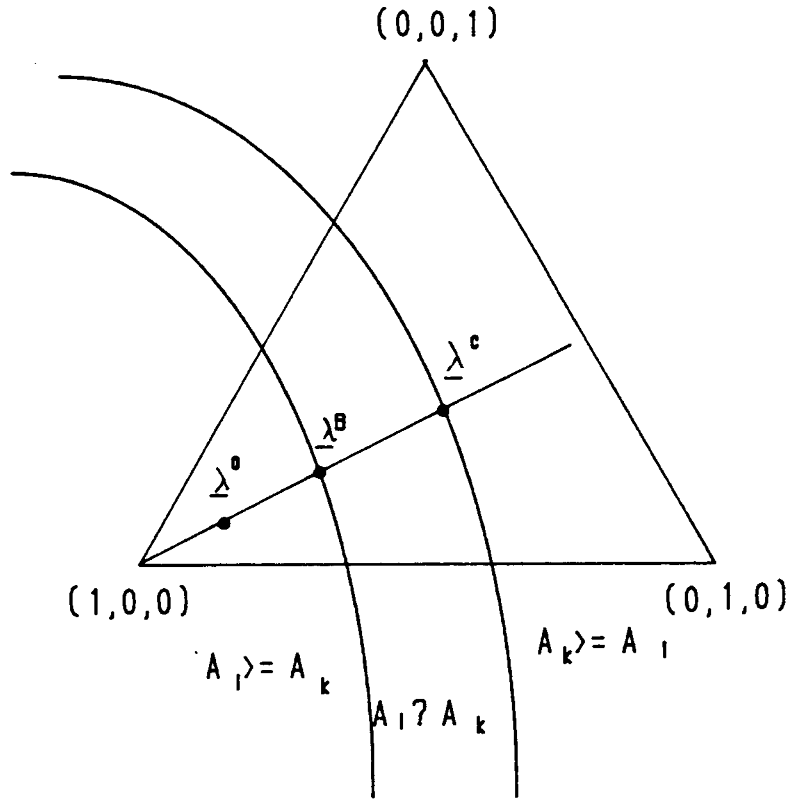

Figure 4. Turning points with incomparable alternatives.

3.3. The nearest turning point: all weights are allowed to vary

A limitation in the method of halving is that all weights except one are assumed to be proportional to the original weights. Thus, one arrives at a point $\underline{\lambda}^A$, which may be far removed from the point $\underline{\lambda}^z$ which is the point on T_{ik} nearest to $\underline{\lambda}^o$ when all weights are allowed to move freely (see Figure 5).

If a linear utility function would be used, $\underline{\lambda}^z$ can be found as the solution of a quadratic programming problem. Since it is our aim to develop a method which is applicable to a much more general class of multicriteria methods, a different approach has to be followed.

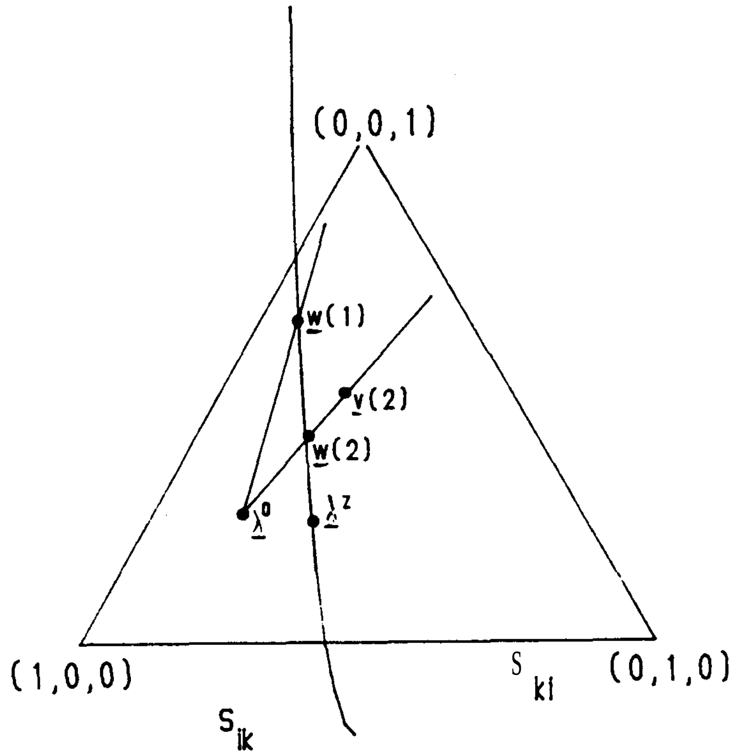

Figure 5. Search procedure for turning point when all weights are allowed to vary.

The structure of the algorithm reads as follows. Let $\underline{\lambda}^o$ be an element of S_{ik}. Then the following steps have to be made in order to find the turning point $\underline{\lambda}^z$ nearest to $\underline{\lambda}^o$.

1. Find a weight vector $\underline{v}$ which is an element of S_{ki} and which consists of nonnegative elements adding up to 1, If no such vector can be found, a turning point does not exist.

2. Use the method of halving based on the vectors $\underline{\lambda}^o$ and $\underline{v}$ to determine the vector $\underline{w}$ which is an element of T_{ki}. Let $d(\underline{\lambda}^o, \underline{w})$ be the distance between the vectors $\underline{\lambda}^o$ and $\underline{w}$.

3. Find in a neighbourhood of $\underline{w}$ a new point $\underline{v}$ with $d(\underline{\lambda}^o, \underline{v}) \le d(\underline{\lambda}^o, \underline{w})$ and where $\underline{v}$ is an element of S_{ki},

4. Return to step 2, but stop when subsequent results of $\underline{v}$ come very near to each other.

Figure 3 gives an illustration of the algorithm for the first two iterations. We will now discuss the four steps in more detail.

Step 1. For each of the extreme points in the weights set (1, 0,..., 0), (0, 1, 0,...0), ... (0,...,0, 1) an investigation is made whether it is an element of S_{ki}. For those extreme points $\underline{v}$ which are indeed an element of S_{ki}, the distance $d(\underline{v}, \underline{\lambda}^o)$ to $\underline{\lambda}^o$ is measured, where $d(\underline{v}, \underline{\lambda}^o)$ is defined as:

$$d = [\sum_j (v_j - \lambda_j^o)^2]^{.5}$$

The extreme point $\underline{v}$ with minimum distance is selected to be used in the second step.

A difficulty is that there may be cases where feasible weight vectors in S_{ki} exist, but where there are no extreme points in S_{ki} (see Figure 6). Several approaches can be followed to counter this problem. First, one may examine in a systematic way points on the faces of the polyhedral set S. For example, when J=4, one can examine the points:
(0, 1/3, 1/3, 1/3), (1/3, 0, 1/3, 1/3), (1/3, 1/3, 0, 1/3), (1/3, 1/3, 1/3, 0), (0, 0, 1/2, 1/2), (0, 1/2, 0, 1/2), (0, 1/2, 1/2, 0), (1/2, 0, 0, 1/2), (1/2, 0, 1/2, 0), (1/2, 1/2, 0, 0).

Another approach to generate in a random way a set of points in S after which for each point it is examined whether it is in S_{ki}. Appendix 1 contains a method to generate random weights which are uniform in S.

Step 2 The method of halving as presented in section 3.2 can be used directly here.

Step 3. A neighbourhood of $\underline{w}$ is defined here as a polyhedral set around $\underline{w}$ which is contained in the set S. Element $\underline{x}$ of such a set can be generated by using the formula:

$$x = b\underline{w} + (1-b)\underline{\delta},$$

where $\underline{\delta}$ is an arbitrary element of the set S. An example of such.a neighbourhood is given in Figure 7 (for J=3 and b=.5). The method presented in Appendix 1 can be used again for generating weights which are uniformly distributed in S. The generation of point $\underline{x}$ continues until a point is found which is nearer to $\underline{\lambda}^0$ than is $\underline{w}$, and which is an element of S_{ki}. If no such point $\underline{x}$ is found, $\underline{w}$ is the optimal solution.

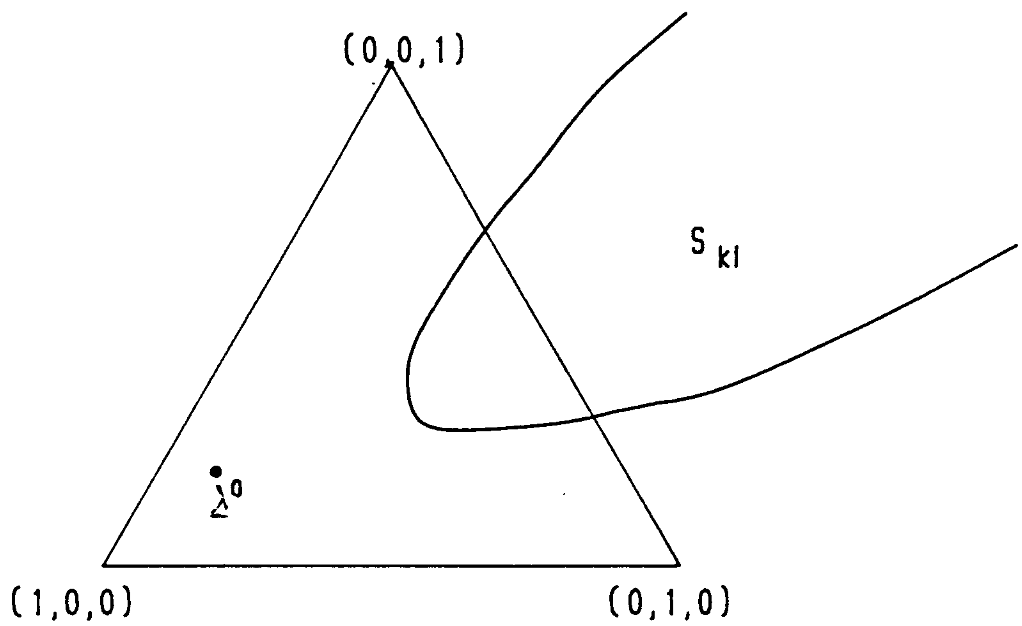

Figure 6. Existence of turning point: a special case.

Step 4. The algorithm stops when for some subsequent iterations t:

$|w_j(t) - w_j(t-1)| < \epsilon\, w_j(t)$,

where ϵ is a suitably choosen smal1 number. In order to deal with the problem of local optima, one may return the algorithm with another starting point in Step 1.

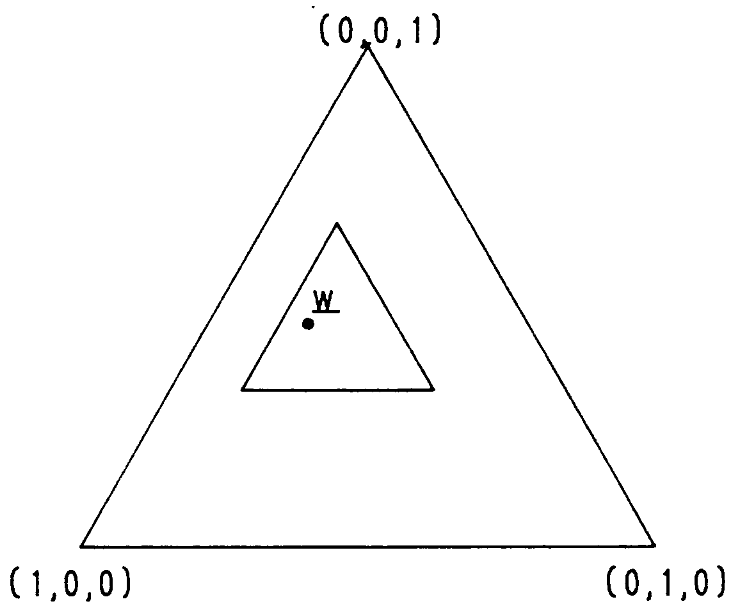

Figure 7. A neighbourhood for w.

The parameters to be fixed in the algorithm are b and ε. When fixing b a compromise must be found between the probability of finding a nearer point, and the expected size of decrease of distance. Further, a maximum level must be set for the number of random vectors x in Step 3 (and Step 1).

In a strict sense the algorithm does not guarantee that the turning point with minimum distance is found. One problem is related to fixing the number of points generated in Step 3 (or Step 1). If the maximum number of interations in Step 3 is small, the algorithm may stop at a point which is far removed from the nearest turning point. The probability that this occurs can be made arbitrary small by increasing the maximum number of iterations. Another problem is that the optimum solution found is not the global one. This probability can be made arbitrary small by restarting the algorithm for a sufficiently large number of starting points. Finally, the algorithm needs some straightforward extensions when multicriteria methods are used which may yield incomplete rankings (cf. section 3.2).

4. AN APPLICATION: THE SELECTION OF THE OPTIMAL LOCATION FOR NUCLEAR PLANTS IN THE NETHERLANDS

4.1. Introduction

The share of nuclear power in total power production is very smal1 at present in the Netherlands. In 1985 the Dutch government expressed the intention to build two new nuclear power plants with a capacity of 1000 MWe each. Given this decision an important decision is where to locate these two plants. After some initial scoping, nine potential locations for the plants were selected (Tweede Kamer 188303, 43-44). These locations are shown in figure 8.

In this section these nine locations will be ranked using 15 appraisal criteria. Our support system for decisions on a finite set of alternatives (DEFINITE) is used to produce this ranking. This will be done by using the following steps:

1. Problem definition.
2. Problem presentation.
3. Problem evaluation.
4. Sensitivity analysis.

Steps 1 to 3 are described only briefly. This section concentrates on the methods for sensitivity analysis discussed in the previous section.

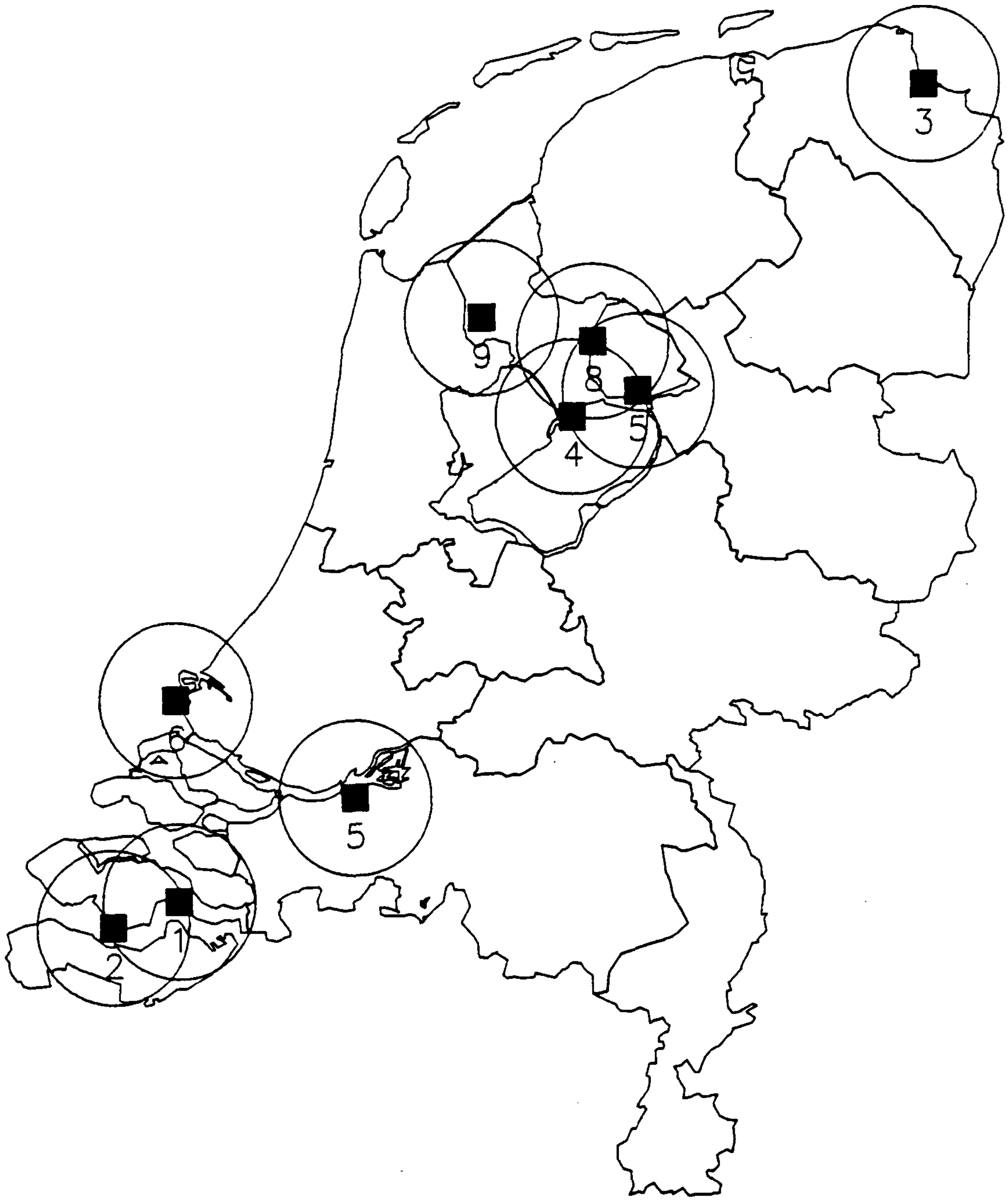

Figure 8. Potential sites for nuclear plants.

0 potential site

0 20 km range around the potential site

1 Bath/Hoedekenskerke
2 Borssele
3 Eems
4 Flevo Noord
5 Ketelmeer
6 Maasvlakte
7 Moerdijk
8 West. NOP-dijk
9 Wieringermeer

4.2. Problem definition

The impact matrix of this evaluation problem is shown in Table 1, where nine potential locations are scored according to 15 criteria. Only the score for population around a site, is measured on a cardinal scale. All other scores are measures on an ordinal scale: a score 1 is assigned to the best alternative, 2 to the second best, etc. (see Appendix 11 for a definition of the criteria).

Table 1 Impact matrix
(Source: Tweede Kamer 18830, 43-44; advice to the government)

	Bath	Borssele	Eems	Flevo	Ketel	Maasvlak	Moerdijk	NOPolder	Wiering
Population	51	49	16	27	30	43	100	19	21
Evacuation	1	2	1	1	1	2	1	1	1
Agricult at risk	2	2	2	2	2	1	3	2	2
Industry at risk	1	4	3	2	1	5	3	1	1
Fr water at risk	1	1	1	2	2	1	2	2	2
Cool-water quant	2	1	1	1	1	1	3	1	1
Cool-water qual	2	1	2	3	3	1	2	3	3
Air pollution	2	2	2	2	2	1	2	2	2
Thermal poll.	3	2	2	2	3	1	2	3	3
Indirect landuse	2	3	3	2	1	2	4	1	1
Landscape	3	1	1	1	3	1	2	3	3
Nat environment	3	1	3	1	2	1	1	1	1
National grid	2	2	3	1	1	2	2	2	3
Infrastructure	2	1	1	2	2	1	1	2	2
Coal-location	3	6	4	3	2	7	5	1	1

4.3. Problem presentation

A graphical presentation of the impact matrix is shown in Figure 9. This figure is derived by standardising all criterion scores between 0 and 1 (see section 2). The highest bar in each row represents a score of 1 corresponding to the best alternative for that row. As a next step the criteria are ordered from most important (top> to least important (bottom). These priorities have been expressed in an ordinal way by experts of the government advisory board on physical planning (Tweede Kamer 18830, 43-44). Using a combination of the expected value method for weights and the weighted summation method (see section 2) the information on priorities and scores can be used to sort the alternatives from best (left) to worst (right). It is clear from this figure that the impact matrix contains many tied scores and that differences between alternative locations are fairly small.

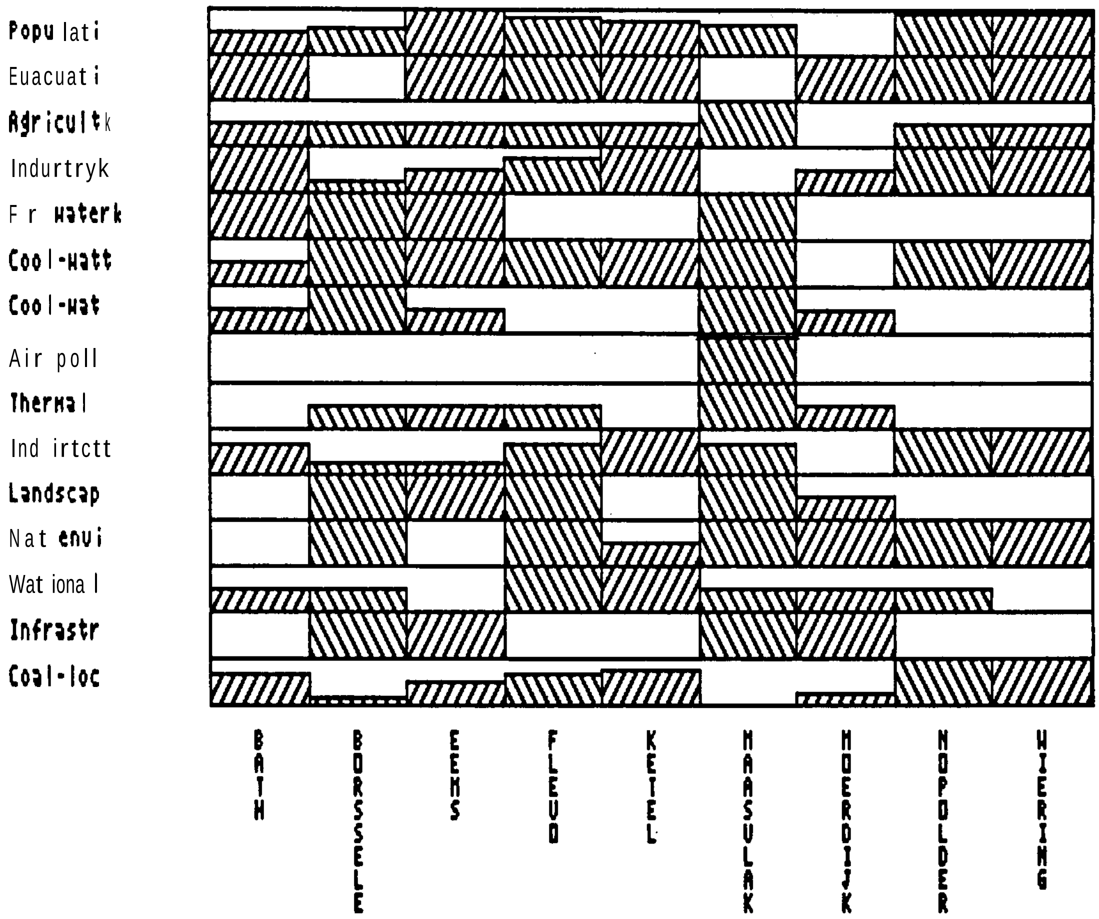

Figure 9. A graphical presentation of the impact matrix.

4.4. Problem evaluation

A variety of evaluation methods can be used to rank these alternatives (see for a short description section 2). In this application the expected value method is used to transform the priority ranking of the criteria to quantitative weights (Table 2). Using these weights both the weighted summation and the Electre method were applied to generate a ranking of the alternatives (Table 2). Both the weighted summation method and the Electre 2 method result in an almost complete ranking of the alternatives. In the results of the weighted summation method, alternatives Maasvlakte and Borsele share the 7th and 8th position and the Electre method results in a tie for alternatives Flevo and Ketelmeer. Note that the methods generate different rankings.

Table 2. Ranking of the alternatives according to the weighted summation and the ELECTRE-2 method.

Weights Ranking	Weights Score	Weighted summation Ranking	Weighted summation Score	ELECTRE 2 Ranking
1: Population	0.221	1: Eems	0.86	1: NOPolder
2: Industry at risk	0.155	2: NOPolder	0.83	2: Eems
3: Agricult at risk	0.110	3: Flevo	0.82	3: Flevo
Fr. water at risk	0.110	4: Wiering	0.81	Ketel
5: Cool-water quant.	0.064	5: Ketel	0.79	5: Maasvlak
Cool-water qual.	0.064	6: Bath	0.76	6: Wiering
Thermal poll.	0.064	7: Maasvlak	0.75	7: Borssele
Coal-location	0.064	Borssele	0.75	8: Bath
9: Air pollution	0.027	9: Moerdijk	0.52	9: Moerdijk
Landscape	0.027			
Nat. environment	0.027			
National grid	0.027			
Infrastructure	0.027			
14: Evacuation	7.OE-03			
Indirect landuse	7.OE-03			

4.5. Sensitivity analysis

The location of nuclear plants is a politically sensitive decision. It is therefore interesting to analyze the relationship between assigned priorities and the ranking of the locations. The genera1 robustness of the derived ranking can be analyzed as a first step. In this case a Monte Carlo analysis shows that if the weights were allowed to vary by ± 5% and assuming the weights are normally distributed, the overall ranking proves uncertain (see also Rietveld 1988). However, the selection of the two best alternatives, proves sufficiently certain. Since the government aims to select two locations this is a useful result. More interesting than the overall stability of the derived ranking is, in cases like this, the sensitivity of the ranking to specific weights and the stability of the ranking of specific alternatives to changes in weights.

Since the governement wishes to select two sites we will try to analyze how firmly the alternatives Eems and Noord-Oost Polder hold the first two positions. Firstly we will use the methods as described in section 3.2 to calculate robustness intervals for these two alternatives and secondly we will use the methods as described in section 3.3 to find the nearest weight combination that brings one of the other alternatives to the first or second position.

Robustness intervals are calculated for the weight of the most important criterion: population living around the site.

As shown in Table 2 weighted summation selects Eems as the best alternative followed by NO Polder. Alternative Flevo ranks on the third place. In Table 3 robustness intervals are calculated for the pairs Eems - Flevo and NO Polder - Flevo. Table 3 shows that Eems ranks higher than Flevo for any weight assigned to the population criterion. Table 3 also shows that if the weight assigned to population is reduced below 0.16 the ranking of Flevo and NO Polder is reversed.

Table 3 Robustness intervals; weighted summation method.

Ranking	Weight of criterion population
Eems ≥ Flevo	0 -> 1
Flevo ≥ Eems	empty
NO Polder ≥ Flevo	0.16 -> 1
Flevo ≥ NO Polder	0 -> 0.16

The ELECTRE-2 method also ranks Eems and NO Polder as the best two alternatives, but in the reversed order. Table 4 shows the required changes in weights to move Eems or NO Polder from their first position. It is shown that with any weight assigned to population, alternative Flevo will not replace alternative Eems or NO Polder from their first two positions. The same can be shown for alternative Ketelmeer. It is interesting to note that alternative Maasvlakte, which is ranked at a fifth position in the initial ranking, ranks higher than NO Polder if the weight assigned to population is lower than 0.13, and higher than Eems if this weight is less than 0.12. A similar procedure can be applied to establish robustness intervals for criterion scores.

Table 4. robustness intervals; ELECTRE-2 method.

Ranking	Weight of criterion population
Eems ≥ Flevo	0 -> 1
Flevo ≥ Eems	empty
NO Polder ≥ Flevo	0 -> 1
Flevo ≥ NO Polder	empty
Eems ≥ Maasvlak	0.12 -> 1
Maasvlak ≥ Eems	0 -> 0.12
NO Polder ≥ Maasvlak	0.13 -> 1
Maasvlak ≥ NO Polder	0 -> 0.13

Only the relative weight of the population criterion to all other criteria was changed in calculating the weight intervals. If we allow all weights to change it becomes clear how sensitive the derived ranking is to overall changes in weights. Using the method described in section 3.3 for each pair of alternatives, the set of weights with the smallest Euclidean distance from the original weights that reverses the ranking of the alternatives can be calculated. Table 5 shows the results for six pairs of alternatives. The smal1 values for distances indicate how sensitive the ranking is to changes in weights.

Table 5. Weight combinations with rank reversal; ELECTIE-2 method.

Criterion	Original	Flevo > Eems	Flevo > NOPolder	Ketel > Eems	Ketel > NOPolder	Maas > Eems
Population	0.21	0.150	0.158	0.157	0.150	0.160
Evacuation	0.007	0.047	0.020	0.020	0.047	0.019
Agricult at risk	0.110	0.075	0.103	0.081	0.075	0.092
Industry at risk	0.155	0.105	0.106	0.230	0.105	0.119
Fr water at risk	0.110	0.075	0.078	0.088	0.075	0.088
Cool-water quant	0.064	0.043	0.070	0.057	0.043	0.061
Cool-water qual	0.064	0.043	0.058	0.051	0.043	0.060
Air pollution	0.027	0.018	0.024	0.040	0.018	0.039
Thermal poll.	0.064	0.043	0.129	0.061	0.043	0.136
Indirect landuse	0.007	0.047	0.032	0.037	0.047	0.020
Landscape	0.027	0.018	0.043	0.025	0.018	0.030
Nat environment	0.027	0.018	0.043	0.024	0.018	0.041
National grid	0.027	0.341	0.052	0.022	0.341	0.041
Infrastructure	0.027	0.018	0.040	0.031	0.018	0.043
Coal-location	0.064	0.043	0.047	0.076	0.043	0.054
Distance to original	0	0.043	0.119	0.113	0.332	0.111

It is already clear from Table 2 that different evaluation methods might result in different rankings of the alternatives. The results of both multicriteria methods are shown in Figure 10. It is clear that both methods rank Eems and NO Polder above all other alternatives. The ranking of the alternatives is similar for both methods. From Figure 10 it can be safely concluded that Eems and NO Polder should be selected as the best two locations.

As a last step DEFINITE offers conclusions on the usefulness of the derived rankings based on the results of sensitivity analysis. The conclusions read as follows:

- The overall ranking is insufficiently certain.
- Eems and NO Polder are the best two alternatives.

This result should prevent the decision maker from publishing the complete ranking. Although our aim to derive a complete ranking has failed the result obtained is useful. Two locations had to be selected: the difference in ranking between the first two alternatives is in this special case irrelevant.

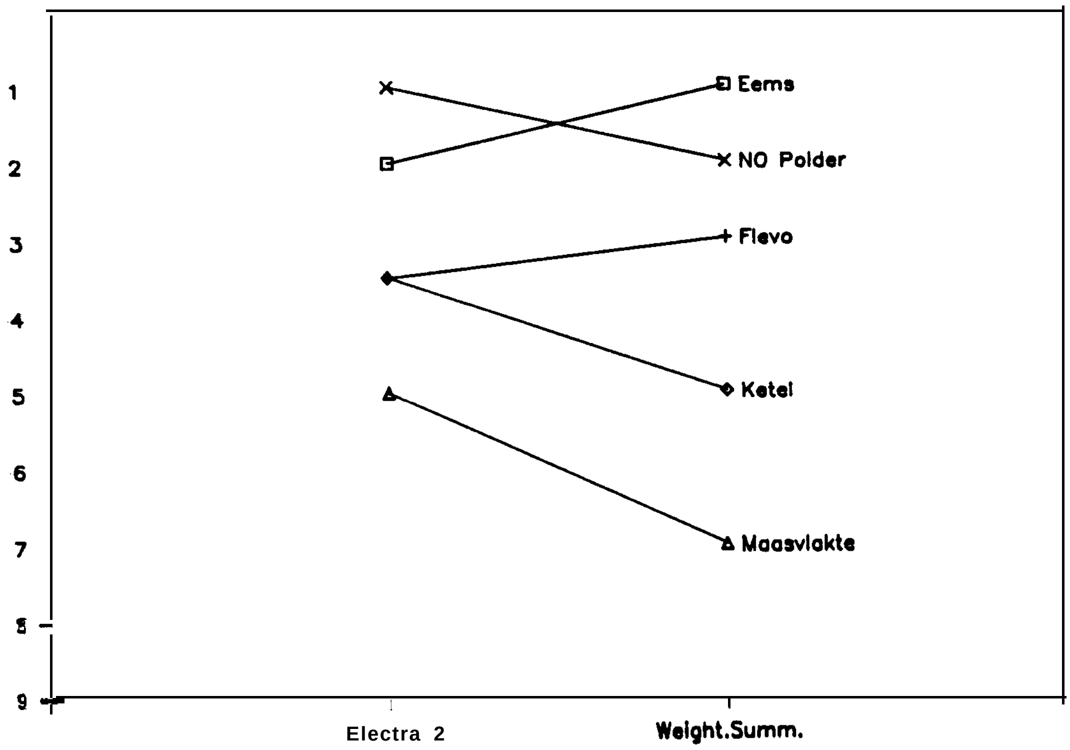

Figure 10. Ranking of five alternatives according to two multicriteria methods.

5. CONCLUDING REMARKS

The availability of various evaluation methods and methods for sensitivity analysis in a decision support system such as DEFINITE has clear advantages. It increases availability of these methods to various types of users; although the exact calculation procedures will not be clear to all users, the results are easy to interpret and unambiguous.
In addition, it allows users to study the sensitivity of outcomes for the choice of a particular method of multicriteria analysis. In the case study presented in this paper, it appears that the ranking produced with the weighted summation method is indeed different from the ranking produced with ELECTRE. For the choice of the two highest ranking alternatives, the methods appear to yield identical outcomes, however.

The methods for sensitivity analysis available in DEFINITE allows one to carry out detailed studies for the sensitivity of evaluation results. For the choice of nuclear power plants, it appears that the choice of location is rather sensitive to the value of the weights. Sensitivity analysis of this type are important because the information on weights is rather soft in many real world applications. This is also the reason in this paper special attention is given to methods for dealing with ordinal information.

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NOTES

1) The difference between (3) and (4) is caused by the different ways of standardizing weights and criterion scores.

2) By decreasing the step size in Step 2 one can increase the probability that in the case of multiple turning points it is the nearest one which is found.

3) Documents presented to the lower house of parliament for discussion.

APPENDIX I

Generating random weights

Consider a weight vector $\underline{\lambda}$ which is uniformly distributed in S as defined in section 2:

$$S = \{\underline{\lambda} \mid 0 \le \lambda_j \le 1 \text{ for all } j=1,\ldots, J, \text{ and } \sum_j \lambda_j = 1\}$$

One might be tempted to generate weight vectors $\underline{\lambda}$ by drawing J random numbers $x_1,\ldots x_J$ from a uniform distribution on the interval $0 \le x_j \le 1$, and setting $\lambda_j = x_j / \Sigma x_j$.

However, this does not lead to weight vectors which are uniformly distributed on S. Therefore, another approach has to be followed.

The joint density of $\underline{\lambda}$ is:

$$g(\lambda_1,\ldots,\lambda_{J-1}) = (J-1)! \text{ for } \begin{array}{l} 0 \le \lambda_1 \le 1 \\ 0 \le \lambda_2 \le 1-\lambda_1 \\ \\ 0 \le \lambda_{J-1} \le 1-\lambda_1-\ldots\lambda_{J-2} \end{array}$$

$$= 0 \quad \text{elsewhere}$$

On the basis of this joint density function one can derive for the density of λ_1:

$$g(\lambda_1) = (J-1)(1-\lambda_1)^{J-2} \text{ for } 0 \le \lambda_1 \le 1$$

$$= 0 \quad \text{elsewhere}$$

Further, the conditional density functions can be shown to read as follows for $j=2,\ldots,J-1$.

$$g(\lambda_j \mid \lambda_1,\ldots,\lambda_{j-1})$$

$$=(J-j)(1-\lambda_1-\ldots-\lambda_j)^{J-j-1}(1-\lambda_1-\ldots\lambda_{j-1})^{j-J}$$

$$\text{for } 0 \le \lambda_j \le 1-\lambda_1-\ldots-\lambda_{j-1}$$

$$=0 \quad \text{elsewhere}$$

Then, a random weight vector can be generated by drawing a value for λ_1 on the basis of $g(\lambda_1)$, followed by drawing a value for λ_2 on the basis of $g(\lambda_2 \mid \lambda_1)$, etc. Finally, λ_J can be computed as $1-\lambda_1-\ldots-\lambda_{J-1}$.

The conditional distributions mentioned above are not included in standard statistical packages. Therefore, random weight vectors cannot be directly created by means of random generators. A solution for this problem is given by the theorema which says that if $G(x)$ is the distribution function of x, then $u=G(x)$ is uniformly distributed on the interval $0<u<1$. (Hogg and Craig, 1970, p. 349).
For the latter uniform distribution, standard random generators are available. then, if u_1 is uniformly distributed on the interval (0,1), $\lambda_1= G^{-1}(u_1)$ can be shown to be distributed according to the density funtion $g(\lambda_1)$ corresponding with the distribution function $G(\lambda_1)$. Thus, random values for λ_1 can be found by using the following transformation.

$$\lambda_1= 1-(1-u_1)^{1/(J-1)}$$

For $\lambda_2,\ldots, \lambda_{J-1}$ the following transformation has to be used:

$$\lambda_j= (1-\lambda_1-\ldots-\lambda_{j-1})(1-(1-u_j)^{1/(J-j)}) \quad j= 2,\ldots,J-1$$

Finally, λ_J can be computed as $1-\lambda_1-\ldots-\lambda_{J-1}$

APPENDIX11

Definition of evaluation criteria

Population.	A weighted sum of population around a location was calculated to quantify this score. The weight assigned decreases with distance. The result is standardized by dividing by the maximum score. A minus sign is added to indicate that the criterion is a cost criterion.
Evacuation.	The score reflects the availability of sufficient transport infrastructure.
Agriculture at risk.	This score reflects the location of agricultural land in the vicinity.
Industry at risk.	This score reflects the size and importance of industry near the location.
Fresh water at risk.	This score reflects the quantity of fresh water that may be affected by a nuclear plant at each location.
Cool-water quantity.	This score represents the quantity of available water for cooling the nuclear plant.
Cool-water quality.	This score represents the capacity of the coolant to flush out pollution originating from a nuclear plant at each location.
Air pollution.	It is assumed that the nuclear plant is an alternative to a conventional coal power plant, and so has the most beneficial effect at the most polluted location.
Thermal pollution.	The amount of pollution is lower if users of the waste heat are available. The score reflects the availability of such users.
Indirect land use.	This score reflects limitations on potential land uses around a nuclear plant.
Landscape.	This score reflects the visual effects of the landscape and the extent to which a nuclear plant fits in with existing activities.
Natura1 environment.	This score reflects expected damage to the natural environment.
National grid.	This score reflects the proximity of high voltage lines and connector stations.
Infrastructure.	This score reflects the availability of transport and other infrastructure near the site.
Coal-location.	It is assumed that the nuclear plant is an alternative to a conventional coal power plant. The score reflects the cost of the lost opportunity to build a coal plant at the site if a nuclear plant is constructed.