In this thesis, algorithms are proposed to solve three different routing problems in which service time windows are taken into account. Service time windows arise in many practical routing problems, for example, parcel, service, and attended home deliveries. Since these markets continue to grow, the corresponding routing and scheduling problems will only become more complex. It is a challenge for logistics companies to keep up with the increasing customer service level and remain cost efficient. This thesis proposed several algorithms to tackle this problem.

First, we developed models to solve routing problems in which customers can indicate multiple time windows in which they are available for service. The service provider can minimize its transportation cost by determining the optimal routes and selecting the optimal time window per customer. Existing algorithms are all based on heuristic solution methods that use approximations of the route duration in the local search. We proposed an efficient exact algorithm to recalculate route durations in local search operations and we developed the first exact solution method for the vehicle routing problem with multiple time windows. Secondly, to improve the service to the customers, we proposed a robust method to determine narrow service time windows within larger time windows proposed by the customers. Existing solution methods assume that the uncertainty in travel and service times are completely known. However, it can be the case that the travel and service time distributions are hard to estimate. Therefore, we are the first to propose a robust approach to simultaneously determined the routes and assign time windows, assuming that only some characteristics of these distributions are known. In this problem, the time of arrival at a customer should be robust and predictable. There are also routing problems in which the arrival time at a customer should be as unpredictable as possible, for example in cash transport. To solve this last research question, we developed a model to determine a routing solution with unpredictable arrival times while keeping the transportation cost low. The proposed algorithm is more powerful, efficient and practical usable than existing methods.

In this last chapter, the main findings and conclusions of the thesis are summarized.
The practical insights are discussed and directions for future research are formulated.

6.1 Findings and contributions

Incorporating service time window constraints in routing problems and developing efficient methods to deal with the additional complexity is the main contribution of this thesis. Routing problems with deterministic and stochastic travel times are examined and exact and heuristic solution methods are proposed. The highlights of our findings will be addressed in this section.

In Chapter 2 and 3, an exact and heuristic solution method are developed to determine a routing solution that minimizes the total duration when every customer has multiple service time windows available. In Chapter 2, an exact polynomial time algorithm is presented to efficiently determine the minimal duration of a given route by finding the optimal start time for servicing each customer in the route. We showed that this algorithm has a reduced worst-case complexity and it is at least twice as fast in terms of average computational time compared to existing route duration minimization algorithms. Furthermore, the proposed algorithm can be used to efficiently recalculate the exact route duration when customers are removed or inserted into a route. To show the benefits of using this exact neighborhood evaluation to solve the VRPMTW, the proposed algorithm is embedded in an adaptive variable neighborhood search. Experimental tests show that the proposed exact local search evaluations resulted in a factor four acceleration compared to using existing exact algorithms in the local search. Furthermore, the proposed exact evaluations significantly improved the solution quality compared to using approximations of the route duration during the local search. The increase in computational time because of the exact evaluations was relatively small. Finally, the proposed algorithm identified new best-known solutions for 22 of the 48 benchmark instances. Therefore, we believe that the proposed algorithm can be useful for solving other routing and scheduling problems involving multiple time windows.

In the presence of multiple time windows per customer, different departure times from the depot can lead to a different selection of time windows and therefore other waiting times at the customer. Therefore, the complexity of the VRPMTW with the objective of minimizing total duration is much higher than in the single time window case. As a result, all existing solution methods rely on heuristics. In Chapter 3, the first exact algorithm for solving this problem is proposed. The algorithm is based on a branch-and-cut-and-price approach to determine the set of routes that minimizes total duration. The master problem is solved using column generation and a tailored bidirectional labeling algorithm solves the pricing problem. To deal with the multiple time windows in the pricing problem, we introduced the start time function that represents the start time of servicing a customer given the departure time from the depot. By exploiting the structure of the start time function, new strong dominance criteria are proposed to make the labeling algorithm tractable.
Computational experiments show that the proposed algorithm solves instances up to 100 customers to optimality within one hour computation time. All instances with 50 customers and a short planning horizon are solved to optimality.

In Chapter 4, a new efficient solution approach is presented to diversify the arrival time at a customer over successive visits. The arrival times are diversified by deleting the arrival intervals of previous visits from the solution space; in this way the problem is transformed in a VRPMTW that is solved in a rolling horizon of one day. As no periodic setting or computationally intensive penalty function are used, the proposed approach is easier, more efficient, and more powerful than existing methods.

Because this problem is inspired by the distribution of cash where the transport is most vulnerable when standing still, waiting times are not allowed. As the algorithms in Chapter 2 and 3 focus on minimizing waiting time in a route, these methods are not efficient for this case. Therefore, a novel algorithm is developed to efficiently determine whether a departure time from the depot exists such that no waiting time will occur during a given route. Furthermore, four different methods for penalizing violations of the waiting time and arrival time diversification constraints are developed and tested. These methods are embedded in an iterated granular tabu search that obtained new best-known solutions for all benchmark instances. The average improvement in total distance was 29% and the computational time decreased by 93% compared to the best-known results.

Until now we have assumed that the travel times are deterministic, but in reality travel times are uncertain. In Chapter 5, the Robust vehicle routing problem with time window assignment (RVRP-TWA) is formulated in which the travel time probability functions are uncertain and only some descriptive statistics such as mean, minimum, and maximum travel times are available. The objective of the RVRP-TWA is to simultaneously determine routes and time window assignments such that the risk of violating the time windows is minimized and the expected travel time is kept below a threshold value. An efficient algorithm to optimize the time window assignments for a given set of routes is proposed. We show that the subproblem of finding the optimal time window assignment for a given route is convex and that the subgradient can be derived. The RVRP-TWA is solved by iteratively generating subgradient cuts from the subproblem that are added in a branch-and-cut fashion. The branch-and-cut algorithm is able to solve instances up to 35 customers.

The solution quality and robustness of the RVRP-TWA model is tested by comparing it to a stochastic variant of the VRP-TWA in which the travel time distributions are known. To the best of our knowledge, we are the first to propose an exact solution method for the SVRP-TWA. Experiments show that the robust solution is close to the optimal solution and the robust approach outperforms the stochastic approach if travel time distributions are uncertain.
6.2 Practical insights

The research results of this thesis provide several important insights for logistics service providers operating under various service time window restrictions. The insights for the logistics service provider in general will be followed by findings for the cash industry in particular.

To keep up with the high service level standards when delivering goods or services the logistics service provider can take the following points into consideration.

- *Let the customer indicate multiple time windows for the service.* Most parcel and service deliveries work with a single time window per customer that is indicated by either the customer or the logistics service provider. In this case, the logistics service provider or the customer has no say in determining the time of service, which leads to high routing cost or low service levels. By giving the customer the opportunity to indicate multiple time windows in which it is available for service, the customer can indicate its preferences and the logistics service provider is flexible in scheduling the services. The proposed methods for the VRP with multiple time windows of Chapter 2 and 3 should allow a logistics service provider to efficiently handle the multiple time windows proposed by the customers.

- *Support small time windows to improve the customer service.* The single time windows used in routing problems are often very wide, such that the logistics service provider has sufficient flexibility and the probability of arriving in the time window is high. However, customers prefer small time windows to better plan their activities. Using the time window assignment methods discussed in Chapter 5, the logistics service provider can indicate a narrow time window (possibly within a wider time window initially chosen by the customer) and communicate these narrow time windows to the customer to improve the customer service. We showed that the naive approach of optimizing routing cost first and assigning time windows second, results in much higher time window violations than when the routing problem and time window assignments are determined simultaneously. Therefore, it is preferred to integrate the decisions following the methods in Chapter 5. However, if also other routing constraints have to be taken into account when constructing the routes, then the algorithm to find the optimal time window assignment for a given route proposed in Chapter 5 can be used. If the data about service and travel time is of good quality, then the distributions can be used to determine the time windows. If the service and travel time distributions are insufficiently known, then the algorithms based on the mean, minimum and maximum values give a more robust solution.

- *Explore the balance between service level and transportation cost.* We showed that there is a trade-off between the probability to be on time and the expected travel time. Allowing the travel time to increase with at most 5% compared to the
minimum travel time, decreases the time window violations by, on average, 33%. Increasing the length of the assigned time windows increases the probability to be on time and reduces the total travel time, but the customer has to “wait” longer. The logistics service provider has to find the best balance and determine which criterion is most important when determining the length of the assigned time windows.

Although the above suggestions also apply to cash transportation and that they can help to reduce costs for Geldmaat and the CITs involved. This thesis also contains findings that are particularly relevant to the cash transport industry.

- **Unpredictable routing can be achieved without a high increase of cost.** We showed that there is a trade-off between unpredictability of the arrival time and the transportation cost. When unpredictability increases, the transportation costs increase as well. However, this relation is not linear: the increase in transportation costs are minor when deviations of the arrival times are relatively small. However, forcing the arrival times to deviate significantly from previous arrivals can increase the total travel distance up to 40%. The cash-in-transit (CIT) company has to determine which level of unpredictability is acceptable. Maybe some areas are more susceptible for raids and require a higher level of uncertainty. Such considerations can be easily handled in the model proposed in Chapter 4 by requiring larger deviations in arrival times for certain customers. Moreover, our proposed model and solution approach have the additional benefits that it is easier to use and can better handle large instances than existing models.

- **Drive robust routes.** If a CIT arrives at a customer outside the agreed time window, the bank employee could be unavailable or safety locks could still be in place, forcing the CIT employee to wait. Since the CIT transport is most vulnerable when it is stationary, this waiting time should be minimized. This can be done by taking uncertainty in travel times into account in determining robust routing solutions. This will increase the security of the transport since waiting at the customers due to arriving earlier or later than scheduled is avoided. By using the proposed algorithms for robust vehicle routing with time window assignments, the optimal time window assignment can be found while keeping the total travel time under a predefined boundary.

### 6.3 Future research directions

This thesis proposed solution methods to optimize routes with service time window constraints. The work can be extended in different directions.

In Chapter 2 and 3, all the time windows indicated by the customer have the same preference or weight. It would be interesting to let the customers indicate their preferences for the time windows, leading to a higher weight for more preferred time
General conclusions and findings

windows. The resulting model will be multi-objective in which the total transportation cost has to be minimized and customer satisfaction maximized.

Furthermore, we assumed that the service of a customer takes place at the same location independently of the chosen time window. However, in some applications, the customer would like to deviate the location per time window, for example that a package can be delivered at work during the day and at home in the evening. This leads to a different problem setting and solution approach, but it highly interesting in the evolving e-commerce market.

We developed an exact and a heuristic solution method for handling multiple time windows, combining these method in a hybrid solution approach could be a promising way to improve the performance of the algorithms.

Following interviews on route security with safety managers at the largest CIT company in the Netherlands, Chapter 4 focused on making route unpredictable by diversifying the arrival times at the customers. Instead of varying the time of arrival at a customer, on could also vary the day of delivery. Nowadays, the replenishments of bank offices are often scheduled at the same time at the same day of the week. By combining constraints on arrival time and replenishment day, the CIT is more flexible and unpredictable at the same time. For example, fluctuating the day of replenishment or keeping the day the same but changing the delivery time, can be a rule to make the routes unpredictable. Another option is to diversify the area through which the CIT is driving. Some areas can be more susceptible for raids and should be avoided as much as possible. The social economic status of the neighborhoods through which the vehicle travels is taken into account in the work of Bozkaya et al. [2017]. Another approach is to diversify the arcs that are used in a route, as proposed by Talarico et al. [2015a].

It would be interesting to investigate the difference in route unpredictability level and transportation cost of these different route diversification mechanisms. This can be done by theoretical research, but also by experimental tests with practitioners in cash transport.

In Chapter 5 is assumed that the mean, minimum and maximum value of an uncertain parameter are known. If there would be more data available, then it could be worthwhile to investigate which other characteristics can be achieved from the data to achieve better decision making. For example taking the standard deviation or a 90% confidence interval of the travel time into account.

In practice there is uncertainty in travel time but there are also daily patterns, such as the morning and evening rush hours. Taking these into account would lead to different travel time distributions for the morning, afternoon and evening time zones. Combining the time dependent routing problem and the stochastic or robust routing problem would be a challenging but very useful extension to practitioners.