Following periodic orbits through bifurcations

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3 Rigorous verification of Hopf bifurcations via desingularization and continuation

3.1 Introduction

In dynamical systems, bifurcations are of key importance to understand global parameter dependence of the dynamics. The analysis of bifurcations by pen and paper is generally restricted to cases where the solution at the bifurcation point is known analytically, and even in such cases it is often not feasible to examine properly the associated eigenvalue problem by hand. Hence numerical methods are applied ubiquitously to study bifurcation diagrams, for example using specialized software such as AUTO [16], MatCont [15], PyDSTool [11], XPP [17] and COCO [13]. This involves both the numerical continuation of solutions as well as the computational analysis of bifurcation points. To make such numerical simulations into rigorous mathematical statements, additional effort is required. For this purpose rigorous verification schemes (sometimes referred to as *a posteriori error analysis*) have been developed for a variety of continuation problems in the past decade, see [1, 5, 21, 65, 74, 70] and the references therein. The analogous methodology for bifurcation problems is much less developed, although some foundational results on pitchfork and saddle-node have been obtained in [1, 27, 38, 41, 75, 77, 82]. Moreover, double turning points [45, 58], period doubling bifurcations [77] and cocoon bifurcations [30] have also been considered.

In this paper we develop a general framework for the rigorous verification