Double Crossing of the Ground Rotational Band and Super Band

Hamilton, J.H.; Peker, L.K.

published in
Physical Review Letters
1978

DOI (link to publisher)
10.1103/PhysRevLett.40.744

document version
Publisher's PDF, also known as Version of record

citation for published version (APA)
Hamilton, J. H., & Peker, L. K. (1978). Double Crossing of the Ground Rotational Band and Super Band. *Physical Review Letters*, *40*(12), 744-747. https://doi.org/10.1103/PhysRevLett.40.744

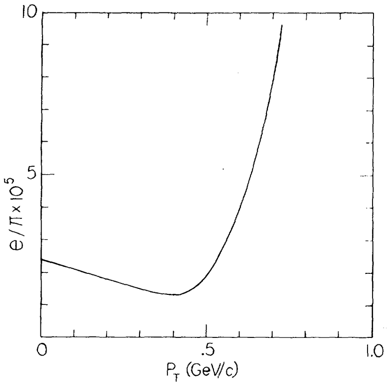

FIG. 2. 90% C. L. upper limit on the ratio $e^{\pm}/\pi^{\pm}$ as a function of the lower limit in p_T accepted in the data.

probability then depends on the sum of the semielectronic branching rations (R) of the charmed particles, and one obtains a 90%-C.L. limit for any charm-production channel cross section σ: $\sigma[R_1+R_2] < 2.4$ μb. In the case of charmed-meson-pair production, the semielectronic branching ratio $(9.4 \pm 1.4)\%$ is indicated by e^+e^- storage ring experiments,[8] and we obtain $\sigma < 13$ μb at 90% C.L.

We wish to thank the SLAC accelerator and bubble-chamber staff for their efficient operation of the equipment and all the scanners for their efforts in searching the film. This work was supported by the U. S. Department of Energy and the United Kingdom Scientific Research Council.

---

[a] Now at Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvette, France.

[b] Now at Michigan State University, East Lansing, Mich. 48823.

[c] Now at Argonne National Laboratory, Argonne, Ill. 60439.

[1] L. M. Lederman, Phys. Rep. 26C, 151 (1976), and references cited therein.

[2] L. Baum *et al.*, Phys. Lett. 60B, 485 (1976).

[3] E. W. Beier *et al.*, Phys. Rev. Lett. 37, 1117 (1976).

[4] J. G. Branson *et al.*, Phys. Rev. Lett. 38, 457 (1977).

[5] J. Ballam and R. D. Watt, SLAC Report No. SLAC-PUB-1954, 1977 (to be published).

[6] H. Messel and D. F. Crawford, *Electron-Photon Shower Distribution Function* (Pergamon, London, 1970); T. Kitagaki *et al.*, Tohoku University Report No. TUHEL-76/3, 1976 (unpublished).

[7] Besides scanning efficiency (96 + 1)%, efficiency factors are as follows: electron identification acceptance (80 ± 2)%; probability of being accompanied only by identifiable tracks of opposite charge (70 ± 2)%. Thus, the efficiency correction is 0.54 ± 0.02.

[8] R. Brandelik *et al.*, Phys. Lett. 70B, 387 (1977); W. Bacino *et al.*, Phys. Rev. Lett. 40, 671 (1978); J. M. Feller *et al.*, Phys. Rev. Lett. 40, 274 (1978).

Double Crossing of the Ground Rotational Band and Super Band

L. K. Peker
Natuurkundig Laboratorium der Vrije Universiteit, Amsterdam, Netherlands

and

J. H. Hamilton
Physics Department, Vanderbilt University, Nashville, Tennessee 37235

(Received 1 November 1977)

We suggest that the crossing of the ground rotational band by a second rotational-aligned band may in fact be a double crossing. From extrapolations of the rotational energy formula, we show that one could expect such a second crossing around $I \approx 30$ in ^{158}Er. This provides an alternative explanation in terms of a double band crossing for the second discontinuity recently observed in the moment of inertia about $I = 28$ in ^{158}Er.

The general understanding of the phenomena of backbending, where there occurs a sudden large increase in the moment of inertia, $\mathcal{I}$, and reversal of the monotonic increase in energy level spacings, is that the ground band is crossed by an excited super band with a larger $\mathcal{I}$ than that of the ground band. The various interpretations of the nature of the second bands have been discussed in several reviews.[1] In the light-mass rare-earth nuclei, the rotational alignment effect[2] involving the $\nu(i_{13/2})^2$ configuration most probably produces the backbending observed around $I = 14^+$ in the yrast cascades (see Stephens, Faessler, and co-workers[1,3]).

For $I < I_{\text{crossing}}$, the levels are considered to belong to the GRB (ground rotational band) and above I_{cross} to the SB (super band). An intriguing question has been what happens for $I \gtrsim 20$. Very recently Lee *et al.*[4] have observed yrast states to spin 28^+ and probably 32^+ in ^{158}Er. They observe a second discontinuity between spin 26 and 28. As possible explanations of this second discontinuity, they considered the sudden collapse of the pairing correlation.[5] While this cannot be ruled out, they point out that calculations[6,7] suggest that pairing effects do not cause sudden changes in $\mathscr{J}$. Since the first backbending in this case is probably from the alignment of a pair of $i_{13/2}$ neutrons, they suggest that the more likely possibility is that at higher spin additional pairs of high-j nucleons are aligned to cause the second discontinuity in ^{158}Er. They suggest that the second pair would be additional $i_{13/2}$ neutrons or $h_{11/2}$ protons.

In this Letter we wish to point out another alternative explanation of the second discontinuity. As we shall see, if the SB is really based on an aligned two-particle (high j)^2 configuration with $I_0 = 2j - 1 \gg 0$, then the SB should cross the GRB not once but twice.

The energy of the GRB levels can be described by the expansion

$$E_{\text{rot}}(I) = \sum_{n=1} \alpha_n \omega_I^{2n}, \quad n = 1, 2, 3 \ldots . \tag{1}$$

Generally a four-term expansion is used[8]

$$E_{\text{rot}}(I) = \alpha\omega_I^{\,2} + \beta\omega_I^{\,4} + \gamma\omega_I^{\,6} + \delta\omega_I^{\,8} \tag{2}$$

with the parameters derived from the experimental energies for $I \leq 10$. In Fig. 1, it is seen that in a nucleus where no backbending or band crossing is observed,[8,9] up to $I = 18$ in ^{176}Hf and $I = 24$ in ^{238}U, the formula nicely describes the experimental data. While it is not known to how high spin Eq. (2) may be useful for a rotational band, the general trends predicted by it may be qualitatively reasonably good up to even higher spins than 18 to 24 for the GRB's based on the ^{176}Hf and ^{238}U data.

We fitted Eq. (2) to the experimental data for $I \leq 10$ and calculated $E_{\text{rot}}(I)$ for the GRB to high spin for comparison with the experimental yrast cascades in three nuclei with strong, medium, and weak backbending, ^{158}Er, ^{158}Dy, and ^{174}Hf.[4,10,11] The results are given in Table I, with the results for ^{158}Er also shown in Fig. 2. In all three cases one finds that there is a double crossing of the GRB and the SB. Also one finds that the stronger the backbending, the higher is I for the second crossing and the larger is $\Delta I = I_{\text{cross 2}} - I_{\text{cross 1}}$. For ^{158}Er, $I_{\text{cross 2}} \approx 30$; for ^{158}Dy, $I_{\text{cross 2}} \approx 26$ and for ^{174}Hf, $I_{\text{cross2}} \approx 22$. For ^{158}Er, this predicted $I_{\text{cross 2}} \approx 30$ is remarkably consistent with the observed[4] experimental second discontinuity about spin 28. Of course, when any backbending effect is small as in ^{158}Dy and ^{174}Hf, the GRB and SB bands are closer and mixing is stronger and can occur for many spins. Thus the predicted second crossings may not be seen until one reaches much higher spin, although an anomaly has already been noted in ^{174}Hf about spin 20 or 22.[12]

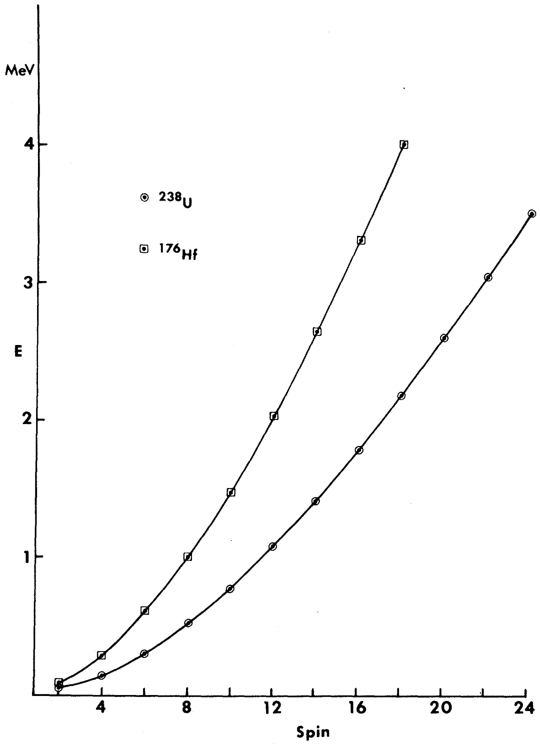

FIG. 1. The experimental level energies as a function of spin for ^{176}Hf and ^{238}U, connected by a theoretical curve derived by fitting Eq. (2) to the experimental energies for $I \leq 10$ for ^{176}Hf and $I \leq 12$ for ^{238}U. The differences between the curves and the data points are ≤ 1 keV for $I \leq 16^+$ and gradually increased to only 17 keV at $I = 24^+$ for ^{238}U and are 2 and 10 keV at 16^+ and 18^+ in ^{176}Hf. These differences are too small to be observed in the figure.

A crucial question is what evidence do we have that the GRB can be described by Eq. (2) above the first crossing? In other regions where two bands with different structures cross with states in both bands observed above and below the crossings such as ^{184,186,188}Hg, the two bands follow the $I(I+1)$ rule away from the crossing remarkably

TABLE I. The experimental yrast levels in ^{158}Er, ^{158}Dy, and ^{174}Hf are given along with the calculated energies (Ref. 8) of the ground-state rotational band as obtained from Eq. (2) fitted to $I^{\pi} \leqslant 10^{+}$. All energies in keV.

	^{159}Er Levels		^{158}Dy levels		^{174}Hf levels	
Spin	Expt.	$\text{GRB}_{\text{calc.}}$	Expt.	$\text{GRB}_{\text{calc.}}$	Expt.	$\text{GRB}_{\text{calc.}}$
2	192	193	98.9	98.9	91.0	91.0
4	527	529	317.5	317.5	297.4	297.5
6	970	972	637.6	637.6	608.4	608.2
8	1493	1497	1044.0	1044.2	1009.4	1009.6
10	2072	2076	1519.9	1519.3	1485.9	1485.9
12	2680	2694	2049.2	2047.2	2023.2	2022.5
14	3190	3339	2612.5	2615.5	2599.5	2606.7
16	3663	4006	3190.5	3215.7	3211.1	3229.3
18	4229	4691	3781.5	3841.8	3859.1	3883.4
20	4887	5391	4407.3	4489.6	4554.0	4563
22	5622	6105	5085.4	5156.1	5295.5	5263
24	6428	6830	5819.9	5838.9		5983
26	7271	7562	6611.2	6540		
28	8126	8302				
30	8997	9050				
32	9899	9810				

well.[13,14] More recently we have observed the GRB above the crossing by two rotational aligned bands in ^{68}Ge and the 8^{+} and (10^{+}) GRB states are nicely predicted by fits of Eq. (2) to the states up to spin 6^{+}.[15] Since this paper was submitted, other new data have become available which clearly support that the expansion is good above the crossing. In neighboring ^{164}Er, Johnson *et al.*[16] observe states above and below the crossing of the ground band and super band at $I = 16$. Fitting the $I \leqslant 10$ states to Eq. (2), we find that the fitted energies agree exactly for $I \leqslant 10$ and are as follows above that with the fitted energies in parentheses: 12^{+}, 2082.8 (2083.0); 14^{+}, 2702.6 (2708.1); 16^{+}, 3411.2 (3383.7); 18^{+}, 4121.2 (4102.0); (20^{+}), 4868.4 (4856.7); (22^{+}), 5651.5 (5642.7) keV. There is a jump of 27.5 keV in the experimental energy over the fitted one at the crossing point where mixing would push up this level. After that the experimental levels are similarly 10–20 keV higher than the fit as may be expected from the pushing up of the 16^{+} level. Thus, the expansion works for the next three states above the first crossing point to offer strong support to our use of it above the first crossing in ^{158}Er.

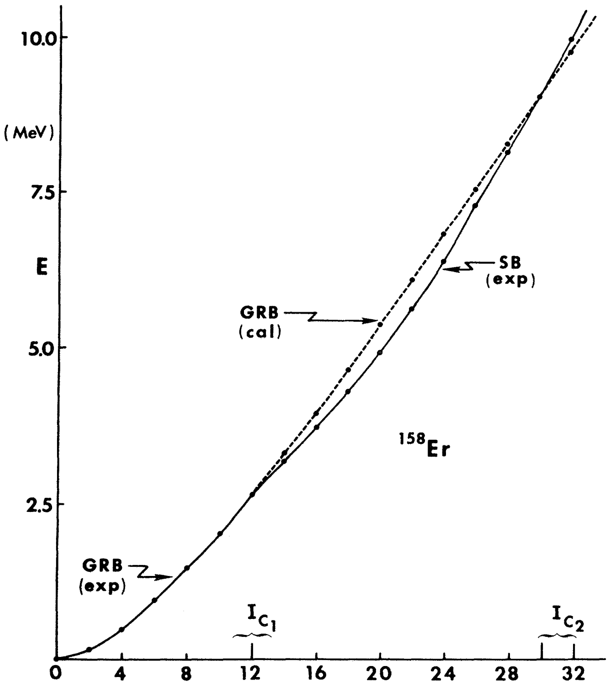

FIG. 2. The experimental yrast level energies for ^{158}Er. The GRB is crossed about $I = 12$ by SB. A fit of the experimental levels with $I \leqslant 10$ to Eq. (2) yielded the theoretical curve for the GRB shown as a dashed curve. The experimental yrast curve is crossed a second time by the calculated GRB curve about $I = 30$.

The reason for the prediction of a second crossing of the GRB and SB is that at high I the γ-ray energies, E_{γ}^{GRB}, for the GRB based on Eq. (2) do not increase as fast as E_{γ}^{SB} for the same I. After $I_{\text{cross }1}$, $E_{\gamma}^{\text{GRB}}(I) > E_{\gamma}^{\text{SB}}(I)$ (because $\mathcal{I}_{\text{SB}} > I^{\text{GRB}}$), but at some $I_{\text{cross }2}$ $E_{\gamma}^{\text{GRB}}(I) \leqslant E_{\gamma}^{\text{SB}}(I)$ again. (This can lead to $\mathcal{I}_{\text{eff}} > \mathcal{I}_{\text{rigid}}$ which is possible because $\mathcal{I}_{\text{eff}}$ is not a real moment of inertia, and there are many reasons to decrease E_{γ} in addition to a real increase in the amount of inertia.) In ^{158}Er up to $I = 32$, $\mathcal{J}_{\rm eff}$ is always less than $\mathcal{J}_{\rm rigid}$, however. If the SB is a rotational aligned one, then the above behavior is expected because of the large difference in the collective (rotational) angular momentum of the states of the same I in each band; e.g., for the GRB, $R = I$ but for an aligned, SB, $R = I - I_0$ with $I_0 = 8-12$ so that the correction ΔE to $E_{\rm rot}^{VMI}(I)$, $\Delta E = \gamma\omega_R^6 + \delta\omega_R^8$ (which experimentally is found to be negative), is much larger for the GRB levels than that for the SB levels at a given I even though this equation should not strictly hold for the SB which is not a pure $K = 0$ band. Thus for a rotational aligned SB, $E_\gamma^{\rm SB}$ can increase more strongly with I at high spin than $E_\gamma^{\rm GRB}$ and lead to a double crossing of the two bands. For ^{158}Er, one can turn the argument around to say the following: If the observed second discontinuity in the yrast cascade for ^{158}Er is a result of a second crossing of the GRB and SB, then the SB is a rotational aligned band. Of course, perturbation of the GRB by the SB should occur to some degree and may be responsible for the difference between the expected second backbending about $I \approx 28$ and our prediction at $I \approx 32$. In any case the above arguments indicate that for weak backbending, when a rotational aligned band crosses the GRB, one should expect that the states above the crossing will be mixed for many spins, and even for strong backbending, the states in the two bands should come back together so that mixtures of these two bands may occur again at higher spin. Our proposed second crossing of the GRB and SB levels can be tested by experimentally seeking to follow the GRB to levels with spins above the first crossing to see how the energies of these levels compare with those predicted by Eq. (2) for ^{158}Er.

This work was supported in part by a grant from the U. S. Energy Research and Development Administration.

---

[1] F. S. Stephens, Rev. Mod. Phys. 47, 43 (1975); D. Ward, in *Reactions Between Complex Nuclei*, edited by R. L. Robinson *et al.* (North-Holland, Amsterdam, 1974); A. Faessler, *ibid.*, p. 437, and A. Faessler, M. Ploszajczak, and K. R. Sandhya Devi, in *International Symposium on High Spin States and Nuclear Structure, Dresden, 1977*, edited by L. Funke (Akademie der Wissenschaften der Deutschen Demokratischen Republik, Rossendorf, 1977), p. 68.

[2] F. S. Stephens and R. S. Simon, Nucl. Phys. A183, 257 (1972).

[3] E. Grosse, F. S. Stephens, and R. M. Diamond, Phys. Rev. Lett. 31, 840 (1973), and 32, 74 (1974).

[4] I. Y. Lee, M. M. Aleonard, M. A. Deleplanque, Y. El-Masri, J. O. Newton, R. S. Simon, R. M. Diamond, and F. S. Stephens, Phys. Rev. Lett. 38, 1454 (1977).

[5] B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. 5, 511 (1960).

[6] H. J. Mang, J. O. Rasmussen, and M. Rho, Phys. Rev. 141, 941 (1966).

[7] A. Faessler, R. K. Sandhya Devi, F. Grümmer, and K. W. Schmid, Nucl. Phys. A256, 106 (1976).

[8] Ø. Saethre, S. A. Hjorth, A. Johnson, S. Jagare, K. Ryde, and Z. Szymanski, Nucl. Phys. A207, 486 (1973); T. L. Khoo, F. M. Berntnal, R. G. N. Robertson, and R. A. Warner, Phys. Rev. Lett. 37, 832 (1976).

[9] E. Grosse, J. de Boer, R. M. Diamond, F. S. Stephens, and P. Tjøm, Phys. Rev. Lett. 35, 565 (1975).

[10] H. Bokemeyer, H. Emling, P. Fuchs, J. S. Greenberg, E. Grosse, D. Schwalm, H. J. Wollersheim, and D. Pelte, in *International Symposium on High Spin States and Nuclear Structure, Dresden, 1977*, edited by L. Funke (Akademie der Wissenschaften der Deutschen Demokratischen Republik, Rossendorf, 1977), pp. 78 and 79; D. Schwalm, private communication.

[11] H. Bohn, T. V. Egidy, Th. Faestermann, F. v. Feilitzsch, P. Kienle, K. J. Körner, and J. P. Schiffer, in Proceedings of the Colloque Franco-Japonais and INS Symposium, Tokyo, 1976 (unpublished), p. 317.

[12] L. K. Peker and J. H. Hamilton, in *Proceedings of the International Conference on Nuclear Structures, Tokyo, Japan, 1977*, edited by the Organizing Committee (International Acedemic Printing Co. Ltd., Tokyo, 1977), p. 110.

[13] J. H. Hamilton *et al.*, Phys. Rev. Lett. 35, 562 (1975); J. D. Cole *et al.*, Phys. Rev. Lett. 37, 1185 (1976).

[14] I. N. Mikhailov, K. Neergard, V. V. Pashkevich, and S. Frauendorf, to be published.

[15] A. de Lima *et al.*, in *Proceedings of the International Conference on Nuclear Structure, Tokyo, Japan, 1977*, edited by the Organizing Committee (International Academic Printing Co. Ltd., Tokyo, 1977), p. 276, and private communication (to be published).

[16] N. R. Johnson, D. Cline, S. W. Yates, F. S. Stephens, L. L. Riedinger, and R. M. Ronningen, Phys. Rev. Lett. 40, 151 (1978).