Introduction

How to distribute goods is a question that is faced daily by many companies and, therefore, these questions are solved regularly in practice either with or without supporting technologies. A general aim is to keep costs low and customer service high, for example, by minimizing delivery costs and making sure that customer demand is satisfied. Increasing resource utilization and transportation efficiency can lead to cost savings and service improvements for all parties involved (e.g., manufacturers, transportation companies and customers). Efficiency can, for example, be enhanced by finding improved distribution plans, i.e., plans with lower costs, or by exploring new distribution strategies. This dissertation intends to provide insight in complex distribution problems, to find more efficient distribution plans, and to analyze the potential benefit of novel distribution strategies.

Academic research on distribution problems relates to industry in several ways. First, scientific studies identify basic optimization problems which are underlying real-life distribution networks. Research focuses on analyzing these basic problems and on developing solution methods to solve them. An example of a fundamental problem for the distribution of goods is the so-called Traveling Salesman Problem (TSP). In this problem there are a vehicle at a depot and some geographically spread customers that need to be visited, the question is to determine the tour with the shortest distance. A fundamental extension of this problem is the Vehicle Routing Problem (VRP) in which each customer requests a number of units of goods (demand) and there are multiple vehicles with a load capacity (i.e., number of units that can be loaded into the vehicle). Again, the customers need to receive their demand and the problem is to decide which vehicle serves which customers and to find the corresponding routes such that the total covered distance is minimized. Another fundamental problem, which does not include determining vehicle tours, is the Joint Replenishment Problem (JRP). In the JRP, deliveries have to be made to customers over a given time horizon. For each period in which a replenishment takes place and for each replenishment to a customer a fixed fee is incurred. The problem is to decide when to deliver goods to each customer and how
many goods while minimizing the delivery costs.

Secondly, literature concerns more integrated problems and examines multiple types of solution approaches for these problems. For instance, consider the case in which there are multiple vehicles available (fleet) to replenish multiple locations that have to satisfy a certain demand every period over a given planning horizon. This problem is known as the Inventory Routing Problem (IRP). The IRP contains the following questions: when to replenish the locations, how much to deliver at each replenishment and how to make the deliveries with the given fleet (routing) such that all demand can be satisfied. Along with these research questions, the IRP usually considers multiple, contradicting, cost components, such as routing and inventory holding costs. Multiple types of solution approaches are present in academic literature for such integrated problems. For the IRP, it is possible to apply a sequential solution method in which first the delivery periods and quantities are decided upon and thereafter, the delivery routes are determined per period (i.e., a VRP is solved per period). This decomposition shows that the VRP is a subproblem of the IRP. Next to sequential solution methods, also iterative and integrated solution methods are proposed, among others, which make use of the knowledge on subproblems as well. The difference in efficiency between solving two problems sequentially or integrating the optimization of the decisions can be substantial (see for example Chandra and Fisher [1994] and Archetti and Speranza [2016]), since integrated methods address the trade-off between costs directly. Therefore, integration of more types of decisions in one optimization problem, towards a ‘systemic focus’, is one of the trends identified for the field of Operations Research [Speranza, 2018].

Finally, academic research investigates advanced distribution strategies which are possibly already applied in the industry. As an illustration, consider the case in which for the distribution of goods to a customer a choice can be made between a privately owned truck and an external carrier. Outsourcing the service can, for example, be beneficial when the total quantity to delivery is higher than the total capacity of the private fleet. Despite the practical applications and the actual use in industry, the relatively recent work by Chu [2005] can be considered the first to examine the VRP combined with the question which customer services to outsource (Vehicle Routing Problem with Private Fleet and Common Carrier (VRPPC)). Compared to the abundant literature on the VRP (over 50 years [Laporte, 2009]) and the IRP (over 30 years [Coelho et al., 2014]), studies that consider the VRPPC are emerging only recently.

Inspired by distribution challenges in cash supply chains, this dissertation aims to contribute to the above three research directions. In order to be able to define the research challenges which this dissertation addresses in more detail, first, short overviews are presented on the IRP, the JRP and the VRPPC in Sections 1.1, 1.2 and 1.3, respectively. The contributions of this dissertation are specified within those sections. Thereafter, an introduction to a main solution method is provided in Section 1.4. The practical motivation that served as inspiration for the research topics is discussed in Section 1.5 before concluding with an overview of this dissertation in Section 1.6.

1.1 Inventory Routing Problems

The Inventory Routing Problem (IRP) is a combination of an inventory management problem and a vehicle routing problem. The most common variant of the problem in the
literature has the following characteristics. There are one or multiple vehicles located at a depot each with a certain load capacity. These vehicles replenish the inventory of a set of geographically spread customers over a given planning horizon of several periods. At the depot a quantity of the product to be distributed becomes available each period of the planning horizon. A customer has a limited inventory capacity, has to satisfy a given demand per period and cannot have a shortage. The IRP entails the decisions on when to replenish each customer, which quantity to deliver and how to combine the visits to the customers in feasible routes while minimizing the total routing and inventory holding cost. This problem arises when a supplier (often denoted by vendor) can decide on the replenishments of its customers which is known as a Vendor Managed Inventory (VMI) setting. It is assumed that the vendor incurs all costs that usually consist of routing costs and inventory holding costs at both the depot and the customers. A good solution to this problem addresses the trade-off between these two cost types as discussed before.

1.1.1 Literature

The IRP is a widely studied class of problems in the literature for over more than thirty years [Coelho et al., 2014]. Bertazzi and Speranza [2012] and Bertazzi and Speranza [2013] introduce the IRP by considering a single customer case with multiple products and a multiple customer case with a single product respectively. In the first case, transportation costs and inventory holding costs are minimized, but only direct routes from the depot to the single customer are possible, hence the transportation cost per replenishment is fixed. In the second case, the transportation cost depends on the combination of customers served by one vehicle. Therefore, a VRP is a subproblem of the IRP in this variant. Bertazzi et al. [2008] provide another introduction to IRPs in which the focus is to examine the influence of holdings costs and inventory holding capacities at the customers, among others.

The two most recent surveys on the IRP each have a different focus. First, Andersson et al. [2010] provide an extensive overview on the industrial aspects and applications of inventory routing. The authors also propose a classification based on seven characteristics that concern the length of the time horizon, demand (deterministic or stochastic), routing, inventory and fleet aspects. The paper contains an extensive literature review divided in three groups based on an instant, finite or infinite planning horizon. Secondly, Coelho et al. [2014] propose a slightly different classification of IRPs than Andersson et al. [2010]. Coelho et al. [2014] suggest to separate the problem structure from the information availability and therefore leave the demand component out. Next to classifying a large number of studies in their classification scheme, Coelho et al. [2014] focus on solution methods for both ‘basic’ IRP variants and for extensions such as the IRP with multiple products.

More recent literature on the IRP includes a novel problem formulation by Desaulniers et al. [2016] including a solution method that gives promising results for the multiple-vehicle IRP. Additionally, Archetti et al. [2017] and Alvarez et al. [2018] present new, competitive heuristic solution methods.
1.1.2 Solution methods

Solution methods for the IRP can be roughly divided into two groups: exact and heuristic solution methods. Exact solution methods result in the optimal solution of the given problem, while heuristic solution methods provide solutions for which there is no guarantee on the quality. Sequential and many iterative solution methods are heuristics by definition. Although these methods can result in the optimal solution, there is no guarantee for that.

For the main exact solution methods, first the problem is formulated as a mixed integer linear program (MILP). This mathematical representation of the problem contains variables that represent the decision to be taken (e.g., the delivery quantity to a customer in a certain period). Based on this formulation branch-and-cut and branch-and-price-and-cut solution methods can be applied. In both methods, the integer decision variables are first relaxed and subsequently forced to be integer sequentially by adding constraints (‘branching’). In a branch-and-cut solution method, next to the branching, additional constraints (valid inequalities) are added in each branching step which are not necessary to find the optimal solution, but will help to reach the optimal solution quicker (‘cutting’). Also sets of constraints that are necessary to find the optimal solution can be added with a similar procedure which is especially useful if the number of constraints is exponential (i.e., enlisting all of them would be cumbersome). Archetti et al. [2007] were the first to apply branch-and-cut to solve the IRP with a single vehicle. They identified multiple families of valid inequalities and also added one type of necessary constraints as cuts. Instances with up to fifty customers and three periods, and thirty customers and six periods were solved to optimality within two hours of running time. Branch-and-cut solution methods for the multi-vehicle IRP were proposed by Coelho and Laporte [2013] and Adulyasak et al. [2014]. The latter solves instances with up to 45 customers, three periods and three vehicles to optimality.

If a problem formulation contains an exponential number of a specific type of decision variables, one can apply branch-and-price-and-cut. This method starts with a limited number of decision variables, and iteratively adds more variables by validating which additional variables would result in a better solution (‘pricing’). Branch-and-price-and-cut was applied to the IRP with multiple vehicles by Desaulniers et al. [2016]. The authors were able to solve instances with up to fifty customers, three periods and five vehicles to optimality. Since branch-and-price-and-cut is applied in multiple chapters in this dissertation, more details on the method will be provided in Section 1.4.

The heuristic solution methods developed to solve the IRP are numerous. Therefore, a few recent examples will be discussed without having the ambition to give a complete overview. For more examples and an extensive discussion on heuristic solution methods for the IRP see Coelho et al. [2014]. First, Campbell and Savelsbergh [2004] develop a sequential, two-phase heuristic. The first phase assigns customer replenishments to periods in the planning horizon. This assignment is based on clusters which allows customers only to be served in the same route if they are in the same cluster. The second phase optimizes the delivery routes with the output of the first phase as suggestion. Secondly, many metaheuristics which are based on local search procedures have been applied to the IRP. Local search means that by making small changes in a non-optimal solution of the problem a better solution is sought. Recent examples of such methods are the iterated local search and simulated annealing heuristics by Alvarez et al. [2018].
With these methods new best solutions were found for almost 300 benchmark instances from the literature with up to 200 customers. Finally, another type of heuristic solution methods are matheuristics which combine exact and heuristic solution methods. Archetti et al. [2017] recently proposed a matheuristic that combines tabu search (a metaheuristic) with the solutions of MILPs. First, a MILP is used to find an initial solution which is not necessarily integer in the routing variables. Then, tabu search is applied to improve the initial solution during which vehicle capacity can be violated and stock-outs at the supplier are allowed. Finally, another MILP is solved which is based on the solutions found during the tabu search. Applying this method to small benchmark instances (up to 50 customers and three periods or 30 customers and six periods) resulted in 48% optimal solutions and 125 improved upper bounds compared to existing literature; for larger benchmark instances (up to 200 customers and six periods) for 92% of the tested instances a better upper bound was established.

1.1.3 Contributions

This dissertation contributes to the area of IRP in the following ways.

Chapter 2 analyses the computational complexity of a variant of the IRP. It is well-known that the IRP as defined earlier is an NP-hard problem since the TSP is an underlying problem which is NP-hard [Karp, 1972]. Hence, one source of complexity in the IRP is the routing part. Therefore, Chapter 2 considers special cases of the studied IRP in which the underlying metric is such that routing does not cause immediate NP-hardness through the TSP. This allows for studying the influence of other aspects than routing on the complexity of the IRP. One main result is proving that the IRP on the half-line with uniform service times and a planning horizon of two periods can be solved in polynomial time. Moreover, it is shown that if the planning horizon is extended to more than two periods, the problem is harder than the Pinwheel Scheduling Problem [Holte et al., 1989], of which the complexity is unknown. Furthermore, almost any variant of the IRP with non-uniform service times is NP-hard and the study shows, equivalently, that the same results holds for an IRP with multiple vehicles each with a vehicle capacity constraint and different demand at the customers. Establishing other sources of complexity than routing could in the future aid the development of solution methods for IRPs.

Chapter 4 incorporates a novel replenishment strategy in the IRP. In the IRP it is assumed that each customer faces a certain demand in each period of the planning horizon which must be satisfied by the customer itself without running out of stock. In Chapter 4, inspired by practice, an extension of the IRP is studied which relaxes the assumption that demand must be satisfied by the customer itself. In the extension it is assumed that if a customer \( i \) is close enough to another customer \( j \), customer \( i \) can satisfy (part of) the demand of customer \( j \) in each period. This implies that a customer does not always have sufficient stock to satisfy its demand, but that it can move part of the demand to another customer. Hence, the required replenishments differ compared to the traditional IRP and costs can be saved on the distribution of the goods. To include this extension in the IRP, the possibility of demand moves is introduced in the IRP, i.e., a customer \( i \) can satisfy the demand of another customer \( j \), since \( j \)'s demand is moved to \( i \). This problem is defined as the Inventory Routing Problem with Demand Moves (IRPDM). To move a unit of demand a service cost
is charged depending on the distance between the involved customers. The objective is to minimize the total inventory holding, routing and service costs. In Chapter 4 a branch-and-price-and-cut solution method is developed to solve the IRPDM. The costs of the solutions are compared to those of the IRP to evaluate the influence of allowing demand moves. Moreover, the impact of changing the service costs and of putting a maximum on the demand that can be moved per customer per period is assessed.

1.2 Joint Replenishment Problems

Joint Replenishment Problems (JRP) concern the distribution of goods without a routing aspect. In the JRP there are multiple customers that have to be replenished over a given time horizon of multiple periods such that the customers can satisfy the demand in each period. The costs consist of replenishment costs and inventory holding costs. The replenishment costs involve two components. First, if any customer is replenished in a period, independent of the locations involved, a fixed reorder/transportation cost (major cost) is incurred. Secondly, a customer specific cost (minor cost) is charged per replenished customer in a period. An example of such a cost structure is given by Anily and Haviv [2007] for the case that a number of retailers outsource their inventory management to an external carrier. If there would be no major cost, it would be most cost efficient to replenish all customers just before they run out of stock. When including major costs, it might be beneficial to replenish a customer earlier to serve it jointly with other customers in order to save on the major costs. Note that replenishing a customer earlier will give slightly higher inventory holding costs and possibly higher minor costs since in the long run the number of replenishments is higher. Another setting which is modeled as JRP is the case when multiple products have to be ordered for one customer and the question is how to combine the different products in one order [Khouja and Goyal, 2008].

1.2.1 Literature and solution methods

The literature on JRPs distinguishes three types of problems which differ in the assumptions on or availability of data on the demand [Khouja and Goyal, 2008]. First, the traditional JRP assumes that the demand is known and constant, i.e., for a customer the demand is the same in each period of the planning horizon. Secondly, the JRP with stochastic demand assumes that demand is stochastic but stationary in the mean. Finally, the Dynamic-Demand JRP (DJRP) considers the case in which the demand is known, but can be different in each period of the planning horizon. The solutions to JRP problems consist of either a plan for a given time horizon or a long term plan which minimizes long term average costs. As in the IRP, a solution method should find a good trade-off between the replenishment and inventory holding costs, however, note that the cost structure is fundamentally different than in the IRP.

For the traditional JRP it is possible to derive analytical expressions that define the minimal total costs for a given replenishment policy. Heuristics have been designed to determine the replenishment policies. These policies are often cyclic for each customer which means there is a fixed time between two replenishments. Khouja and Goyal [2008] provides a thorough overview of heuristics for the JRP including the RAND algorithm, the power-of-two policy, in which the time between replenishments is restricted to $2^p$
times the cycle time with \( p \) a positive integer, and genetic algorithms. A systematic review of the literature published between 2006 and 2015 is given by Bastos et al. [2017].

Two main policies for the stochastic JRP are the periodic review policy and the can-order policy. In a can-order policy there is an indicated inventory level at which a replenishment must take place, but also a can-order level at which a replenishment is optional which provides flexibility in the ordering process. A periodic review policy implies a replenishment order at fixed moments in time, similar to the cyclic policies for the JRP. More details on the policies and heuristics to determine the parameters are provided in Khouja and Goyal [2008]; references to more recent literature can be found in Bastos et al. [2017].

For the JRP with dynamic demand, cyclic replenishment policies are not as applicable as for the other JRP variants. Still, cyclic policies have been studied for the DJRP because of the easiness of implementation [Webb et al., 1997]. Boctor et al. [2004] proposes multiple MILP formulations for the DJRP which are compared in performance by implementing them in CPLEX. Moreover, the authors compare the performance of eight heuristics which are, for instance, based on dynamic programming and greedy planning of replenishments. Exact algorithms are based on dynamic programming, branch-and-bound, branch-and-cut and column generation according to Boctor et al. [2004]. Tighter DJRP problem formulations were proposed and tested by Narayanan and Robinson [2006]. Additional heuristics were tested by Robinson et al. [2007]. Robinson et al. [2009] study formulations for the DJRP but denote the problem by ‘coordinated uncapacitated lot-sizing problem’ and review heuristic solution methods classified as specialized heuristics, metaheuristics and mathematical programming-based heuristics.

## 1.2.2 Contributions

Chapter 3 proposes a model to incorporate customer locations in the replenishment decisions in the DJRP without aiming at optimizing the actual routing of the vehicles. This study follows the tendency to integrate optimization of multiple decisions. The locations of the customers are not taken into account in the decisions on replenishments in traditional JRPs. Hence, in an optimal distribution plan, it can occur that two customers that are in close proximity are served in subsequent periods. This might not be desirable when considering that an actual vehicle has to execute the deliveries. Hence, replenishments determined by the DJRP can lead to high actual transportation costs. Moreover, the number of customers that can be served per period can be unnecessarily low caused by higher travel time between served customers. Still, the cost structure in the JRP is a realistic one in practice, for example if all deliveries are outsourced to a common carrier. Given the JRP cost structure, the supplier of the customers has no incentive to consider customer locations when making the replenishment decisions. Thereafter, given the replenishment orders for one period, the common carrier can only solve the routing problem for this single period. Therefore, Chapter 3 introduces the Dynamic-Demand Joint Replenishment Problem with Approximated Transportation Costs (DJRP-AT) which accounts for customer locations while solving the DJRP. Instead of optimizing the delivery routes, which is not a task in the DJRP, the transportation costs are approximated. The DJRP-AT is solved with a branch-and-
price-and-cut solution method to provide insight in the problem structure and solution structure. Novel dominance conditions are introduced in order to discard labels in the exact labeling algorithm that is used to solve the pricing problem.

1.3 Vehicle Routing Problems with Outsourcing

If there are both a private fleet and a common fleet available to deliver goods to customers in the VRP, the literature refers to this feature as outsourcing or as problems with private fleet and common carrier. In the VRPPC there are multiple geographically dispersed customers that each have a certain demand and there is usually a limited number of privately owned vehicles available to deliver goods to a customer. Delivery can also be outsourced to a common carrier. Outsourcing services to a common carrier is suitable if the total demand exceeds the total vehicle capacity or if it is unprofitable to serve certain customers with the private fleet. A cost is incurred for outsourcing a customer service to the common carrier. This outsourcing cost can, for example, be a fixed fee per service, potentially customer dependent, a fixed fee per unit of demand, or a cost dependent on the total outsourced quantity with a discount structure. Routing costs and, often, a fixed setup cost per vehicle are incurred for the private fleet. The objective in the VRPPC is to minimize routing, setup and outsourcing costs.

1.3.1 Literature and solution methods

The VRPPC is first introduced by Chu [2005] in a practical case. The authors propose a heuristic based on a modified savings algorithm to solve the problem. More heuristics have been proposed for the problem subsequently. Bolduc et al. [2008] propose a perturbation-based metaheuristic and Côté and Potvin [2009] define a tabu search metaheuristic. Potvin and Naud [2011] use an ejection-chain neighborhood within a tabu search heuristic. Huijink [2016] develop large neighborhood search heuristics and introduce new local search moves. Several different outsourcing and private fleet cost structures are considered by Gahm et al. [2017] for the VRPPC with heterogeneous fleet and the authors present a variable neighborhood search to solve the VRPPCs. The VRPPC is also a variant of the VRP with profits for which Vidal et al. [2016] study a unified hybrid genetic search framework and local search metaheuristics. For a survey on the VRP with profits, see Archetti et al. [2014b]. Goeke et al. [2018] propose a large neighborhood search heuristic with a post-processing step which solves an ILP that selects the best routes from all routes encountered during the search. The multi-depot version of the VRPPC is introduced by Stenger et al. [2013] and the authors develop a variable neighborhood search algorithm. Only two studies that solve the VRPPC with an exact solution method have appeared very recently. Dabia et al. [2019] proposes a branch-and-price-and-cut approach for the VRPPC with heterogeneous fleet and a discount outsourcing cost structure. Independently, Goeke et al. [2018] proposes a similar method for the VRPPC with customer-dependent, fixed fees as outsourcing costs and a heterogeneous fleet.
1.3.2 Contributions

In the VRPPC it is assumed that service to a customer can either be performed by a private vehicle or that it can be outsourced to a common carrier. Chapter 5 proposes to relax this assumption and allows that the service to a customer can be split over one private vehicle and the common carrier. This problem is denoted by Vehicle Routing Problem with Partial Outsourcing (VRPPO). Combining outsourcing and split delivery aspects in VRPs has received increasing attention in the literature. There are several studies considering split delivery and outsourcing distribution strategies in practical cases for which heuristic solution methods are developed (e.g., Bolduc et al. [2010], Lee and Kim [2015]). Chapter 5 contributes to the literature by formally defining the VRPPO. Thereafter, branch-and-price-and-cut solution approaches are developed for two different problem formulations, including two different exact pricing mechanisms for each formulation. The aim is to explore the potential cost improvement of the VRPPO over the VRPPC and to examine how solutions change compared to the VRPPC in which a split of the service to a customer is not allowed.

1.4 Solution Methods

For the optimization of distribution decisions many approaches have been used in the literature. A distribution problem can be formulated as a MILP which provides insight in the structure of the problem and offers the starting point for an exact solution method. Although exact solution methods can usually only solve instances of limited size, the obtained optimal solutions do reveal the solution structure and moreover, the exact solution method can serve as a base for a matheuristic capable of solving larger instances. In this section it is not the intention to give a complete overview of all possible exact solution methods, but to highlight one method that is at the core of this dissertation. Therefore, Section 1.4.1 introduces branch-and-price-and-cut which is explained with the help of the VRP.

1.4.1 Branch-and-Price-and-Cut

Consider a VRP in which a set of vehicles is available to serve a set of customers that each have to receive a given number of units of goods in the one-day planning horizon. One type of MILP formulations for the VRP is based on the connections between the customers (arc flow formulations). This formulation explicitly considers whether a vehicle route visits customer \( j \) after customer \( i \) and hence, a decision variable representing this option is present in the model, for each pair of customers. Another type of MILP formulations for the VRP is not based on the arcs between customers, but on the set of vehicle routes (route-based formulation). This means that complete routes, which start and end at the depot and that satisfy all route constraints such as vehicle capacity, should already be available. Then, instead of modeling the construction of the routes in the linear program, one has to model the selection of the routes such that all customers are visited exactly once while respecting the fleet size. These two examples of VRP formulations show that there is often not just one possible MILP representation of a problem. Laporte [2009] provides several linear programming formulations for the VRP, Archetti et al. [2014a] compare several formulations for the IRP and Narayanan
and Robinson [2006] analyze several JRP formulations.

It is possible to deduce a route-based VRP formulation from an arc flow formulation by applying the so-called Dantzig-Wolfe decomposition which was introduced by Dantzig and Wolfe [1960]. For a thorough technical description of the decomposition method see e.g., Lübbecke and Desrosiers [2005], and for some examples see Barnhart et al. [1998].

The number of routes in a route-based VRP formulation can become quite substantial since the model has to consider all possible routes (i.e., all customer sequences) that respect the route constraints (e.g., vehicle capacity). To avoid enumeration of all possible routes upfront, a method called column generation can be applied (Desaulniers et al. [2005]). First, one formulates the problem with a limited number of routes and solves this MILP. The solution of the program is not necessarily the overall optimal solution since not all possible routes were included in the program. Then, given the current solution of the program, additional routes (columns) are generated by solving a so-called pricing problem. The pricing problem identifies routes that are likely to steer the program in the direction of the overall optimal solution. This process is continued until no more routes are identified that will give a better solution. The additional routes can for example be generated with a labeling algorithm which iteratively extends a path from the depot to all possible subsequent customers (see for example Feillet et al. [2004], Righini and Salani [2006] and Tilk et al. [2017]). Heuristics can be used to generate multiple routes quickly before solving a labeling algorithm.

When solving MILPs, the integer variables make the problem much harder to solve since these variables must be integer. Therefore, when applying column generation, the integer variables are usually relaxed to be continuous variables and integrality is enforced later. Hence, a VRP solution that is obtained with the column generation procedure is not necessarily integral, for example, the solution is to perform a route half. This is not feasible in practice, therefore, the column generation method is incorporated in a branch-and-bound framework to enforce integrality. By applying branching, a variable with a fractional value is selected and two branches are created. In one branch a constraint is added which limits the value of the variable from above by its rounded down value, and in the other branch a constraint requires that the variable has at least a value higher than its rounded up value. Column generation is applied in each branch before repeating the branching step. The combination of column generation and branch-and-bound is denoted by branch-and-price.

Finally, in a MILP formulation for the VRP only necessary constraints are present initially. Leaving out one of these constraints, may result in an infeasible solution to the overall problem. Next to these necessary constraints, it is possible to identify constraints that are not required to find a feasible optimal solution, but that can be useful during the solution method since they eliminate fractional solutions. This type of constraints are called cutting planes or valid inequalities. Applying valid inequalities in the above described branch-and-price framework results in a solution approach called branch-and-price-and-cut (see e.g., Nemhauser and Park [1991] and Lübbecke and Desrosiers [2005]). When applying branch-and-price-and-cut, some technical considerations have been kept in mind. For example, branching can change the structure of the pricing problem and adding valid inequalities can imply that the pricing problem has to be expressed differently, see for instance Jepsen et al. [2008] and Dabia et al. [2019]. Branch-and-price-and-cut is applied in Chapters 3, 4 and 5.
1.5 Practical Motivation

The research in this dissertation is motivated and inspired by cash management in the Netherlands and research questions have been established in consultation with the business partner Geldmaat (Geldservice Nederland until January 1, 2019). Although cash is still essential for the economy and access to cash for all inhabitants is required by the government, the use of cash is diminishing in recent years. In the Netherlands, three commercial banks are responsible for the distribution of bank notes to bank offices and Automated Teller Machines (ATMs). According to Van Anholt [2014], the banks were not fully aware of the impact of the costs of this task and they did not focus on efficient replenishment. In 2008 this view changed, influenced by the shift in use of payment methods (less cash money) and the financial crisis. In order to organize the supply of cash more efficiently Geldmaat was founded to support collaboration between the banks while securing the accessibility of cash throughout the Netherlands. Van Anholt [2014] thoroughly describes the cash supply chain and involved parties. Only the important aspects for this dissertation are summarized here.

In the current situation in the Netherlands, Geldmaat determines each day which ATMs to replenish during the next day based on a predictive model. A third-party transportation company, specifically referred to as Cash-in-Transit company (CIT) in this field, performs the actual ATM replenishments. The orders based on the prediction model are packed in the Geldmaat cash center, subsequently these packages are shipped to a CIT cash center. In the CIT cash center, the packages are distributed to armored vehicles that will perform the actual routes serving the ATMs and potentially other customers such as retailers. The CIT has full control over the performed routes and carries out its own optimization.

High costs are involved in the replenishment of the ATMs for Geldmaat. Therefore, in multiple ways, the company is in search for more efficient or more collaborative ways to replenish the ATMs. The quest for more efficiency is intensified by the fact that less and less cash is used and hence, the cost per banknote is increasing which is not desirable. In this dissertation several possible directions for more efficiency are explored with the purpose to support Geldmaat to improve their business. Chapter 3 facilitates the discussion between Geldmaat and the CITs by providing insight in the consequences of certain collaborative cost structures. To this end, in Chapter 3 transportation costs are taken into account when deciding on the replenishments of the ATMs. Chapter 4 supports the search for a more efficient distribution strategy by exploring the option to not replenish fully all ATMs but allowing for redirecting users between ATMs. Both research questions were established in close collaboration with Geldmaat and the latter one was a future research topic in the dissertation by Van Anholt [2014].

The dissertation by Van Anholt [2014] contains a thorough literature review on cash supply chains up to 2014. Studies discussed in Van Anholt [2014] which are directly relevant for the work in this dissertation and newer studies that contribute to the stream of research on cash supply chains are highlighted here. Van Anholt et al. [2016] consider a combined inventory management and routing problem for so-called Recirculation ATMs (RATM). At an RATM, an ATM-user can both withdraw and deposit money, hence, the IRP-like solutions contain both delivery and pick-up activities. Money that is picked up at one ATM can be used for a replenishment of another ATM. Bati and
Gözüpek [2017] study an IRP for a network containing both traditional ATMs and RATMs combined with the optimization of which ATMs to change to RATMs, given that withdrawal and deposit amounts are known. Larrain et al. [2017] consider an IRP that allows for stock-outs and the replenishment policy consists of swapping new cassettes of a chosen amount for the current cassettes that can still contain bank notes which are returned to the depot. Geismar et al. [2017] provide an overview on currency supply chains by reviewing studies that look into the cash supply chain from the supply side (national banks), the demand side (commercial banks and ATM networks), and the private sector logistics providers’ side. In their analysis on ATM replenishment-related literature, Geismar et al. [2017] mention the study by Van Anholt et al. [2016] on RATMs and suggest for future research to investigate possible incentives to rebalance RATM inventories by steering users to a certain RATM (either withdraw from a full RATM or deposit at an empty RATM). They suggest a premium as incentive for making a deposit at a certain RATM and these premiums can be reviewed online by the user. Chapter 4 investigates the possible gain in supply chain costs when implementing a similar idea for regular ATMs. Other recent studies consider different issues in the cash supply chains such as vault location and size optimization by the central bank [Huang et al., 2017], combined optimization of inventory management and the denomination mix issued at a cash withdrawal [Van der Heide et al., 2017], and combined demand forecasting and replenishment policy optimization [Lázaro et al., 2018].

1.6 Dissertation Outline and Research Output

Chapter 2 studies a variant of the IRP in which routing is easy with the aim to study the computational complexity of the problem and to identify other sources of complexity than routing. Chapter 3 studies the DJRP-AT and proposes a branch-and-price-and-cut solution method. Incorporating demand moves in the IRP is explored in Chapter 4 and a branch-and-price-and-cut solution approach is developed to solve the IRPDM. Chapter 5 formally introduces the VRPPO in which it is not only possible to outsource some services to a common carrier, but also to split the service to one customer between a single private vehicle and the common carrier. To solve the problem, a branch-and-price-and-cut solution method is designed for two path-based formulations. In each chapter the notation is consistent with that in closely related literature of that chapter, resulting in different notation throughout the chapters in this dissertation. Finally, Chapter 6 concludes with an overview of the main findings and suggestions for future research.

An overview of the research output of this dissertation is presented in Table 1.1. For each of the chapters the table contains the title, the research questions and the status of the publication status (i.e., published, revision, submitted or in preparation for journal submission).
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Research Questions</th>
<th>Journal Publication Status</th>
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<tbody>
<tr>
<td>2</td>
<td>On the complexity of Inventory Routing Problems when routing is easy</td>
<td>What factors influence the computational complexity of the IRP? Can a borderline between easy and hard problems be defined?</td>
<td>Accepted for publication in <em>Networks</em> [Baller et al., 2019d]</td>
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<tr>
<td>3</td>
<td>The Dynamic-Demand Joint Replenishment Problem with Approximated Transportation Costs</td>
<td>What is the potential improvement by considering transportation costs in a DJRP setting? How close are the resulting costs to an IRP solution with optimal routes?</td>
<td>Published in <em>European Journal of Operational Research</em> [Baller et al., 2019b]</td>
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<tr>
<td>4</td>
<td>The Inventory Routing Problem with Demand Moves</td>
<td>What is the potential cost improvement of allowing for demand moves in the IRP? How do several factors influence the cost improvement?</td>
<td>In preparation for submission [Baller et al., 2019a]</td>
</tr>
<tr>
<td>5</td>
<td>The Vehicle Routing Problem with Partial Outsourcing</td>
<td>How to model a VRP in which a customer can either be served by a single private vehicle, by a common carrier, or by both a single private vehicle and a common carrier? What is the potential cost improvement compared with only allowing fully outsourcing customers and not allowing for splits?</td>
<td>Accepted for publication in <em>Transportation Science</em> [Baller et al., 2019c]</td>
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