This dissertation explored several distribution problems inspired by industry practice. Four separate studies were presented in Chapters 2 to 5. Although these studies relate to different practical problems, they each contribute to the understanding of distribution problems and help to increase efficiency in such problems. The studies focus on gaining insight in fundamental distribution problems, developing efficient distribution strategies and analyzing the benefit of novel distribution strategies. This concluding chapter summarizes the main findings, discusses implications and reflects on limitations and further research.

6.1 Summary of main findings

The following sections summarize the main findings of chapters 2 to 5 by focusing on the key issues addressed and by highlighting the main results obtained.

6.1.1 Understanding the Computational Complexity of the Inventory Routing Problem

To understand the difficulties that arise in solving distribution problems, it is important to comprehend the underlying computational complexity of a problem. This allows to reveal the structure of a problem which contributes to developing solution methods that exploit this structure. Therefore, Chapter 2 investigates the sources of computational complexity of the Inventory Routing Problem (IRP) by looking for complexity proofs for different variants of the problem. Since the Traveling Salesman Problem (TSP) is an NP-hard problem and it is a special case of the IRP, it can be concluded immediately that the IRP is NP-hard [Karp, 1972]. However, the underlying routing problem is not necessarily the only complicating aspect of the IRP. Therefore, Chapter 2 studies the IRP on metrics on which the TSP is easy or even trivial and, hence, NP-hardness through the TSP is avoided. The IRP on a point (the depot and all customers at
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The analysis shows that, next to routing, also the time horizon, the service times, the customer demand combined with vehicle capacity, and the number of vehicles contribute to the complexity of the IRP. The main finding is a polynomial time algorithm for the studied IRP on the half-line with uniform service times and a planning horizon of two days. Since this variant is a borderline problem, this result implies that the studied IRP becomes hard or has an open complexity if one of the features is generalized. Moreover, analysis shows that almost any IRP variant with arbitrary service times (i.e., the service times are different per customer) is NP-hard. The same result holds if vehicle capacity is considered and the customers have arbitrary demand. If the planning horizon is extended to an arbitrary number of days while all other aspects of the problem are simple (i.e., not implying NP-hardness of the problem), it turns out that the Pinwheel Scheduling Problem [Holte et al., 1989] determines the complexity of the IRP. Unfortunately, the complexity of this problem has not been established so far, but it is unlikely that this problem is easy to solve [Jacobs and Longo, 2014].

Finally, Chapter 2 considers a variant of the IRP in the Euclidean plane for which the objective is to minimize the total tour length. The tour length is not given by optimized TSP tours, but rather by approximations. The approximation of the tour length takes the number of locations and an area in which these locations are spread as input [Beardwood et al., 1959, Chien, 1992]. In the literature several types of areas have been considered. In Chapter 2 the convex hull of the involved locations is used as area. This approximation of the TSP allows for showing NP-hardness of the considered IRP in the Euclidean plane. Therefore, this chapter shows NP-hardness of a generalization of the Pinwheel Scheduling Problem in which tasks are executed at different locations without using the hardness of TSP.

6.1.2 Considering delivery aspects when taking ordering decisions

Chapter 3 considers the case in which a supplier outsources its customers replenishment deliveries to an external carrier. The business partner of this Ph.D. project is such a supplier. The business partner Geldmaat decides upon the replenishment of ATMs in the Netherlands and issues replenishment orders to Cash-in-Transit companies. In the terminology of the replenishment literature, Geldmaat acts as a supplier that outsources delivery of goods. Different cost structures can be applicable for outsourcing the distribution of goods. Chapter 3 considers a cost structure in which a fixed fee is incurred per customer replenishment and per day on which at least one replenishment takes place. Given this cost structure, the optimization problem faced by the supplier is a Dynamic-Demand Joint Replenishment Problem (DJRP) [Khouja and Goyal, 2008] in which it has to be determined in which period to replenish each customer. Chapter 3 argues that solving a DJRP does not generate efficient distribution plans from a transportation or supply chain point of view. In practice, however, suppliers do face a DJRP and likely solve this problem to optimize their business, because, the supplier cannot take customer locations into account when deciding on the replenishments given the
fixed fee-cost structure. As a result, the carrier is forced to perform inefficient delivery routes which leads to higher transportation costs which will result in higher fixed transportation fees in future contracts. Moreover, if the carrier has a limited fleet, it can occur that not all customers can be served on the same day due to longer travel times between distant locations.

To address this shortcoming of the DJRP, Chapter 3 studies an extension of a DJRP by including transportation costs. This extension allows to assess the efficiency improvement if customer locations would be taken into account by the supplier when deciding on which customers the carrier has to service. To this end, the Dynamic-Demand Joint Replenishment Problem with Approximated Transportation Costs (DJRP-AT) is defined and a solution method based on branch-and-price-and-cut is developed. The transportation costs are computed as an approximation of the optimal tour length with the number of locations and an area containing the locations as inputs [Beardwood et al., 1959]. A similar tour length approximation was considered in Chapter 2. At first, in Chapter 3, the convex hull covering all locations was used as area similar to Chapter 2, but eventually Chapter 3 adopted the more straightforward computation of the smallest rectangle covering all locations to establish the area. For solving the DJRP-AT, using approximations for the transportation costs implies that sets of customers that are visited by one vehicle on one day have to be generated. Generating customer sets is expected to be easier than generating actual vehicle routes since the sequence of the customers is not important. However, although problem specific dominance rules were developed to discard labels, the DJRP-AT showed to be hard to solve. This will be discussed in more detail in Section 6.3. Still, the solutions found with the designed solution method could support the analysis and could prove the point that was being made.

The distribution plans and costs found with the DJRP and the DJRP-AT are compared. The results show that significant cost savings can be achieved by deviating from the traditional DJRP cost structure. It is shown that a collaborative ordering strategy can be beneficial to both the supplier and the carrier and this insight provides support for future negotiations between the involved parties. Also, since approximated transportation costs are used, the results can be compared to a variant of the IRP. This IRP contains different constraints than the IRP typically addressed in the literature and hence, there is no state-of-the-art solution method for this variant of the IRP. Therefore, an IRP equivalent to the DJRP-AT with optimized routing is implemented which can be solved for the smallest instances. Comparison with the DJRP-AT shows that the average deviation of the DJRP-AT from the IRP is small, and hence, that using route length approximations results in solutions close to those found by fully optimizing the delivery routes.

6.1.3 More Efficient Replenishment by Introducing Demand Moves in the Inventory Routing Problem

Chapter 4 addresses an approach for inventory replenishment in which customers can fulfill (part of) the demand of a nearby customer. As an example, consider ATMs that are often located in close proximity of each other in urban areas which provides the opportunity to redirect or proactively steer ATM-users (end-users) to a certain ATM to make a cash withdrawal. An ATM-user can, for instance, be redirected when arriving
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at an ATM or be informed upfront via a mobile application, possibly incurring a reward or penalty for using a certain ATM. This redirection option can be incorporated in the optimization of ATM replenishment. Therefore, Chapter 4 extends the IRP with demand moves which precisely represent the redirections of end-users between customers which leads to the definition of the Inventory Routing Problem with Demand Moves (IRPDM). The aim of the chapter is to introduce the novel concept of demand moves, to model the demand moves in the IRP and to assess the impact on the solutions and costs compared to the traditional IRP. For each demand move a service fee/cost is incurred which depends on the distance between the involved customers and quantity moved.

To model the IRPDM, a problem formulation for the IRP from the literature [Desaulniers et al., 2016] is extended and a branch-and-price-and-cut solution approach is developed. The technical analysis of the IRPDM showed that using the initial inventory at the customers to satisfy the moved demand added complexity to the use of valid inequalities in the solution method. Therefore, the solution method in Chapter 4 is restricted to the case in which the initial inventory at a customer can only be used to satisfy the demand of the customer itself. Multiple families of valid inequalities have been developed for the IRP in existing literature. However, only one of these families is directly applicable to the IRPDM, for the others non-trivial adjustments are required. The adjustments imply that the valid inequalities are not as strong as the original ones, this is for example caused by the fact that there are more possibilities to satisfy a customer’s demand. An IRPDM solution can result in the situation that a certain customer is never replenished by the vehicle since all demand (after consuming the initial inventory) is moved to another customer. This might not be desirable in practice, hence, in the model the option is included to limit the percentage of demand of a customer that can be moved per day. The results show that substantial cost improvements can be achieved if there is no maximum imposed on the demand that can be moved per customer per day. The results also indicate that only a limited number of demand moves per day is applied in the solutions. Hence, to achieve these substantial cost improvements not many customers have to be involved, but are rather only a few customers per day are affected. Furthermore, sensitivity analysis is performed on both the demand move fee and the maximum percentage of demand that can be moved. Limiting the demand that can be moved to 75% approximately halves the potential cost improvement, which seems a large reduction. However, even by allowing only 25% of the demand of a customer to be moved per day still results in cost improvements that are worthwhile in practice.

6.1.4 The Vehicle Routing Problem with Partial Outsourcing

Chapter 5 formally defines the Vehicle Routing Problem with Partial Outsourcing (VRPPO) which is an extension of the Vehicle Routing Problem with Time Windows. In the VRPPO, a customer can either be served by a single private vehicle, by a common carrier or the service can be split between a private vehicle and the common carrier. For outsourcing, a fixed fee per unit of goods is paid which is independent of the customer. Both VRPs with split deliveries (SDVRP) (see for example Archetti and Speranza [2012]) and VRPs with outsourcing (VRPPC) (e.g., Chu [2005] and Dabia et al. [2019]) have been defined and studied before in the literature, but a formal
definition of a VRP with both these distribution strategies was not present in the literature.

To address the VRPPO, Chapter 5 proposes two formulations and designs branch-and-price-and-cut solution methods, with for each formulation two different exact pricing mechanisms. The aim of the chapter is to analyze the two problem formulations and the corresponding solution methods. Besides that, the solutions and the associated costs are compared to those of the VRPPC in which a split between the delivery options is not possible. Comparison of the solution methods, clearly shows that, per problem formulation, one exact pricing mechanism is more efficient than the other pricing mechanism. Moreover, it shows that the difference between the problem formulations is small for the best pricing mechanisms. Testing on two sets of instances to analyze the cost difference between the VRPPO and the VRPPC shows higher cost improvements of the VRPPO over the VRPPC if customer demand is close to or higher than the vehicle capacity. Also, higher outsourcing costs results in higher cost improvements of the VRPPO over the VRPPC than lower outsourcing costs. A possible explanation is that if outsourcing costs are low, then it is more beneficial to outsource all units of one customer and hence, with low outsourcing costs, the benefit of allowing for splits declines. Finally, if customers are located in clusters, cost improvements are lower than if customers are randomly spread over an area. Visualization of some solutions shows that a VRPPO solution can contain completely different routes than the corresponding VRPPC solution.

6.2 Implications

Solution methods for the IRP are often focused on hardness of the underlying routing problem. Chapter 2 shows that other aspects should receive sufficient attention as well since routing is not the only factor determining the computational complexity of the IRP. For example, selecting sets of customers to be served together each by one vehicle on one day combined with vehicle constraints is difficult. This is because bin packing aspects are present in the problem and bin packing is a hard problem. Another example is the question on which days to serve a customer, since this causes difficulties through the relation with the pinwheel scheduling problem. Chapter 2 suggests that it is worthwhile to give sufficient attention to other aspects than routing when developing solution methods for the IRP, for both exact and heuristic methods.

Some solution methods in both the literature and this thesis already incorporate these observations. For example, Desaulniers [2010] studies the SDVRP and during the construction of the potential vehicle routes, the delivery quantity for the split customer is not decided upon until finishing the route construction. Hence, the vehicle capacity and delivery quantities are directly considered in the solution method. For the IRP, Desaulniers et al. [2016] use delivery patterns representing all possible combinations of delivery quantities addressing the vehicle capacity issue. Moreover, several delivery patterns can be discarded in a clever way by applying dominance criteria which reduces the number of possible delivery quantities and hence improves the performance of the solution method. The same idea is applied for the IRPDM in Chapter 4 of this thesis. Also in Chapter 5, the delivery quantities and vehicle capacities are addressed explicitly for the VRPPO. In the firstly proposed pricing algorithms for both problem formulations, the potential delivery quantity by the private vehicle for a split customer
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is stored during the execution of the algorithm and is therefore directly considered. In the second proposed pricing algorithms, the privately delivered quantity for the split customer is only explicitly considered at the end of a route creation. Although it cannot be decided upfront which of the pricing algorithms is more efficient (among others because of a trade-off between number of labels and the length of the created routes), the computational results clearly show that the second type of pricing algorithm outperforms the first.

In recent literature, only limited attention was paid to dynamic-demand joint replenishment problems. This is shown by the systematic literature review by Bastos et al. [2017] in which the authors report only two published studies on the DJRP in the years 2006-2015, while they found more studies for the static (36) and stochastic (19) demand variants of the JRP. Still, the DJRP is a very relevant problem in practice since the cost structure is present in many business applications that face varying demand, for example in ATM replenishment and for retailers that outsource their storage and replenishment activities. Chapter 3 contributes to the insight that more traditional problems, such as the DJRP, can provide a good starting point for decision support in practice and remain important in academic research. Therefore, it can be beneficial to both research and practice to determine the underlying fundamental problems faced by the industry and to study these problems in more detail.

Chapter 3 shows that splitting the optimization of ordering and of delivery decisions is insufficient to find good solutions for practical supply chain problems. However, complete integrated optimization is not always necessary, nor possible, in practice. Chapter 3 shows that the supply chain can benefit if delivery aspects are considered when making ordering decisions. This has multiple implications. First, it is important, both for research and practice, to analyze which decisions can be taken by each party under which cost structure. This provides insight in which formal problem, such as the DJRP, relates to the practical setting. Secondly, to improve the supply chain, one can consider integrating aspects of the business partner’s objectives when optimizing one party’s decisions. In the DJRP-AT in Chapter 3, the locations of the customers are explicitly considered when optimizing the ordering decisions at the supplier. This allows logistics service providers to construct better delivery routes which lowers costs and increases resource utilization. It is crucial that in contracts between the different parties, the inclusion of the transportation aspects in the ordering process is reflected in the cost structure. If a contract does not contain mutual incentives to cooperate, the parties in a supply chain will most likely not collaborate since there is no benefit for them. Future research can support the process of collaboration by providing insight in the consequences of alternative cost structures.

Chapter 4 shows that incorporating demand moves in the IRP is useful for reducing replenishment costs. It should be noted that implementing demand moves in practice, for example for ATM replenishment, is rather involved. It has to be determined under which circumstances demand moves are appropriate, which includes establishing the service cost for the end-user and appropriate ‘neighbor’ sets. This latter aspect implicates the question what distance between locations is permissible to redirect an end-user. Hence, the results on the IRPDM in Chapter 4 pose interesting follow-up
questions both for research and practice. These will be discussed in more detail in Section 6.3.

Novel distribution strategies are considered in this dissertation, such as demand moves in the IRP and allowing for splits between a private vehicle and outsourcing in the VRPPO. New distribution strategies are considered to enhance efficiency in order to save costs. Sensitivity analysis, such as in Chapters 4 and 5, shows that it is important to analyze under which circumstances these distribution strategies actually provide improvements in practice. For example, if the loss in service due to demand moves (Chapter 4) is very high, then it may not be beneficial to consider demand moves, while even at a maximum of 25% demand moved per customer per day cost reductions can be achieved that are substantial in practice. And Chapter 5 showed that for certain data sets using splits between private and common vehicles does not result in a cost improvement over not using splits. Hence, a careful analysis is necessary before implementing innovative distribution strategies in practice to see whether a novel strategy is actually profitable.

6.3 Limitations and Further Research

Chapter 3 considers an extension of the DJRP which includes transportation costs. The transportation costs are computed by approximating the length of the tour that visits a given sets of customers. By using an approximation of the tour length which uses the combination of customers, the sequence of the customers is not important. It was expected that generating customer sets would result in relatively easy to solve pricing problems. This expectation is based on the fact that a limited number of labels is needed for finding customer sets compared to solving an actual routing problem which requires determining the actual customer sequence. However, the dominance criteria that can be used in generating vehicle routes are not applicable to the DJRP-AT pricing problems. Problem specific dominance rules are developed for the designed pricing problems, but these dominance criteria are not strong which makes the solution method less efficient since many labels are kept during the labeling algorithm. Also, the integrality gaps for the DJRP-AT master problem are large despite using two families of valid inequalities and hence, many iterations between solving the master and pricing problems are required to close the gaps. The results in Chapter 3 show that calculating approximated transportation costs gives solutions close to the ones achieved by calculating actual routing costs. Therefore, the above observations can inspire future research to design more efficient solution methods for the DJRP-AT by potentially developing different solution methods for the pricing problems and also by designing new valid inequalities to reduce the integrality gaps.

Chapter 4 is the first research on including the novel concept of demand moves in the IRP. Being the first study comes with multiple limitations and therefore various directions for further research can be identified.

First, because of algorithmic issues, the initial inventory at a customer at the beginning of the planning horizon is not used to fulfill moved demand in the solutions found in Chapter 4. Including the option to satisfy moved demand from initial inventory can further reduce the costs of the IRPDM compared to the IRP. Hence, further research
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on the IRPDM includes the design of a solution method that does include the option to fulfill moved demand from initial inventory. Secondly, Chapter 4 analyses the impact of imposing a maximum on the demand that can be moved per customer per day. This represents the practical aspect that it is probably not desirable that all demand of a customer is moved to another customer every day. Currently, this chapter only considers to have the same maximum for each customer on all days. In practice, it can be useful to consider a more flexible system. For example, on a limited number of days it is allowed to move all demand but in the following days the demand that can be moved is limited. Thirdly, in the model it is assumed that a demand move can only take place if there is no inventory left. Hence, only out-of-stock situations are exploited while keeping customer service sufficiently high. If one wants to influence end-user behavior, e.g., by suggesting alternative ATMs for cash withdrawals or by offering financial benefits for using a different ATM than the preferred option, then the model can be extended to include the case in which customers are not necessarily out of stock when demand is moved. Finally, the modeled IRPDM is designed to take decisions on the total volume delivered to a customer in each period, and how much demand is satisfied by each customer. This implies that decisions are not taken on end-user level and hence, an IRPDM solution does not indicate which end-users to redirect to another customer. The IRPDM can be extended to include decisions on end-user level, which can, e.g., be used in a mobile application to inform the end-users.

Furthermore, several implementation issues arise when demand moves are going to be used in practice. These issues also came up during discussions with the business partner in the case of ATM replenishment. Currently, users of Dutch ATMs are accustomed to having the possibility of withdrawing cash free of charge from each ATM. Therefore, further research can investigate appropriate stimuli (i.e., rewards/penalties for cash withdrawals at certain ATMs) to influence end-user behavior while considering that the change should be limited with respect to the current situation faced by the end-user. Moreover, safety is always an important aspect in cash supply chains, therefore, when implementing a reward system it should be considered how to prevent abuse.

This dissertation contains several chapters that study problems that are inspired by practice. When investigating problems inspired by real-life supply chains, it is important to analyze which party takes what decisions and to keep in mind from which perspective the studied problem is to be modeled. Moreover, it is interesting to study what the consequences are of one party’s decisions on the choices of other parties in the chain. Chapter 3 on the DJRP-AT explicitly considers the situation in which one party in the supply chain cannot directly influence all decisions in the chain. If the considered supplier would take customer locations into account when taking the ordering decisions, a cost reduction can be achieved by the carrier which could potentially increase the efficiency of the whole supply chain. Although it is clear the supply chain can benefit, it remains to be studied for the DJRP-AT how the supplier can benefit from considering transportation costs when making ordering decisions, i.e., how can a party in a supply chain benefit from considering multiple parts of the chain in their decision making. By contrast, Chapters 4 and 5 assume that one party can take all decisions that are in scope. Concluding, in academic research that is inspired by practice, careful consideration is needed on which parties take what decisions and what processes each party affects. An informed choice should be made which decisions to
include in a model. In practice it is important that a party realizes which decisions it can take and how these decisions influence their business partner. And also the other way around: how do the decisions taken by a partner influence your business.

As stated above the research in this dissertation is inspired by practice, at the same time a deliberate choice is made to apply exact solution methods. In some cases it is possible to solve real-life problems via an exact method, however, more often exact methods are incapable of solving real-life sized problems to optimality. Exact approaches do, however, provide insights that heuristic approaches cannot.

First, several exact approaches require a mathematical formulation for the considered problem which provides insight in the structure of the problem which aids the development of solution methods. Chapter 5 also underlines that it is important for some exact solution methods to analyze several problem formulations and corresponding solution methods, since efficiency can differ significantly. Secondly, optimal solutions can reveal structural solution aspects. For example, the number of locations involved in demand moves in the IRPDM in Chapter 4 and the circumstances under which splitting deliveries between a private and common vehicles in the VRPPO in Chapter 5 is beneficial. Thirdly, solutions that are guaranteed to be optimal can be used to assess the quality of a heuristic method, which cannot be fully evaluated otherwise. Additionally, in some cases it is sufficient to develop an exact solution method to prove a point or to show a potential improvement. Finally, an exact approach can provide a starting point for developing a matheuristic which can make use of very fast exact MILP solvers and combines the strengths of exact and heuristic solution methods.

Although exact solution approaches are a good starting point to address problems, it is evident that the new problems introduced in this thesis could benefit from the development of heuristic solution approaches, in particular for solving larger problem instances. Several insights can be obtained from this thesis that are useful in the development of heuristic solution methods. Chapter 2 raises awareness that other aspects than routing cause complexity in the IRP, which can be useful knowledge when designing heuristic solution methods. For example, sufficient emphasis should be put on when a customer is replenished during the planning horizon. Chapter 3 uses an approximation to represent the transportation costs involved in visiting a set of locations instead of finding the optimal route. The comparison of the DJRP-AT with the equivalent IRP shows that the deviation of a heuristic method, which cannot be fully evaluated otherwise. Hence, although an approximation of the route length is used, the model is capable of balancing the different costs. The insight that an approximation of certain costs can be sufficiently good can be used in the development of heuristic solution methods, for example if many long routes have to be optimized. Furthermore, the solutions in Chapter 4 for the IRPDM show that the optimal solutions do not necessarily contain many demand moves, and also, on average, approximately half of the demand of a customer in one period is moved if a demand move takes place. These observations which are based on optimal solutions can be exploited when designing heuristic solution methods by not having too many demand moves in a heuristically obtained solution and if there is a demand move, not all demand of a period should necessarily be moved. An approach for a construction heuristic for the VRPPO in Chapter 5 could use a VRPPC solution as starting point and subsequently add customers to the routes in that solution, possibly with a split delivery between a private and common vehicles. However, the
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solutions obtained in Chapter 5 show that some routes in an optimal VRPPO solution are rather different than in the corresponding VRPPC solution. Therefore, this insight can be used to design heuristics that base the routes less on a VRPPC solution.

Concluding, this dissertation studies the computational complexity of a class of distribution problems, models both fundamental and more practical distribution problems, and develops exact solution methods for such problems. The problems are inspired by real-life optimization problems from a cash supply chain, but are more widely applicable. The studies provide insight in problem structures and solution aspects and contribute to the development of alternative solution methods.