Following periodic orbits through bifurcations

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Summary

The goal of validated numerics is to prove, for a variety of problems, the existence of a solution in the neigborhood of a numerical approximation. Given a numerical approximation, the procedure used in this thesis involves creating a bound r such that there exists a solution to the given problem that is at most r away from the numerical approximation. In this thesis, the approach used is the radii polynomial approach, based on the Banach contraction theorem. The global idea is to rewrite the initial problem into a fixed point problem and use the Banach contraction theorem on a neighbourhood of the numerical approximation to prove the existence of a solution.

In Chapter 2 of this thesis, this method is applied to proving the existence of periodic orbits of polynomial ODEs, both in the case of fixed parameters and variable ones.

In Chapter 3, we use the previous construction to prove the existence of two types of bifurcations: the saddle node and the Hopf bifurcations. In both cases numerical evidence is useful to start the procedure. The existence proof of the saddle node is achieved by checking that one parameters reached a local maximum or minimum. This is done by appending two equations to the initial ODE, these appended equations compute the first and second derivative of the given parameter with respect to the archlength. The Hopf bifurcation case requires the use of a blow up approach, this is a technique that that allows us to desingularise the system in the neighborhood of the bifurcation by rescaling the periodic orbit.

The last Chapter of this thesis takes the latest results on the validation of Hopf bifurcations in ODEs and extend them to the Kuramoto-Sivashinky PDE

$$\partial_t u = -\partial_x^4 u - \gamma \partial_x^2 u + \partial_x(u^2),$$

where γ is a real parameter. This Chapter proves the existence of a Hopf bifurcation from a non-trivial space-periodic stationary solution in an small interval of the parameter γ.