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Gamow-Teller (p,n) and (n,p) strength in a dressed extended random phase approximation

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Induced forces of second order in the nucleon-nucleon G-matrix interaction are included in a random phase approximation (RPA) type formalism with dressed single-particle propagators. The predictions of a considerable amount of Gamow-Teller (n,p) strength for magic nuclei with neutron excess, found in earlier dressed RPA calculations, are not modified by the induced forces. Also the distribution of (p,n) strength is only a little affected by the induced interactions. It is therefore concluded that the relatively simple dressed RPA method is a suitable one to calculate excitation strengths.

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For magic nuclei with neutron excess, such as ^{48}Ca, the Gamow-Teller (n,p) strength is completely due to ground state correlations. In particle-hole random phase approximation (RPA) calculations this strength is predicted to be negligible, even in an extended version (ERPA) in which induced forces that originate from medium polarization are included [1]. This result is due to the fact that in these RPA methods only particle-hole and hole-particle excitation amplitudes are considered. The bulk of the Gamow-Teller (n,p) strength in magic nuclei comes from amplitudes with shell model orbits for the proton and neutron on the same side of the Fermi level [2]. In accordance with Refs. [2–4], these are called particle-particle (pp) or hole-hole (hh) excitation amplitudes; not to be confused with amplitudes for two nucleon addition or removal reactions. Such pp and hh excitation amplitudes immediately arise in a formalism in which the partial occupancy of shells, even for magic nuclei, is taken serious. This partial depletion of shells below and partial filling of shells above the Fermi level has recently been studied in detail with (e,$e'p$) reactions [5–7]. The ratios of spectroscopic factors for the strongly excited quasi-hole state and other states with smaller spectroscopic factors may be reproduced in calculations in which the propagation of a hole or particle is coupled to excitations of the residual nucleus [8]. This coupling is most easily described by solving a Dyson equation, graphically represented in Fig. 1. If the self-energy Σ^* is expanded to second or higher order in the (G-matrix) interaction V, one finds the aforementioned smearing of the Fermi surface which finds expression in nonzero particle addition as well as removal amplitudes for a given orbit α in the single-particle propagator [8–10]:

$$g_{\alpha\beta}(\omega) = \sum_n \frac{\langle\Psi_0^A|c_\alpha|\Psi_n^{A+1}\rangle\langle\Psi_n^{A+1}|c_\beta^\dagger|\Psi_0^A\rangle}{\omega-(E_n^{A+1}-E_0^A)+i\eta} + \sum_m \frac{\langle\Psi_0^A|c_\beta^\dagger|\Psi_m^{A-1}\rangle\langle\Psi_m^{A-1}|c_\alpha|\Psi_0^A\rangle}{\omega-(E_0^A-E_n^{A-1})-i\eta} \,. \tag{1}$$

One finds [1,2] that for a suitably chosen set of shell model orbits α, this propagator is to a very good approximation diagonal in that model space:

$$g_{\alpha\beta}(\omega) = \delta_{\alpha\beta}\left[\sum_i \frac{S_{i_\alpha}^{\alpha,p}}{\omega-E_{i_\alpha}^{\alpha,\mathrm{p}}+i\eta} + \sum_j \frac{S_{j_\alpha}^{\alpha,h}}{\omega-E_{j_\alpha}^{\alpha,\mathrm{h}}-i\eta}\right]. \tag{2}$$

If one assumes that particles and holes propagate in full interaction with the nuclear medium but without any direct or indirect interaction with each other (dressed independent particle approximation (DIPA) in Ref. [2]) the polarization propagator [2,9–11] (here for charge exchange excitations)

$$L_{\alpha\beta;\gamma\delta}(\omega) = \sum_{n\neq 0}\left[\frac{\langle\Psi_0|c_\beta^\dagger c_\alpha|\tilde\Psi_n\rangle\langle\tilde\Psi_n|c_\gamma^\dagger c_\delta|\Psi_0\rangle}{\omega-E^n+i\eta}\right] - \sum_{m\neq 0}\left[\frac{\langle\Psi_0|c_\gamma^\dagger c_\delta|\tilde\Psi_m\rangle\langle\tilde\Psi_m|c_\beta^\dagger c_\alpha|\Psi_0\rangle}{\omega+E^m-i\eta}\right], \tag{3}$$

where $\tilde\Psi$ denotes a state of the $(N\pm 1, Z\mp 1)$ nucleus, takes the simple form

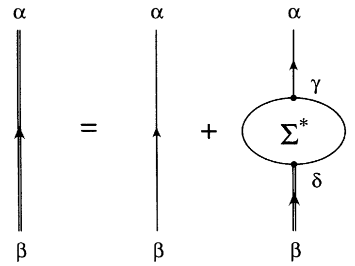

FIG. 1. Graphical representation of the Dyson equation for the single-particle propagator with irreducible self-energy Σ^*, which acts as an effective dynamical shell model potential.

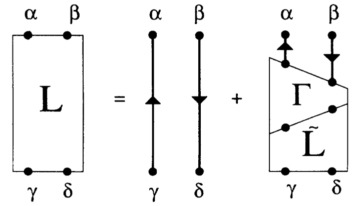

FIG. 2. Graphical representation of the Bethe-Salpeter equation (5) for the polarization propagator.

$$L^{\mathrm{DIPA}}_{\alpha\beta\gamma\delta}(\omega) = \delta_{\alpha\gamma}\delta_{\beta\delta}\left[\frac{S^{\alpha,p}_{i_\alpha}S^{\beta,h}_{j_\beta}}{\omega-(E^{\alpha,p}_{i_\alpha}-E^{\beta,h}_{j_\beta})+i\eta}-\frac{S^{\alpha,h}_{j_\alpha}S^{\beta,p}_{i_\beta}}{\omega-(E^{\alpha,h}_{j_\alpha}-E^{\beta,p}_{i_\beta})-i\eta}\right]. \quad (4)$$

From this expression it is clear that there are nonzero contributions even if indices α and β belong to orbits on the same side of the Fermi level.

An exact expression for the (two-times) polarization propagator is given by the exact Bethe-Salpeter (BS) equation [9–11], as depicted in Fig. 2:

$$L_{\alpha\beta;\gamma\delta}(t_1-t_2) = -ig_{\alpha\gamma}(t_1-t_2)g_{\delta\beta}(t_2-t_1') + \int dt_3dt_4dt_5dt_6\Big[-i\,g_{\alpha\mu}(t_1-t_3)g_{\nu\beta}(t_4-t_1')$$
$$\times i\Gamma_{\mu\nu;\kappa\lambda}(t_3,t_4;t_5,t_6)\,\tilde{L}_{\kappa\lambda;\gamma\delta}(t_5-t_2,t_6-t_2)\Big], \quad (5)$$

which is very complicated because it contains an effective interaction Γ, which depends on four times (when Fourier transformed three energy variables), while on the rhs also the more general three-times polarization propagator $\tilde{L}$ appears. In practice, approximations have to be made. If Γ is neglected altogether, one has the DIPA. With the approximation $\Gamma = V$, with V a static interaction, one has the dressed RPA (DRPA) of Ref. [2]. In Ref. [2] it was shown that it makes little difference for the calculated total (p,n) and (n,p) strength whether one adopts the approximation $\Gamma = 0$ or $\Gamma = V$; only a certain redistribution of strength at low energy is caused by coherence effects when V is used as an effective interaction. Therefore it was suggested that this total calculated strength would hardly change when terms of higher order in V would be included in the effective interaction Γ. Here we report on a check of this supposition by explicit calculations in which induced interactions by medium polarization to second order in V are included in Γ. Thereby the treatment of the effective interaction Γ is more in balance with that of the single-particle potential Σ^*, the dynamic nature of which also to second order in V gives rise to the fragmentation of spectroscopic strength in the single-particle propagator (1). Such a more consistent treatment of self-energy Σ^* and effective interaction Γ may be required to satisfy conservation laws and thereby sum rules for total excitation strengths [11,12]. This total amount of strength, especially for Gamow-Teller excitations, has been the subject of many investigations as it might show the influence of delta excitations of the nucleon at low nuclear excitation energies [13].

The induced effective interaction that was included in the present study is of second order in V and with dressed single-particle propagators, see diagrams of Fig. 3. The approximation implies that Γ depends on only one time difference. All terms of L up to second order in V are included and higher-order terms in some approximation. These interaction diagrams are reminiscent of the "screening" and "ladder" diagrams that were introduced in the extended RPA (ERPA) of Ref. [1], but for the sake of consistency the lines in the diagram are now dressed propagators [2]. A consequence of this feature is that with this approximation, in contrast to the RPA, ERPA, and DRPA, the BS equation for the polarization propagator is no longer of the form (with summation over indices appearing twice)

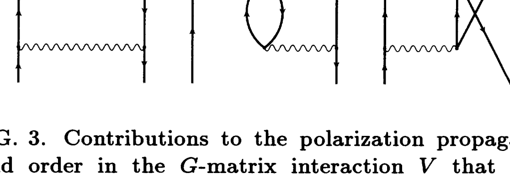

FIG. 3. Contributions to the polarization propagator to second order in the G-matrix interaction V that are included by the adopted approximation for the effective particle-hole interaction Γ. The double lines represent dressed single-particle propagators of the form Eq. (2). The diagram on the left is already included in DRPA. The other two are the induced interactions referred to in the text.

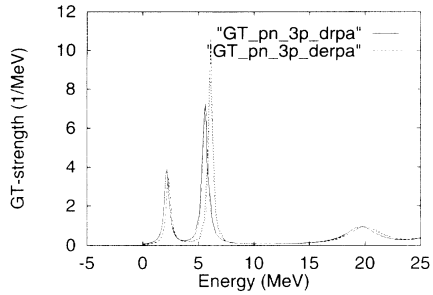

FIG. 4. Gamow-Teller (p,n) strength for ^{48}Ca, using a three-term approximation for the single-particle propagators (2) in DRPA, i.e., adopting $\Gamma = V$ and in DERPA, i.e., including the induced interactions, cf. Fig. 3.

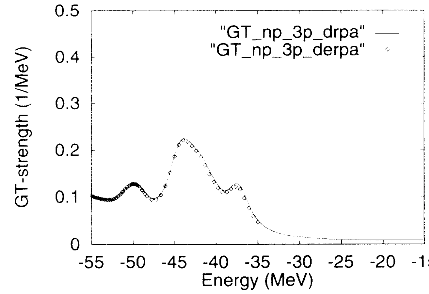

FIG. 5. Same comparison as in Fig. 4 for the (n,p) strength.

$$L^{\rm (D)(E)RPA}_{\alpha\beta\gamma\delta}(\omega) = L^{f}_{\alpha\beta\gamma\delta}(\omega) + L^{f}_{\alpha\beta\kappa\lambda}(\omega)\Gamma^{\rm (D)(E)RPA}_{\kappa\lambda\mu\nu}(\omega)L^{\rm (D)(E)RPA}_{\mu\nu\gamma\delta}(\omega) \tag{6}$$

and therefore cannot be solved by a standard matrix inversion procedure. However, a matrix equation form (6) can be retained if instead of the orbit labels α, β, etc., the labels $\{\alpha, i_\alpha, \beta, j_\beta\}$ of each fragment in the propagator (4) are used as indices of the matrices L and Γ: $L_{\{\alpha,i_\alpha,\beta,j_\beta\};\ldots}$.

Supposing that the single-particle propagator (2) contains 100 terms for each α, this implies that the matrices to be handled now have typically a 10 000 times larger dimension than in RPA, ERPA, of DRPA. This becomes untractable and because it is only our aim to check whether the inclusion of the "screening" and "ladder" diagrams of Fig. 3 have a large effect, we have limited the computational effort drastically by representing each single-particle propagator (2) with only three terms. In order to maintain the essential physical features, these three terms were so chosen, that besides the main quasiparticle or quasihole pole, with large residue, the other two poles were chosen on each side of the Fermi energy. Their residues were the sums of the small residues on each side of the Fermi energy and their positions the energy-weighted averages. In this way, the essential features of fragmentation of the strength and of partial depletion and occupation of the orbits is retained. The point of our investigation is now whether with such a three-pole approximation of the single-particle propagator it makes a big difference whether the induced interaction (Fig. 3) are included or not.

The results of such a comparison are displayed in Fig. 4 for the (p,n) strength and in Fig. 5 for the (n,p) strength for the earlier studied case of ^{48}Ca. Shell model space and G-matrix interaction are the same as in Ref. [2]. The only difference between DRPA results of [2] and those of Figs. 4 and 5 is that the pole strength of weak poles of the single-particle propagator has been concentrated in one pole on both sides of the Fermi level. As an artefact of this treatment the bumps at 20 MeV and -43 MeV appear, which in a less drastic approximation with more poles are smeared out more equally over the energy region between 8 and 20 MeV and between -20 and -45 MeV, respectively. In this respect the results of [2] are more realistic. The point to be tested is the effect of the induced interactions, i.e., the comparison with the DRPA and DERPA results, both with the same three-pole approximation for the single-particle propagators. One notes that for the (n,p) strength this effect is quite negligible. This means that in the configurations which are predominantly of "particle-particle" and "hole-hole" type the induced forces do not yield extra coherence. For the particle-hole amplitudes, which are the main contributors in the lowest (pn) peaks there is some increase in the repulsive coherence which shifts strength from the lowest to the higher peak. The total integrated strength up to 25 MeV as well as up to 150 MeV changes by less than a percent however.

It may therefore be concluded that the induced forces of second order in the (G-matrix) interaction are not essential, except for details, for charge-exchange RPA calculations with dressed single-particle propagators. The DRPA method, applied in Ref. [2], is a valid tool to study (n,p) strengths in magic nuclei and their consequences for sum rules. Only for the precise distribution of strength among states at low energy, the inclusion of these induced forces has some effect, increasing the coherence already present due to the bare interaction.

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