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2009

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-Approximating Perfect Knowledge-

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Huge amounts of data can nowadays easily be stored. But the simple availability of detailed information does not automatically lead to more precise descriptions, conclusions or predictions of a quantity of interest. In order to tap the potential of the data, one needs to choose a suitable mathematical model.
Natural restrictions sometimes prevent us from observing a specific quantity. For instance, think of observations that can be decomposed into signal plus noise and where the object of interest is the uncorrupted signal. Knowing this specific setting and interpreting the available data as observations of the signal, introduces an error that could be avoided. It can be accurately taken into account by using a model that is specifically designed for situations where one can only observe a quantity that is related to the quantity of interest by some known relation.
One specific example of such an inverse model is discussed in this thesis: the *deconvolution model*. There, an observation is the sum of the variable of interest and some independent random error.
We focus on the *asymptotics* of nonparametric distribution function estimators. In the deconvolution setting we aimed at deriving asymptotic properties of the maximum likelihood estimator (MLE), and in particular the pointwise limit distribution of the estimator evaluated at a fixed point, originally given as a conjecture in **?**.

In Chapter 2-5 we study the MLE in a class of deconvolution models with decreasing noise densities satisfying certain smoothness conditions. The only very implicit characterization of this estimator in terms of Fenchel optimality conditions, makes it hard to straightforwardly derive its properties. Nevertheless, we prove various (global and local) asymptotic results for the MLE.
We succeeded in showing that the MLE is a well defined, piecewise constant estimator that only has points of jump at observation points and can be computed using iterative methods as can be found in the literature. It converges uniformly to the underlying distribution function F_0 (provided that F_0 is continuous) and is therefore consistent with respect to the uniform metric. Assuming bounded support of the noise density, its last point of jump is shown to stay away from the upper support point of the sampling density. Moreover, the MLE converges at rate $n^{-1/3}$ to F_0, globally with respect to the L_2-metric and locally uniformly in a neighborhood of fixed and shrinking length around some x_0. The latter result implies that also the distance between two successive points of jump of the MLE converges at rate $n^{-1/3}$.
Apart from the intrinsic interest of the asymptotic results, they are important ingredients of our strategy to derive the pointwise limit distribution. Under the conjecture that specific functionals of the MLE converge at rate $n^{-1/2}$ to their true values, we discuss how to derive that the limit equals the derivative of the concave majorant at zero of a

Brownian motion with negative quadratic drift. We do believe that the conjecture holds since it can be verified in specific cases, but is still being investigated.